Stability and chaos diagnosis of 3-D Rabinovich-Fabrikant (R-F) system with adaptive control and synchronization

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Abstract

In this paper, the Rabinovich-Fabrikant (R-F), three dimension continuous-time dynamical system is dealt. The equilibrium points, stability analysis, chaos detection, and Adaptive control technique were used to analyze the (R-F) system (1). Findings revealed that the (R-F) system unstable hyperbolic equilibrium points. The Lyapunov exponent $L_1 = 0.289887, L_2 = 0.075847, L_3 = -0.411733$. which indicate that the (R-F) system highly chaotic. By means of adaptive control and synchronization, it was found that(R-F) system (1) is stable, and the adaptive synchronization achieved good results. Finally, the electronic circuit is design for implementation of chaotic (R-F) system by Multisim (14.2)

Keywords: Stability, multistability, Dissipativity, Lyapunov exponent**,** adaptive Control and Synchronization, electronic circuit

1-INTRODUCTION

Since the expanding boundaries of chaotic application in the fields of science and engineering research on chaotic phenomena has drastically grown during the past few years. The system is defined as chaotic if three condition are met, first: the orbit is bounded in a finite region of space, second: the orbit is aperiodic, third: the orbit is sensitive to initial condition [2]. The Lapiynuov exponente is one of the basic methods for determining the chaoticness of a system, if one of the values is positive, the system is chaotic ([3],[4],[5]). Scientists have introduced many chaotic dynamical systems. Edward Lorenz is one of the earliest and most influential pioneers of chaos theory, in 1963 Lorenz presented chaotic system of three ordinary differential equation ([6],[7],[8]). In1977 Rössler, was ablate extract simple asymmetric attracting structures from the Lorenz attractor ([6],[9],[10]), Chua system ([6],[11]) Chen system ([12],[13]). In 1979 the Rabinovich-Fabrikant chaotic system was developed by Mikhail Rabinovich and Anatoly Fabrikant, ([14],[15],[16]). Describing modulation instability in a non-equilibrium dissipative medium, this finite-dimensional mode can describe various physical system such as wind waves on water, Tollmien-schlichting wave in hydrodynamic flows, Langmuir waves in plasma, and waves in chemical media with diffusion [8]. There are multiple ways to control a chaotic system or convert it to a stable system like feedback control, adaptive control, active control, and sliding mode control ([17],[18],[19]). One way to control chaos is to synchronize tow osystans, the drive system and response system [20], Synchronization between two identical or non- identical systems has attracted a great deal of interest in various fields and engineering [21].[22],[23]. This paper, consist of six sections: section (1) present (R-F) system with ten simple terms include five nonlinear terms. Section (2) system analysis, via equilibrium points stability, Dissipativity, Graphical and numerical analysis Multi stability, Lyapunove dimension. Section (3) Adaptive control technique. Section (4) Adaptive synchronization strategy. Section (5) Electronic Circuit, and section (6) the conclusion.

(2) System Description

A three _ dimensional dynamical (R-F) system (1)[14] , consist of three ordinary differential equation and tow _ parameters :

$$\dot{x} = y(z - 1 + x^2) + ax$$
$$\dot{y} = x(3z + 1 - x^2) + ay \qquad (1)$$
$$\dot{z} = -2z(b + xy)$$

(x, y, z) are state variables and (a ,b) are parameters Where

$a = 0.077$, $b = 0.1$

(3) System Analysis

To analyze the dynamical system, We determine the (R-F) system's equilibrium points. ,this equates the equation to zero. So the equilibrium points are

$E_0(0,0,0)$, $E_{1,2}(\pm 1.5378, \mp 0.065, -0.26909)$, $E_{3,4}(\pm 0.0438, \mp 2.2831, 1.7635)$

