# Establishment of mathematical models to estimate the concentration of Uranium in the soil with water and oil in The Qayyarah refinery

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**A B S T R A C T**

The research aims to build mathematical models to estimate the concentration of Uranium in the soil with water and oil using numerical methods such as Neville and Hermite, where we got mathematical models implicitly through the effective amount of Uranium concentration in the soil. We got on miniatures mistake percentage almost non-existent

**Keywords:** Uranium Graph theory Least Square, Mathematical Models, Neville Method, Hermit Method

## 1. Introduction

Natural radioactive elements Like Uranium and thorium are found in the soil [1]. Where Uranium is one of the essential natural radioactive elements, it is found in nature in the form of three isotopes, which are U238, whose percentage in nature is estimated at 99%, U235, which is estimated at 0.720%; and U234 at 0.005%. U238 is one of the most common elements in the Earth's crust, with a long half-life of 4.6 billion years [2]. Uranium is decomposed into radium, which is the main cause of cancer. Uranium is present in oil and gas reservoirs, where it accompanies oil and water during its extraction to the surface and is then transported to oil equipment and tanks [3]. Several research and recommendations have been published by the International Atomic Energy Agency on the possibility of the presence of radioactive elements in the oil industry, as well as methods of treatment and safety [4].

Researchers conducted A study in Turkey by collecting soil samples, crude oil, sludge and water, measuring the radiation level and determining its danger [5]. Our study aims to estimate Uranium concentration in the soil of the Qayyarah refinery with water and oil in Qayyarah refinery using some numerical methods like Neville, Hermite and Least Square methods.

### Field of Application

The study depends on the mathematical modelling process using graph theory and numerical methods like Neville to perform concentration calculations of Uranium in the soil. This search was established to estimate the mathematical modelling through accreditation on graph theory methods like Neville, Hermite and Least Square.

### Neville's Method

The main idea of this method of Neville is to approximate the value of a polynomial at a particular point without first finding all of the coefficients of the polynomial. The method of Neville can be defined as follows: Let f be a function that has values at the n points X0, X1….Xn. Let {m1, m2, … mk} be a set of k distinct integers from the set (0, 1.2, .... n}. Let

Pm1, m2.... mk (x) represent the Lagrange polynomial that agrees with the f function at k points xm1, xm2, ... xmk, …. i.e. Pm1, m2, ..., mk (xm1 ) = f(m1), Pm1. M2,... mk (xm2 ) = f(xm2 ),..., Pm1. M2... mk (xmk ) = f(xmk ). Obviously, Pm1, m2,…., mk (x) is the only degree polynomial (k- 1) that passes within the k points (xm1 . f(xm1)),...., (xmk . f(xmk)). The idea of Neville's method is to recursively use Lagrange polynomials of lower powers to calculate Lagrange polynomials of higher power relationships. This can be useful; for instance, if we have the Lagrange polynomial based on some data set points (xi, f(xk)), k= 0, 1,...,n, and we can get a new data point, (xn+1, f(xn+1)) [6].

$$C_u = \frac{(P - P_0)C_{u_1} - (P - P_1)C_{u_0}}{P_1 - P_0},$$

$$P_0 = 847 \qquad C_{u_0} = 0.585$$

$$=1970 \qquad C_{u_1} = 1.371 P_1$$

$$C_u = \frac{(P - 847)1.371 - (P - 1970)0.585}{1970 - 847}$$

$$= \frac{1.371\,P - 1161237 - 0.585\,P + 115245}{1123}$$

$$= \frac{0.786\,P - 8787}{1123}$$

(1) $$= 0.000699911\,p - 0.007824577$$

Table 1: Calculate uranium concentration in the soil using the Neville method

| No. | Location | P $Tr.cm^{-2}$ | Cu Exp. ppm | CuCal | E | E^2 |
|---|---|---|---|---|---|---|
| 1 | Qayyarah refinery | 847 | 0.585 | 0.585000 | 0.000000 | 0.0000000 |
| 2 | Qayyarah refinery | 1001 | 0.692 | 0.692786 | 0.000786 | 0.0000006 |
| 3 | Qayyarah refinery | 1294 | 0.870 | 0.897860 | 0.027860 | 0.0007762 |
| 4 | Qayyarah refinery | 1372 | 0.922 | 0.952453 | 0.030453 | 0.0009274 |
| 5 | Qayyarah refinery | 1467 | 0.986 | 1.018945 | 0.032945 | 0.0010854 |
| 6 | Qayyarah refinery | 1565 | 1.072 | 1.087536 | 0.015536 | 0.0002414 |
| 7 | Qayyarah refinery | 1742 | 1.180 | 1.211420 | 0.031420 | 0.0009872 |
| 8 | Qayyarah refinery | 1778 | 1.195 | 1.236617 | 0.041617 | 0.0017320 |
| 9 | Qayyarah refinery | 1970 | 1.371 | 1.371000 | 0.000000 | 0.0000000 |
| 10 | Qayyarah refinery | 2308 | 1.587 | 1.607570 | 0.020570 | 0.0004231 |
| 11 | Qayyarah refinery | 2035 | 1.457 | 1.416494 | 0.040506 | 0.0016407 |
| 12 | Qayyarah refinery | 2029 | 1.383 | 1.412295 | 0.029295 | 0.0008582 |
| 13 | Qayyarah refinery | 2055 | 1.369 | 1.430493 | 0.061493 | 0.0037813 |
| | Σ | | | | | **0.0124535376** |

