M-Projectine Curvature Tensor of Locally Conformal Kahler Manifold

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Abstract

In this paper we stady the relationship between tensor is an algebraic curvature tensor , M-Projective Curvature Tensor of a Lokally Conformal Kahler manifold w_4., i . e. "it has a classical symmetry properties of the Riemann carvatur tensor" of a Lokally Conformal Kahler manifold w_4.has been examined in this research.

The typical Riemannian curvature symmetry features of this tensor were demonstrated. In the L.C.K-manifold, calculate the M- Projective tensor (M- tensor) components. Some observations and relationships among them were obtained, and links between the tensor components of this manifold were constructed. With obtaining a neutral equation for each of the eight these components.

Keywords: Locally conformal Kahler manifold w_4., M- projective tensor, conformal curvature tensor.

1.Introduction

Conformal transformations of Riemannian structures are the important object of differential geometry, where this "transformations which keeping the property of smooth harmonic function .It is" known, that such transformations have tensor in variant so-called M- Projective Curvature Tensor, In this paper we investigated the M- Projective Curvature Tensor of a Lokally Conformal Kahler manifold w4."

The M-projective curvature tensor :[8]on AH-manifold M is a tensor of type (4,0) and satisfied the relation $e^{-2f}M(A,B,C,D) = M(A,B,C,D)$, which is defined as the form:

$$M(A,B,C,D) = R(A,B,C,D) - \frac{1}{2(n-1)}[S(B,C)g(A,D) - S(A,C)g(B,D) + g(B,C)S(A,D) - g(A,C)S(B,D)]$$

Where R is Riemannian curvature tensor, g is the Riemannian metric and A is the scalar curvature $A,B,C,D \in X(M)$, where X(M) is the Lie algebra of M^{∞} vector field of M.

The locally conformal Kahler manifold which are going to be dealt with in this study, is one of the sixteen classes of almost Hermitian manifold. The first study on locally conformal Kahler manifold was conducted by Libermann 1955 [12]. Vaisman, in 1981 put down some geometrical conditions for locally conformal Kahler manifold [19]. Letter on in1982, Tricerri mentioned different examples about the locally conformal Kahler manifold[18].

In 1993, Banaru [1] From the Banaru's classification of L.C.K-manifold. The class locally conformal kahler manifold statistics the following conditions:

$B^{abc} = 0$,$B^{ab}_{c} = \alpha^{[a}\delta^{b]}_{c}$

2.Preliminaries

Let M be a smooth 2n dimension manifold,$C^{\infty}(M)$- soft function algebra onM; α(M)vector fields of smoothness module on "manifold" of M; g = <.,.> - Riemannian metrics is a Riemannian metrics link g onM, d: the element of distinction from the outside. The smooth class is assumed for all manifolds, Tensor fields, and other objecets C in the following. The structure of NK ("nearlykahler") on the "(manifold M)" is a pair "(Q,g)" where Q: represents the structure of the almost complicated ("$Q^2 = id$") onM, g=<.,.> represents the Riemannian "(pseudo)" metric onM, where in this case <Qα,Qβ> =<α, β> ; α, $\beta \in \alpha$(M).

Let M be a n1-dimensional 2n-dimensional smooth manifold. On M, C(M) is a smooth function algebra., and α(M) is the vector field module on M. The Riemannian conection of the metric is denoted by g, while the exterior differentiation is denoted by d.

3. The Structure Equation of locally conformal kahler manifold

In the beginning, deals with the construction of class focally conformal kahler manifold in the ad joint G-structure space.

Definition 3.1:[6]

Let g and $\tilde{g}$be two Riemannian metrics on smooth manifold M, we say that on M given a conformal transformation metric if there is a smooth function $f \in C^{\infty}(m)$ such that $\tilde{g} = e^{2f}g$

Let $\{M.J.g =<,>\}$ be an L.C.K-manifold. If there exists a conformal transformation of the metric g into the metric $\tilde{g}$. Then, $\{M.J.\tilde{g} = e^{2f}g\}$ will be L.C.K-manifold. In this case, we say that, on smooth manifold M given conformal transformation of L.C.K-structure, denoted by $\widetilde{M_f}$.