(3-1) Analysis of stability **(3-1-1)**
formula for characteristic roots

The (R-F) system (1) is stable, provided that the eigenvalues own negative real numbers. The Jacobi matrix of(R-F) system at $E_0(0,0,0)$ is:

$$= \begin{bmatrix} 0.077 & -1 & 0 \\ 1 & 0.077 & 0 \\ 0 & 0 & -0.2 \end{bmatrix}$$

It's characteristic equation

$$\lambda^3 + 0.046\lambda^2 + 0.975129\lambda + 0.2011858 = 0 \qquad (2)$$

The roots of equation (2) are :

$\lambda_1 = -0.2$, $\lambda_2 = 0.077 + i$, $\lambda_3 = 0.077 - i$

Proposition 1: by root of characteristic equation, since λ_2, λ_3 are complex and reel part are positive numbers, Therefor the (R-F) system (1) is not stable

(3-1-2) Criteria of Routh stability

Routh criteria states that(R-F) system (1) is steady, that each term in Routh Array table's first column are positive values, is a necessary and sufficient condition.

$a_0 = 0.2011858$, $a_1 = 0.975129$, $a_2 = 0.046$, $a_3 = 1$, $b_1 = \frac{a_2 a_1 - a_3 a_0}{a_2} = -3.3984753$

Table (1) Routh arrays for the(R-F) system (1)

λ^3	1	0.975129
λ^2	0.046	0.2011858
λ^1	-3.3984753	0
λ^0	0.2011858	0

Proposition 2. Since there is negative element in Table (1)'s first column. Therefor the Routh stability condition is not met. The (R-F) system (1) is hence unstable.

(3-1-3) Hurwitz stability criteria

Using coefficients of the characteristic equation, we form square matrix. If the Hurwitz determinants are all positive the system is stable.

$$\begin{bmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{bmatrix} = \begin{bmatrix} 0.046 & 0.2011858 & 0 \\ 1 & 0.975129 & 0 \\ 0 & 0.046 & 0.2011858 \end{bmatrix}$$

$\Delta_1 = a_{n-1} = 0.046 > 0$,

$$\Delta_2 = \begin{vmatrix} a_{n-1} & a_{n-3} \\ a_n & a_{n-2} \end{vmatrix} = -0.1563298 < 0 \quad , \Delta_3 = \begin{vmatrix} a_{n-1} & a_{n-3} & a_{n-5} \\ a_n & a_{n-2} & a_{n-4} \\ 0 & a_{n-1} & a_{n-3} \end{vmatrix} = -0.0314513 < 0$$

Proposition 3. By Hurwitz stability, Since Δ_2 ,Δ_3 are less than zero, so the system (1) is unstable.

(3-1-4) Lyapunov function

Let the Lyapunov function of R-F system (1) is

$V = (x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$

$\dot{V}(x, y, z) = x\dot{x} + y\dot{y} + z\dot{z}$ (3)

By substituting the R-F system (1) in (3) we give

$\dot{V}(x, y, z) = ax^2 + ay^2 - 2bz^2 + 4xyz - 2xyz^2$ (4)

Proposition 4. By substituting the initial value in equation (4) we get

$\dot{V}(x, y, z) = 2404.1629 > 0$, Then (R-F) system (1) is not stable

Table (2) stability analysis

Equilibrium points	Characteristic equation roots	Routh stability	Hurwitz stability	Stability
$E_0(0,0,0)$	$\lambda_1 = -2.2$ $\lambda_2 = 0.07 + i$ $\lambda_3 = 0.07 - i$	$a_0 = 0.2011$ $a_1 = 0.9651$ $a_2 = 0.046$ $a_3 = 1$ $b_1 = -3.398$	$\Delta_1 = 0.046$ $\Delta_2 = -0.1563$ $\Delta_3 = -0.0314$	Unstable
$E_{1,2} = (\pm 1.5378, \mp 0.65, -0.26909)$	$\lambda_1 = 2.8065$ $\lambda_2 = -1.42 + 2.45i$ $\lambda_3 = -1.42 - 2.45i$	$a_0 = -22.6$ $a_1 = 0.0492$ $a_2 = 0.046$ $a_3 = 1$ $b_1 = 491.45$	$\Delta_1 = 0.046$ $\Delta_2 = 22.6071$ $\Delta_3 = -511.02$	Unstable