Fig.1 shows estimated and real values for the uranium concentration in the soil using the Neville method is as follows:

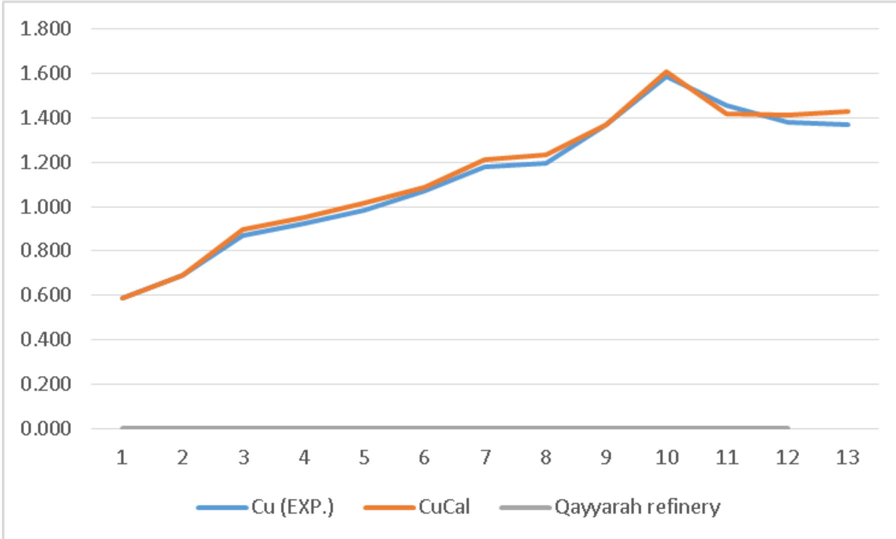


Fig. 1 The graph of estimated values for the concentration of uranium on soil using neville's method

## 4. Hermit Method

The Hermite polynomials are given when the case is mi=1, for each i=0, 1, …, n. These polynomials agree with f at x0, x1, …, xn for a given function f. Also, they have the same form as the function at (xi, f(xi)) because their first derivatives agree with those of f in the sense that the tangent lines are agreed to the polynomial and the function [7].

Using the Hermite method, we will estimate the concentration of Uranium in the soil of Qayyarah City with water and oil in the Qayyarah refinery. By using Hermite's method, the rule can be written as follows [8]:

$P_0 = 847$ , $F(P_0) = 0.585$

$P_1 = 1001$ , $F(P_1) = 0.692$

$P_2 = 1294$ , $F(P_2) = 0.870$

$P_3 = 1372$ , $F(P_3) = 0.922$

$$F[P_0, P_1] = \frac{F(P_1) - F(P_0)}{P_1 - P_0}$$

$$= \frac{0.692 - 0.585}{1001 - 847} = \frac{0.107}{154} = 0.0006$$

$$F[P_1, P_2] = \frac{F(P_2) - F(P_1)}{P_2 - P_1}$$

$$= \frac{0.870 - 0.692}{1294 - 1001}$$

$$= \frac{0.178}{293} = 0.0006$$

$$F[P_2, P_3] = \frac{F(P_3) - F(P_2)}{P_3 - P_2}$$

$$= \frac{0.922 - 0.870}{1372 - 1294}$$

$$= \frac{0.052}{78} = 0.0006$$

$$F[P_0, P_1, P_2] = \frac{F[P_1,P_2] - F[P_0,P_1]}{P_2 - P_0}$$

$$= \frac{0.0006 - 0.0006}{1294 - 847} = \frac{0}{447} = 0$$

$$F[P_1, P_2, P_3] = \frac{F[P_2,P_3] - F[P_1,P_2]}{P_3 - P_1}$$

$$= \frac{0.0006 - 0.0006}{1372 - 1001} = \frac{0}{371} = 0$$

$$F[P_0, P_1, P_2, P_3] = \frac{F[P_1,P_2,P_3] - F[P_0,P_1,P_2]}{P_3 - P_0}$$

$$= \frac{0 - 0}{1372 - 847} = \frac{0}{525} = 0$$

$$Cu = F[P0] + F[P_0, P_1](P - P_0) + F[P_0, P_1, P_2](P - P_0)(P - P_1) + F[P_0, P_1, P_2, P_3](P - P_0)(P - P_1)(P - P_2)$$