Definition 3.2:[3]

An L.C.K-manifold M is called a locally conformal kahler manifold, if for each point m ∈ M there exists an open neighborhood U of this point and there exists $f \in C^{\infty}(U)$ such that $\widetilde{U_f}$ is kahler manifold.

Remark 3.3:

We shall denoted to the locally conformal kahler manifold L.C.K-manifold

Definition 3.4: [2]

Let M be L.C.K-manifold, the from which is given by the following relation

$$a = \frac{1}{n-1}\ \delta F \circ J$$

Is called a Lie form, where 8 represents the coderivative if F is r-form. Then, its coderivative is (r-1). Form and its dual is a vector, which is called a Lie vector.

Remark 3.5: [1]

1- From the Banaru's classification of L.C.K-manifold. The class locally conformal kahler manifold statistics the following conditions:

$$B^{abc} = 0\ , B_c^{ab} = \alpha^{[a}\delta_c^{b]}$$

2- The value of Riemannian metric is g denoted by the form

i- $g_{ab} = g_{\hat{a}\hat{b}} = 0$

ii- $g_{\hat{a}b} = \delta_b^a$

iii- $g_{a\hat{b}} = \delta_a^b$

The structure equation of L.C.K-manifold provide by the following theorem.

Theorem 3.6: [7]

The collection of the structure equation of L.C.K-manifold in the ad joint G-structure space has the following:

i- $dw^a = w_b^a \wedge w^b + \beta_c^{ab} w^c \wedge w_b$

ii- $dw_a = -w_a^b \wedge w_b + \beta_{ab}^c w_c \wedge w^b$

iii- $dw_b^a = w_c^a \wedge w_b^c + A_{bc}^{ad} w^c \wedge w_d + \left\{\frac{1}{2}\alpha^{a[c}\delta_b^d + \frac{1}{4}\alpha^a \alpha^{[c}\delta_b^{d]}\right\} w_c \wedge w_d$

Where $\{w^i\}$ are the components of mixture form, $w_j^i$are the components of Riemannian connection of metric g and $\{A_{bc}^{ad}\}$ are system function in the ad joint G-structure space.

Theorem 3.7: [7]

The component of the Riemannian curvature tensor of L.C.K-manifold in the ad joint G-structure space are given as the following forms:

i- $R_{abcd} = 0$

ii- $R_{\hat{a}\hat{b}\hat{c}\hat{d}} = 0$

iii-$R_{\hat{a}bcd} = \alpha_{a[c}\delta_{d]}^b + \frac{1}{2}\alpha_a \alpha_{[c}\delta_{d]}^b$

iv-$R_{a\hat{b}cd} = -\alpha_{a[c}\delta_{d]}^b - \frac{1}{2}\alpha_a \alpha_{[c}\delta_{d]}^b$

v- $R_{ab\hat{c}d} = \alpha_{[a|d|}\delta_{b]}^c - \alpha_{[a}\delta_{b]}^h \alpha_{[h}\delta_{d]}^c$

vi- $R_{abc\hat{d}} = \alpha_{[a|c|}\delta_{b]}^{d} - \alpha_{[a}\delta_{b]}^{h}\alpha_{[h}\delta_{c]}^{d}$

vii- $R_{\hat{a}\hat{b}cd} = -2\,\alpha_{[c}^{[a}\delta_{d]}^{b]}$

viii- $R_{ab\hat{c}\hat{d}} = 2\,\alpha_{[a}^{[c}\delta_{b]}^{d]}$

ix- $R_{\hat{a}b\hat{c}d} = A_{bd}^{ac} - \alpha^{[a}\delta_{d}^{h]}\alpha_{[b}\delta_{h]}^{c}$

x- $R_{\hat{a}bc\hat{d}} = A_{bc}^{ad} - \alpha^{[a}\delta_{c}^{h]}\alpha_{[h}\delta_{b]}^{d}$