$E_{3,4} = (\pm 0.0438, \mp 2.2831, 1.7635)$	$\lambda_1 = 0.32$ $\lambda_2 = -0.18 + 3.69i$ $\lambda_3 = -0.18 - 3.69i$	$a_0 = -4.39$ $a_1 = 13.587$ $a_2 = 0.046$ $a_3 = 1$ $b_1 = 109.13$	$\Delta_1 = 0.046$ $\Delta_2 = 5.02$ $\Delta_3 = -22.064$	Unstable

(3-2) Dissipativity

Let $f_1 = \dot{x}$, $f_2 = \dot{y}$ and $f_3 = \dot{z}$ in the system (1)

Then $(\dot{x}, \dot{y}, \dot{z})^T = (f_1, f_2, f_3)^T$

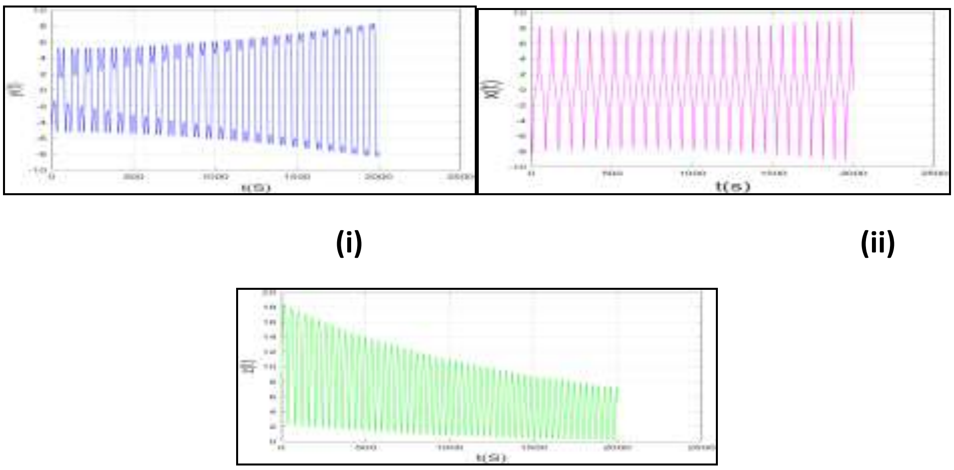
$\nabla.(\dot{x}, \dot{y}, \dot{z})^T = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = 2(a - b) = -0.046$ Not, that $f = 2(a - b) < 0$, so the (R-F) system (1) is dissipative if ($b > a$). The system (1) is conservative if (a=b)

(3-3) Graphical numerical analysis

System (1) has been solved by the Rungl-Kutta technique of fourth order with initial values $x_0 = 3.6548, y_0 = -5.1083$, $z_0 = 9.1086$

(3-1-1) Wave form analysis

Wave-form of (x, y, z) with time , for the system (1) is illustrated irregularly in figure (1), it is one of the basic ways to describe the chaotic dynamic system.

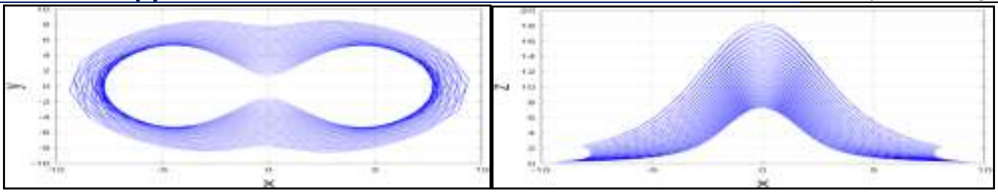


(i) (ii) (iii)

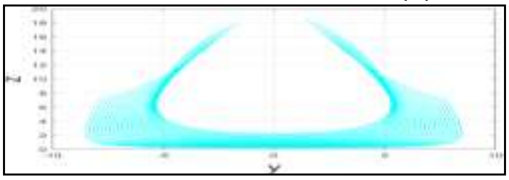
Figure (1) Wave-form of(R-F) system (1): (i) x-t, (ii) y-t(iii) z-t

(3-3-2) phase portrait of (R-F) system (1)

The strange attractors are shown in the phase picture of the (R-F) system (1) in Figure (2). in (x, y), (x, z), (y, z) plane, and figure (3) exhibits chaotic attractor in (x, y, z), (y, x, z) space.

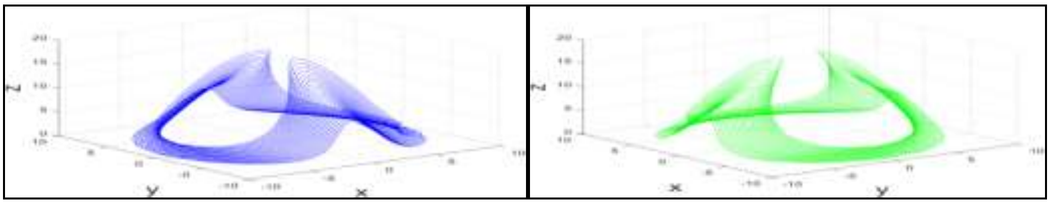


(i) (ii)



(iii) **Figure (2)**

The phase portraits (i) in x, y . (ii) in x, z . (iii) in y, z



(i) **(ii)**

Figure (3) The phase portraits of 3D Rabinovich-Fabrikant chaotic system in (i) (x, y, z)space , (ii) (y, x, z) space.

(3-4) Multistability

Multistability (coexisting attractors) of nonlinear dynamical system mean, by changing the initial condition for same parameters of the system is achieve the coexistence of many aspects according to the table (4), as in figure (4)

Table (4) coexistence with different parameters and different initial condition.

Initial conditions	Parameters	Color	Figures
$x_0 = 3, y_0 = -5, z_0 = 9$ $x_0 = -8, y_0 = 10, z_0 = 5$ $x_0 = -7, y_0 = 5, z_0 = 3$	a=0.077 b=0.1	Green Red Blue	Figure (4) (i)
$x_0 = 3, y_0 = -5, z_0 = 9$ $x_0 = -8, y_0 = 10, z_0 = 5$ $x_0 = -7, y_0 = 5, z_0 = 3$	a=-1 b=-0.287	Green Red Blue	Figure (4) (ii)

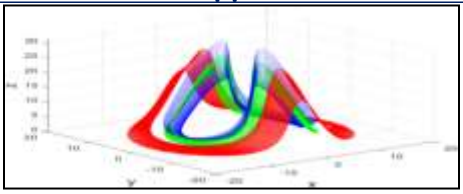


Figure (4)(i)

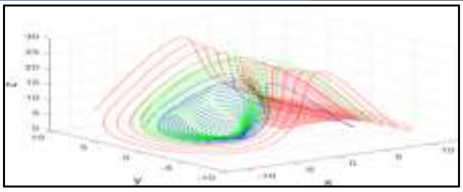


Figure (4) (ii)

Figure (4): Multistability of three attractors with three initial condition corresponding to table (4)

(3-5) Lyapunov Exponent

The Lyapunov exponents are a means of rate at which nearby trajectories in the phase space diverge from each other. They can be used to determine whether the system is chaotic or not. The values of Lyapunov exponents are ($L_1 = 0.289887$, $L_2 = 0.075847$, $L_3 = -0.411733$). And $\sum_{I=1}^{3} L_1 = -0.0459$Therefore, the Lapiynuov dimension is $D_L = 2 + \frac{L_1+L_2}{|L_3|} = 2.888279$,Hence system (1) is Hyper chaotic system since two necessary and sufficient conditions are satisfied: First, two Lyapunov exponents are positive from the three Lyapunov exponents. Second, the sum of all Lyapunov exponent is less than zero. **Figure (5)** illustrate these Lyapunove exponent