xi- $R_{a\hat{b}\hat{c}d} = -A_{ad}^{bc} + \alpha^{[h}\delta_{d}^{b]}\alpha_{[a}\delta_{h]}^{c}$

xii- $R_{a\hat{b}c\hat{d}} = -A_{ac}^{bd} + \alpha^{[b}\delta_{c}^{h]}\alpha_{[a}\delta_{h]}^{d}$

xiii- $R_{\hat{a}\hat{b}\hat{c}d} = -\alpha^{[a|c|}\delta_{a}^{b]} + \alpha^{[a}\delta_{h}^{b]}\alpha^{[h}\delta_{d}^{c]}$

xiv- $R_{\hat{a}\hat{b}c\hat{d}} = -\alpha^{[a|d|}\delta_{c}^{b]} + \alpha^{[a}\delta_{h}^{b]}\alpha^{[h}\delta_{c}^{d]}$

xv- $R_{\hat{a}b\hat{c}\hat{d}} = \alpha^{a[c}\delta_{b}^{d]} + \frac{1}{2}\alpha^{a}\alpha^{[c}\delta_{b}^{d]}$

xvi- $R_{a\hat{b}\hat{c}\hat{d}} = -\alpha^{a[c}\delta_{b}^{d]} - \frac{1}{2}\alpha^{a}\alpha^{[c}\delta_{b}^{d]}$

Definition3.8:[6]

A tensor of type (2,0) which is defined as $r_{ij} = R_{ijk}^{k} = g^{k1}R_{kij1}$, is called a Ricci tensor.

Theorem 3.9:

The component of Ricci tensor of L.C.K-manifold in the ad joint G-structure space are given by the following forms:

i- $r_{ab} = \alpha c_{[b}\delta_{c]}^{a} + \frac{1}{2}\alpha c\alpha_{[b}\delta_{c]}^{a} + \alpha_{[c|b|}\delta_{a]}^{c} - \alpha_{[c}\delta_{a]}^{h}\alpha_{[h}\delta_{c]}^{b}$

ii- $r_{\hat{a}b} = -2\,\alpha_{[b}^{[c}\delta_{c]}^{a]} - A_{cb}^{ac} + \alpha^{[a}\delta_{b}^{h]}\alpha_{[c}\delta_{h]}^{c}$

iii- $r_{a\hat{b}} = A_{ac}^{cb} - \alpha^{[c}\delta_{c}^{h]}\alpha_{[a}\delta_{h]}^{c} + 2\,\alpha_{[c}^{[b}\delta_{a]}^{c]}$

iv- $r_{\hat{a}\hat{b}} = -\alpha^{[c|b|}\delta_{c}^{a]} + \alpha^{[c}\delta_{h}^{a]}\alpha^{[h}\delta_{c}^{b]} - \alpha^{c[b}\delta_{a}^{c]} - \frac{1}{2}\alpha^{c}\alpha^{[b}\delta_{a}^{c]}$

4. M-Projective Curvature Tensor of Locally Kahler

In this section, we study the class of locally kahler manifold and M-projective tensor of locally kahler manifold.

Definition 4.1:[9]

Let g and $\tilde{g}$be two Riemannian metrics on smooth manifold M, we say that on M given a conformal transformation metric if there is a smooth function $f \in M^{\infty}(M)$ such that $\tilde{g} = e^{2f}g$.

Definition 4.2:[8]

The M-projective curvature tensor on NK-manifold M is a tensor of type (4,0) and satisfied the relation $e^{-2f}M(A,B,C,D) = M(A,B,C,D)$, which is defined as the form:

$$M(A,B,C,D) = R(A,B,C,D) - \frac{1}{2(n-1)}[S(B,C)g(A,D) - S(A,C)g(B,D) + g(B,C)S(A,D) - g(A,C)S(B,D)]$$

Where R is Riemannian curvature tensor, g is the Riemannian metric and A is the scalar curvature $A, B, C, D \in X(M)$, where X(M) is the Lie algebra of M^{∞} vector field of M.