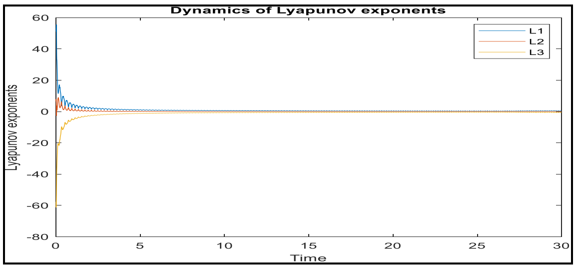


Figure (5) Lyapunov exponent (L_1, L_2, L_3) of Rabinovich-Fabrikant system (1)

(4) Adaptive control technique (4-1)
Theoretical results

To maintain the stability of chaotic (R-F) system (1) with unknown parameter b

The response system is

$$\dot{x} = yz - y + x^2 y + 0.077x + u_1(t)$$
$$\dot{y} = 3xz + x + x^3 + 0.077y + u_2(t) \qquad (5)$$
$$\dot{z} = -2bz - 2xyz + u_1(t)$$

Where (x ,y, z) are state variables , and (u_1, u_2, u_3) are the feedback controllers

The adaptive control function are

$$u_1 = -yz + y - x^2 y - 0.077x - \mu_1 x$$
$$u_2 = -3xz - x + x^3 - 0.077y - \mu_1 y$$
$$u_3 = 2\hat{b}z + 2xyz - \mu_3 z$$

Where (β_1 ,β_2 ,β_3) are positive, and($\hat{b}$ =1)is the parameter estimate of(b)

$$\dot{x} = -\beta_1 x$$
$$\dot{y} = -\beta_2 y \qquad (6)$$
$$\dot{z} = -2(b - \hat{b})z - \beta_3 z$$

Let the parameter estimation error $e_b = b - \hat{b}$ (7) Substituting (7) in (6 we given

$$\dot{x} = -\beta_1 x$$

$\dot{y} = -\beta_2 y$ (8)

$\dot{z} = -2e_b z - \beta_3 z$ The Lyapunov function of new system (8) is

$V(x,y,z) = \frac{1}{2}(x^2 + y^2 + z^2 + e_b^2)$ Differentiate (2-16) we get: $\dot{V} = x\dot{x} + y\dot{y} + z\dot{z} + e_b\dot{e}_b$

(9) And let $\dot{e}_b = -\dot{\hat{b}}$ (10)

We substituting the equations (8) and (10) in (9) we given as:

$\dot{V} = -\beta_1 x^2 - \beta_2 y^2 - \beta_3 z^2 - e_b(z^2 + \dot{\hat{b}})$ (11) Suppose that $\dot{\hat{b}} = -z^2 + \beta_4 e_b$ (12) Substitute (21) in (11) we get :

$\dot{V} = -\beta_1 x^2 - \beta_2 y^2 - \beta_3 z^2 - \beta_4 e_b^2$ Where $\beta_4 > 0$, then $\dot{V}(x,y,z) < 0$ **Proposition 5** : By Adaptive control , the eigenvalues, Routh stability criterion, Hurwitz stability, and Lapiynuov stability are stabilized So, the system (13) is stable.

(8-2) numerical results

Rung-kutta for controlled chaotic system (16) with initial values $(x_0, y_0, z_0) = [3,2,-4]$ and $[\beta_1 = 20,\ \beta_2 = 30,\ \beta_3 = 10]$. The controlled state trajectories of the (R-F) system (1) are illustrated in Figure (6).

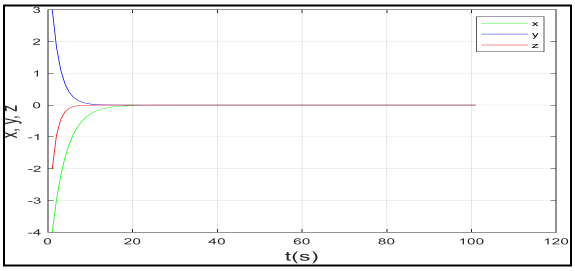


Figure (6); a path of state variables (x, y, z) for the controlled system (1)

(5) Strategy for Adaptive Synchronization. **(5.1) Theoretical results**

This section covers the synchronisation processes between two Rabinovich-Fabrikant (R–F) systems when b is unknown parameter

The first : the drive system, the chaotic (R-F) dynamics described by

$\dot{x}_1 = y_1 z_1 - y_1 + x_1^2 y_1 + 0.077 x_1$

$\dot{y}_1 = 3x_1 z_1 + x_1 - x_1^3 + 0.077 y_1$ (13)

$\dot{z}_1 = -2bz_1 - 2x_1 y_1 z_1$ Where (x_1, y_1, z_1) are state variable and the parameter (b) is unknown

The second : response system, the controlled (R-F) dynamics represented as

$\dot{x}_2 = y_2 z_2 - y_2 + x_2^2 y_2 + 0.077 x_2 + u_1(t)$

$\dot{y}_2 = 3x_2 y_2 + x_2 - x_2^3 + 0.077 y_2 + u_2(t)$ (14)

$\dot{z}_2 = -2bz_2 - 2x_2 y_2 z_2 + u_3(t)$ Where(x_2, y_2, z_2) are state variable and the Adaptive control input $((u_1(t), u_2(t), u_3(t))$ and (b)is parameter unknown .

Adaptive synchronization error gets by:

$e_1 = x_1 - x_1 \quad , e_2 = y_2 - y_1 \quad , e_3 = z_2 - z_1$

So, Adaptive synchronization error dynamics given by

$$\dot{e}_1 = e_2e_3 + z_1e_2 + y_1e_3 - e_2 + e_1^2e_2 + y_1e_1 + x_1^2e_2 + 0.077e_1 + u_1(t)$$
$$\dot{e}_2 = 3(e_1e_3 + z_1e_1 + x_1e_3) + e_1 - e_1^3 + 0.077e_2 + u_2(t) \qquad (15)$$
$$\dot{e}_3 = -2be_3 - 2(e_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3) + u_3(t)$$

We define the adaptive control function $u_1(t), u_2(t), u_3(t)$ by

$$u_1 = -e_2ez_3 - z_1e_2 - y_1e_3 + e_1 - e_1^2e_2 - y_1e_1 - x_1^2e_2 - 0.077e_1 - \beta_1e_1$$
$$u_2 = -3(e_1e_3 + z_1e_1 + x_1e_3) - e_1 - e_1^3 - 0.077y_1 - \beta_2e_2 \qquad (16)$$
$$u_3 = 2\hat{b}e_3 + 2(e_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3 + z_1e_1e_2 + y_1e_1e_3 + x_1e_2e_3) - \beta_3e_3$$

Where $(\beta_1, \beta_2, \beta_3)$ are positive real values and $\hat{b}$ is astimater of parameter unknown b

Substituting (16) into (15) we get dynamic system (R-F) of synchronization error

$$\dot{e}_1 = -\beta_1e_1$$
$$\dot{e}_2 = -\beta_2e_2 \qquad (17)$$
$$\dot{e}_3 = -2(b - \hat{b})e_3 - \beta_3e_3$$

Assume the parameter estimator's error is:

$$e_b = b - \hat{b} \qquad (18)$$

Substitute (18) into (17) then the dynamic of synchronization error :

$$\dot{e}_1 = -\beta_1e_1$$
$$\dot{e}_2 = -\beta_2e_2 \qquad (19)$$
$$\dot{e}_3 = -2e_be_3 - \beta_3e_3$$