Definition 4.3:

The M-projective curvature tensor on L.C.K-manifold M is a tensor of type (4,0) and satisfied the relation $e^{-2f} M(A, B, C, D) = M(A, B, C, D)$, which is defined as the form:

$$M(A,B,C,D) = R(A,B,C,D) - \frac{1}{2(n-1)}[S(B,C)g(A,D) - S(A,C)g(B,D) + g(B,C)S(A,D) - g(A,C)S(B,D)]$$

Where R is Riemannian curvature tensor, g is the Riemannian metric and A is the scalar curvature $A, B, C, D \in X(M)$, where X(M) is the Lie algebra of M^{∞} vector field of M.

Definition 4.4:[8]

The form defines the M-projective curvature tensor

$$M_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}[S_{jk}g_{il} - S_{ik}g_{jl} + g_{jk}S_{il} - g_{ik}S_{jl}]$$

The Riemannian curvature tensor and the Ricci tensor respectively are R and S.

Definition 4.5:[5]

The Riemannian manifold is called an Einstein manifold, if the components of Ricci tensor satisfies the equation $r_{ij} = eg_{ij}$, where e and g are respective an Einstein constant and Riemannian metric.

Let's consider properties of the M-projective curvature tensor.

Remark 4.6:

Thus, the projective tensor satisfies all of the algebraic curvature tensor's characteristics:

i- $M(a,b,c,d) = -M(b,a,c,d)$
ii- $M(a,b,c,d) = -(a,b,d,c)$
iii- $M(a,b,c,d) + M(b,c,a,d) + M(c,a,,b,d) = 0$
iv- $M(a,b,c,d) = M(c,d,a,b).\, a,b,c,d \in X(M)$

Proof:

We shall prove(i)

i- $M(a,b,c,d) = R(a,b,c,d) - \frac{1}{2(n-1)}[S(b,c)g(a,d) - S(a,c)g(b,d) + g(b,c)S(a,d) - g(a,c)S(b,d)]$

$$= -R(b,a,c,d) + \frac{1}{2(n-1)}[-S(b,c)g(a,d) + S(a,c)g(b,d) - g(b,c)S(a,d) + g(a,c)S(b,d)$$

$$= -M(b,a,c,d)$$

Properties are similarly proved.

ii- $M(a,b,c,d) = -(a,b,d,c)$

iii- $M(a,b,c,d) + M(b,c,a,d) + M(c,a,,b,d) = 0$

iv- $M(a,b,c,d) = M(c,d,a,b)$.

Covariant projective tensor M type (3,1) have form

$$M(a,b)c = R(a,b)c + \frac{1}{2N-1}\{< a,c > b - < b,c > a\}$$

Where R is the Riemannian curvature tensor and a is the scalar curvature a, b, c ∈ A(M)

By definition of a spectrum tensor

$$M(a,b)c = M_0(a,b)c + M_1(a,b)c + M_2(a,b)c + M_3(a,b)c + M_4(a,b)c + M_5(a,b)c + M_6(a,b)c + M_7(a,b)c, \quad a,b,c \in A(M).$$

Tensor $M_0(a,b)c$ as non-zero the component can have only components of the form

$$\{M^{a}_{0bcd}, M^{\hat{a}}_{0\hat{b}\hat{c}\hat{d}}\} = \{M^{a}_{bcd}, M^{\hat{a}}_{\hat{b}\hat{c}\hat{d}}\}$$

Tensor $M_1(a,b)c$-components of the form

$$\{M^{a}_{1bc\hat{d}}, M^{\hat{a}}_{1\hat{b}\hat{c}d}\} = \{M^{a}_{bc\hat{d}}, M^{\hat{a}}_{\hat{b}\hat{c}d}\}$$

Tensor $M_2(a,b)c$-components of the form

$$\{M^{a}_{2b\hat{c}d}, M^{\hat{a}}_{2\hat{b}c\hat{d}}\} = \{M^{a}_{b\hat{c}d}, M^{\hat{a}}_{\hat{b}c\hat{d}}\}$$

Tensor $M_3(a,b)c$-components of the form

$$\{M^{a}_{3b\hat{c}\hat{d}}, M^{\hat{a}}_{3\hat{b}cd}\} = \{M^{a}_{b\hat{c}\hat{d}}, M^{\hat{a}}_{\hat{b}cd}\}$$