According to the Lyapunov function the quadratic Lyapunov functions

$$V(e_1,\ e_2,\ e_3,\ e_b) = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2 + \frac{1}{2}e_3^2 + \frac{1}{2}e_b^2 \qquad (20)$$

Not that

$$\dot{e}_b = -\dot{\hat{b}} \qquad (21)$$

Deriving the equation (20)) and substitute the system(19) and equation (21) we get:

$$\dot{V} = -\beta_1e_1 - \beta_2e_2 - \beta_3e_3 - e_b(2e_3^2 + \dot{\hat{b}}) \qquad (22)$$

Hence, new parameter estimation $\dot{\hat{b}} = -2e_3^2 + \beta_4e_b$ (23)

Where $\beta_4 > 0$

Substituting the equation (23) into the equation (22) we obtain:

$$\dot{V} = -\beta_1e_1 - \beta_2e_3 - \beta_2e_3 - \beta_4e_b$$

Hence, $\dot{V}(e_1, e_2, e_3, e_b) < 0$,the Lapiynuov stability. Thus the dynamic of the synchronization error is stability

Proposition6 : by Adaptive synchronization), where $\dot{e}_b = \dot{\hat{b}}$ and $\dot{\hat{b}} = -2e_3^2 + \beta_4e_b$ and $\beta_1, \beta_2, \beta_3, \beta_4$ are positive constant, The chaotic (R-F) drive system (22) and the response system (23) is stabilized.

(5.2) Numerical results

Use the 4th-order Runge-kutta method to solve system of synchronization error (19) We take initial value $x_0 = 6$, $y_0 = 2$, $z_0 = -8$ and $\beta_i = 5$, $i = 1,2,3$ in Figure (7)

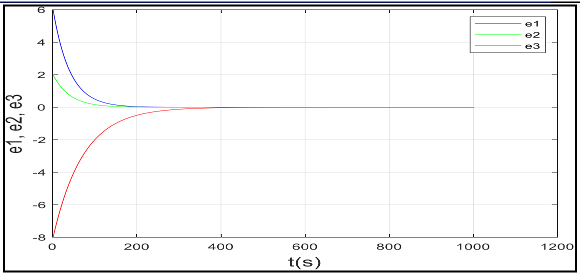


Figure (7) convergent of error dynamic (19)

(6) Electronic circuit

This section shows the electronic circuit design of The Rabinovich-Fabrikant (R-F). By applying Kirchhoff's law we form the electric equation.

$$\frac{dV_x}{dt} = \frac{1}{R_1C_1}V_yV_z - \frac{1}{R_2C_1}V_y + \frac{dV}{R_3C_1}V_x{}^2V_y + \frac{1}{R_4C_1}V_x$$
$$\frac{dV_y}{dt} = \frac{1}{R_5C_2}3V_xV_y + \frac{1}{R_6C_2}V_x - \frac{1}{R_7C_2}V_x{}^3 + \frac{1}{R_8C_2}V_y \qquad (24)$$
$$\frac{dV_z}{dt} = \frac{1}{R_9C_3}(-2V_z) - \frac{1}{R_{10}C_3}2V_xV_yV_z$$

Where V_x, V_y, V_z are the output voltages and

$K_m = 10V$,*the outputs are* $V_{yz} = V_yV_z/K_m, V_{yx^2} = V_x{}^2V_y/K_m, V_{xy} = V_xV_y/K_m, V_{xyz} = V_xV_yV_z/K_m$

Voltages and time normalized by dimensionless states variables

$$V_x = 1V.x\,, V_y = 1V.y\,, V_z = 1V, z \;\;, t' = \tau.t = 100t \qquad (25)$$

substituting (24)in (25) we get.