Tensor $M_4(a,b)c$-components of the form

$$\{M^{a}_{4\hat{b}cd}, M^{\hat{a}}_{4b\hat{c}\hat{d}}\} = \{M^{a}_{\hat{b}cd}, M^{\hat{a}}_{b\hat{c}\hat{d}}\}$$

Tensor $M_5(a,b)c$-components of the form

$$\{M^{a}_{5\hat{b}c\hat{d}}, M^{\hat{a}}_{5b\hat{c}d}\} = \{M^{a}_{\hat{b}c\hat{d}}, M^{\hat{a}}_{b\hat{c}d}\}$$

Tensor $M_6(a,b)c$-components of the form

$$\{M^{a}_{6\hat{b}\hat{c}d}, M^{\hat{a}}_{6bc\hat{d}}\} = \{M^{a}_{\hat{b}\hat{c}d}, M^{\hat{a}}_{bc\hat{d}}\}$$

Tensor $M_7(a,b)c$-components of the form

$$\{M^{a}_{7\hat{b}\hat{c}\hat{d}}, M^{\hat{a}}_{7bcd}\} = \{M^{a}_{\hat{b}\hat{c}\hat{d}}, M^{\hat{a}}_{bcd}\}$$

Tensors $M_0 = M_0(a,b)c, M_1 = M_1(a,b)c, \ldots, M_7 = M_7(a,b)c$

The basic invariants projective AH-manifold will be named.

Definition 4.7:

Locally kahler manifold for which $M_i = 0$ is locally kahler manifold of class M_i , i=0,1, …, 7.

Theorem 4.8:

i- Locally kahler manifold of class M_0 characterized by identity $M(a,b)c - M(a,Jb)Jc - M(Ja,b)Jc - M(Ja,Jb)c - JM(a,b)Jc - JM(a,Jb)c - JM(Ja,b)c + JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

ii- Locally kahler manifold of class M_1 characterized by identity $M(a,b)c + M(a,Jb)Jc - M(Ja,b)Jc + M(Ja,Jb)c + JM(a,b)Jc - JM(a,Jb)c - JM(Ja,b)c - JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

iii- Locally kahler manifold of class M_2 characterized by identity $M(a,b)c - M(a,Jb)Jc + M(Ja,b)Jc + M(Ja,Jb)c - JM(a,b)Jc - JM(a,Jb)c + JM(Ja,b)c - JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

iv- Locally kahler manifold of class M_3 characterized by identity $M(a,b)c + M(a,Jb)Jc + M(Ja,b)Jc - M(Ja,Jb)c - JM(a,b)Jc + JM(a,Jb)c + JM(Ja,b)c + JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

v- Locally kahler manifold of class M_4 characterized by identity $M(a,b)c + M(a,Jb)Jc + M(Ja,b)Jc - M(Ja,Jb)c + JM(a,b)Jc - JM(a,Jb)c - JM(Ja,b)c - JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

vi- Locally kahler manifold of class M_5 characterized by identity $M(a,b)c - M(a,Jb)Jc + M(Ja,b)Jc + M(Ja,Jb)c + JM(a,b)Jc + JM(a,Jb)c - JM(Ja,b)c + JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

vii- Locally kahler manifold of class M_6 characterized by identity $M(a,b)c + M(a,Jb)Jc - M(Ja,b)Jc + M(Ja,Jb)c + JM(a,b)Jc - JM(a,Jb)c + JM(Ja,b)c + JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M)$

viii- Locally kahler manifold of class M_7 characterized by identity $M(a,b)c - M(a,Jb)Jc - M(Ja,b)Jc - M(Ja,Jb)c + JM(a,b)Jc + JM(a,Jb)c + JM(Ja,b)c - JM(Ja,Jb)Jc = 0, \quad a,b,c \in X(M$

Theorem 4.9:

We have the following inclusion relations

i- $M_0 = M_3 = M_4 = M_5 = M_6 = M_7$,

ii- $M_1 = -M_2$.

Theorem 4.10:

The following equation describes the components of the projective tensor of L.C.K-manifold in the ad joint G-structure:

i- $M_{\hat{a}b\hat{c}d} = \beta^{adc}\beta_{bdh} - A^{ac}_{bd} - \frac{1}{2n-1}[(3\beta_{dbh}\beta^{dch} - A^{dc}_{bd})\delta^a_d + (3\beta_{bah}\beta^{bdh} - A^{ad}_{ab})\delta^b_d]$

ii- $M_{\hat{a}bc\hat{d}} = \beta^{adh}\beta_{hbc} + A^{ad}_{bc} - \frac{1}{2n-1}[(-3\beta^{bah}\beta_{hbc} + A^{ab}_{bc})\delta^d_b - (3\beta_{abh}\beta^{adh} - A^{ad}_{bd})\delta^a_c]$

And the others are either conjugate of the above components or equal to zero.

Proof:

By using theorem 3.7, we compute the components of projective tensor as the following:

1- Put i=a , j = b , k = c , I = d

$$M_{abcd} = R_{abcd} - \frac{1}{2(n-1)}[S_{bc}g_{ad} - S_{ac}g_{bd} + g_{bc}S_{ad} - g_{ac}S_{bd}]$$

$$M_{abcd} = 0$$

2- Put $i = \hat{a}, j = b, k = c, I = d$

$$M_{\hat{a}bcd} = R_{\hat{a}bcd} - \frac{1}{2(n-1)}[S_{bc}g_{\hat{a}d} - S_{\hat{a}c}g_{bd} + g_{bc}S_{\hat{a}d} - g_{\hat{a}c}S_{bd}]$$

$$M_{\hat{a}bcd} = \alpha_{a[c}\delta^b_{d]} + \frac{1}{2}\alpha_a\alpha_{[c}\delta^b_{d]}$$

3- Put $i = a, j = \hat{b}, k = c, I = d$

$$M_{a\hat{b}cd} = R_{a\hat{b}cd} - \frac{1}{2(n-1)}[S_{\hat{b}c}g_{ad} - S_{ac}g_{\hat{b}d} + g_{\hat{b}c}S_{ad} - g_{ac}S_{\hat{b}d}]$$

$$M_{a\hat{b}cd} = -\alpha_{a[c}\delta^b_{d]} - \frac{1}{2}\alpha_a\alpha_{[c}\delta^b_{d]}$$

4- $i = a, j = b, k = \hat{c}, I = d$

$$M_{ab\hat{c}d} = R_{ab\hat{c}d} - \frac{1}{2(n-1)}[S_{b\hat{c}}g_{ad} - S_{a\hat{c}}g_{bd} + g_{b\hat{c}}S_{ad} - g_{a\hat{c}}S_{bd}]$$

$$M_{ab\hat{c}d} = \alpha_{[a|d|}\delta^c_{b]} - \alpha_{[a}\delta^h_{b]}\alpha_{[h}\delta^c_{d]}$$

5- $i = a, j = b, k = c, I = \hat{d}$

$$M_{abc\hat{d}} = R_{abc\hat{d}} - \frac{1}{2(n-1)}[S_{bc}g_{a\hat{d}} - S_{ac}g_{b\hat{d}} + g_{bc}S_{a\hat{d}} - g_{ac}S_{b\hat{d}}]$$

$$M_{abc\hat{d}} = \alpha_{[a|c|}\delta^d_{b]} - \alpha_{[a}\delta^h_{b]}\alpha_{[h}\delta^d_{c]}$$