$$\frac{dx}{dt'} = \frac{\tau}{R_1C_1}xy - \frac{\tau}{R_2C_1}y + \frac{\tau}{R_3C_1}x^2y + \frac{\tau}{R_4C_1}x$$
$$\frac{dy}{dt'} = \frac{\tau}{R_5C_2}3xz + \frac{\tau}{R_6C_2}x - \frac{\tau}{R_7C_2}x^3 + \frac{\tau}{R_8C_2}y \qquad (26)$$
$$\frac{dz}{dt'} = -\frac{\tau}{R_9C_3}2z - \frac{\tau}{R_{10}C_3}2xyz$$

comparing system (26) with system (24) we get

$$\frac{\tau}{R_1C_1} = \frac{\tau}{R_2C_1} = \frac{\tau}{R_3C_1} = \frac{\tau}{R_5C_2} = \frac{\tau}{R_6C_2} = 1\,, \frac{\tau}{R_4C_1} = \frac{\tau}{R_8C_2} = a \;\;, \frac{\tau}{R_5C_2} = 3\,, \frac{\tau}{R_9C_3} = 2b, \frac{\tau}{R_{10}C_3} = 2 \;.$$

When $C_1 = C_2 = C_3 = 1$,

$R_1 = R_2 = R_3 = R_6 = R_7 = 100\Omega$,$R_4, R_8 = 1298.7\Omega$,$R_5 = 33.33\Omega$,$R_9 = 500\,\Omega, R_{10} = 50\Omega$.

Hence the experimental electronic circuit (26) for system (1) was simulated by electronic simulation Multi SIM (14.2)

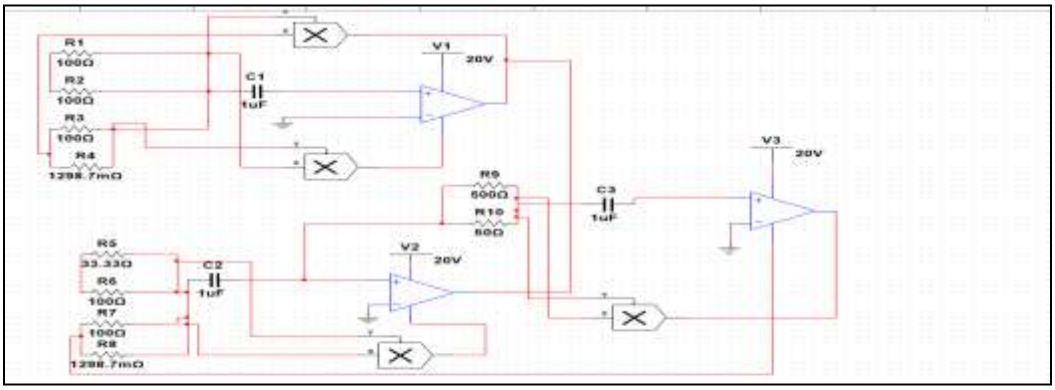


Figure (8): sketch of designed circuit of chaotic system (1).

(7) CONCLUSION

This paper presents a comprehensive analysis of the Rabinovich-Fabrikant (R-F) three-dimensional continuous-time dynamical system, It's solution through fourth order Runge-Kutta method. The study covers various aspects of the system including equilibrium points, stability analysis, chaos detection, adaptive control, and adaptive synchronization. The investigation reveals that the (R-F) system has unstable hyperbolic equilibrium points by all method such as (characteristic equation roots, Routh criteria, Hurwitz stability, and Lyapunov function). Dissipativity shows that R-F system dissipative such that (b>a) for all positive values of the parameters (b,a), and conservative if $a = b$. From the graphical analysis, multi stability and values of Lyapunov exponents ($L_1 = 0.289887$, $L_2 = 0.075847$, $L_3 = -0.411733$) and Lyapunov dimension $D_L = 2.888279$ indicates that the system is highly chaotic. However, through the application of adaptive control and synchronization techniques, the system can be stabilized, and good results can be achieved. Furthermore, the Multisim (14.2) program is used to create an electronic circuit to implement the chaotic (R-F) system. Overall, the findings of this study contribute to a better understanding of the R-F system and its potential applications in various fields.

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