6- Put $i = \hat{a}, j = \hat{b}, k = c, I = d$

$$M_{\hat{a}\hat{b}cd} = R_{\hat{a}\hat{b}cd} - \frac{1}{2(n-1)}[S_{\hat{b}c}g_{\hat{a}d} - S_{\hat{a}c}g_{\hat{b}d} + g_{\hat{b}c}S_{\hat{a}d} - g_{\hat{a}c}S_{\hat{b}d}]$$

$$M_{\hat{a}\hat{b}cd} = -2\alpha^{[a}_{[c}\delta^{b]}_{d]} - \frac{1}{2(n-1)}[\left(-2\alpha\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}\right)\delta^{a}_{b} - \left(-2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}\right)\delta^{a}_{b} + \left(-2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}\right)\delta^{a}_{b} - \left(-2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}\right)\delta^{a}_{b}$$

$$M_{\hat{a}\hat{b}cd} = -2\alpha^{[a}_{[c}\delta^{b]}_{d]}$$

7- Put $i = \hat{a}, j = b, k = \hat{c}, I = d$

$$M_{\hat{a}b\hat{c}d} = R_{\hat{a}b\hat{c}d} - \frac{1}{2(n-1)}[S_{b\hat{c}}g_{\hat{a}d} - S_{\hat{a}\hat{c}}g_{bd} + g_{b\hat{c}}S_{\hat{a}d} - g_{\hat{a}\hat{c}}S_{bd}]$$

$$M_{\hat{a}b\hat{c}d} = A^{ac}_{bd} - \alpha^{[a}\delta^{h]}_{d}\alpha_{[b}\delta^{c}_{h]} - \frac{1}{2(n-1)}\left(A^{cb}_{ac} - \alpha^{[c}\delta^{h]}_{c}\alpha_{[a}\delta^{c}_{h]} + 2\alpha^{[b}_{[c}\delta^{c]}_{a]}\right)\delta^{a}_{b} + (A^{cb}_{ac} - \alpha^{[c}\delta^{h]}_{c}\alpha_{[a}\delta^{c}_{h]} + 2\alpha^{[b}_{[c}\delta^{c]}_{a]})\delta^{a}_{b}$$

$$M_{\hat{a}b\hat{c}d} = A^{ac}_{bd} - \alpha^{[a}\delta^{h]}_{d}\alpha_{[b}\delta^{c}_{h]} - \frac{1}{(n-1)}(A^{cb}_{ac} - \alpha^{[c}\delta^{h]}_{c}\alpha_{[a}\delta^{c}_{h]} + 2\alpha^{[b}_{[c}\delta^{c]}_{a]}\delta^{a}_{b})$$

8- Put $i = \hat{a}, j = b, k = c, I = \hat{d}$

$$M_{\hat{a}bc\hat{d}} = R_{\hat{a}bc\hat{d}} - \frac{1}{2(n-1)}[S_{bc}g_{\hat{a}\hat{d}} - S_{\hat{a}c}g_{b\hat{d}} + g_{bc}S_{\hat{a}\hat{d}} - g_{\hat{a}c}S_{b\hat{d}}]$$

$$M_{\hat{a}bc\hat{d}} = A^{ad}_{bc} - \alpha^{[a}\delta^{h]}_{c}\alpha_{[h}\delta^{d}_{b]} - \frac{1}{2(n-1)} + \left(2\alpha^{[c}_{[b}\delta^{a]}_{c]} - A^{ac}_{cb} + \alpha^{[a}\delta^{h]}_{b}\alpha_{[c}\delta^{c}_{h]}\right)\delta^{b}_{a} - (A^{cb}_{ac} - \alpha^{[c}\delta^{h]}_{c}\alpha_{[a}\delta^{c}_{h]} + 2\alpha^{[b}_{[c}\delta^{c]}_{a]})\delta^{a}_{b}$$

The above theorem calculated components projective tensor curvature on space of the ad joint G-structure projective tensor of L.C.K-manifolds and M_1 and M_2 have only other components projective tensor are equal to zero.

For L.C.K-manifold only two projective tensor don't equal zero. M_1 with component $\{M^{a}_{bc\hat{d}}, M^{a}_{\hat{b}\hat{c}d}\}$ and W_2 with component $\{M^{a}_{\hat{b}\hat{c}d}, M^{\hat{a}}_{\hat{b}c\hat{d}}\}$.

Definition 4.2.11:[9][4]

According to properties well-know that M ⊂ R_1⊂ R_2⊂ R_3. Consider that, $(M.J.g)$ locally kahler manifold of dimension 2n, M-projective tensor.

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