$$= 0.585 + 0.0006948052\,(P-847) + 0\,(P - 847)(P - 1001) + 0\,(P - 847)(P - 1001)(P - 1294)$$

$$= 0.585 + 0.0006948052P - 0.5885000044 + 0 + 0$$

$$= 0.0006948052P - 0.0035000044 \qquad (2)$$

Table 2: Calculate uranium concentration in THe soil using Hermit method

| No. | Location | P $Tr.cm^{-2}$ | Cu Exp. ppm | CuCal | E | E^2 |
|---|---|---|---|---|---|---|
| 1 | Qayyarah refinery | 847 | 0.585 | 0.585000 | 0.000000 | 0.0000000 |
| 2 | Qayyarah refinery | 1001 | 0.692 | 0.692000 | 0.000000 | 0.0000000 |
| 3 | Qayyarah refinery | 1294 | 0.870 | 0.895578 | 0.025578 | 0.0006542 |
| 4 | Qayyarah refinery | 1372 | 0.922 | 0.949773 | 0.027773 | 0.0007713 |
| 5 | Qayyarah refinery | 1467 | 0.986 | 1.015779 | 0.029779 | 0.0008868 |
| 6 | Qayyarah refinery | 1565 | 1.072 | 1.083870 | 0.011870 | 0.0001409 |
| 7 | Qayyarah refinery | 1742 | 1.180 | 1.206851 | 0.026851 | 0.0007210 |
| 8 | Qayyarah refinery | 1778 | 1.195 | 1.231864 | 0.036864 | 0.0013589 |
| 9 | Qayyarah refinery | 1970 | 1.371 | 1.365266 | 0.005734 | 0.0000329 |
| 10 | Qayyarah refinery | 2308 | 1.587 | 1.600110 | 0.013110 | 0.0001719 |
| 11 | Qayyarah refinery | 2035 | 1.457 | 1.410429 | 0.046571 | 0.0021689 |
| 12 | Qayyarah refinery | 2029 | 1.383 | 1.406260 | 0.023260 | 0.0005410 |
| 13 | Qayyarah refinery | 2055 | 1.369 | 1.424325 | 0.055325 | 0.0030608 |
| | Σ | | | | | **0.0105086** |

Using Hermite's method, Fig.2 shows estimated values for the concentration of URANIUM on the soil below.

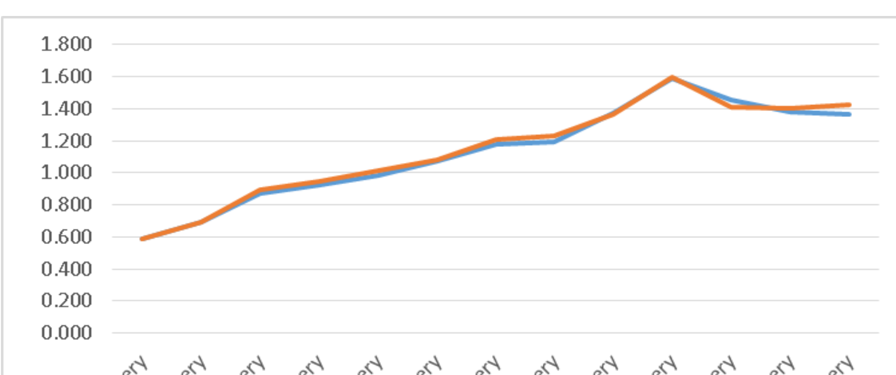

Fig.2 The graph of estimated values for the concentration of URANIUM in the soil by using Hermite method

**5. Least Square Method**

It is an essential statistical method, mainly for regression analysis. It aims to estimate the trendline that minimizes the total of the significant mistakes or deviations in the analysis line of observed points. It is a typical approximation approach used to solve equations systems where the number of equations exceeds the variables' number to get a solution that attempts to lower the value of the mean of the squares of the error produced from estimating each equation [9].

We used the Least squares method to estimate the concentration of Uranium in the soil of Qayyarah refinery. When using the Least squares method, the rule can be formed as shown below [10]:

$N * a_0 + a_1 \sum P = \sum Cu$

$13\ a_0 + 21463 a_1 = 14.669$

$a_0 * \sum P + a_1 \sum p^2 = \sum P * Cu$

$21463\ a_0 + 37762247 a_1 = 25831.426$

$\pm 13\ a_0 \pm 21463 a_1 = \pm 14.669$

$21450\ a_0 + 37740784\ a_1 = 25816.757$

$21450\ a_0 = 25816.757 - 37740784\ a_1$

$a_0 = \frac{25816.757 - 37740784\ a1}{21450}$

$a_0 = 1.2035784149 - 1759.4771095571 a_1$

$13\ a_0 + 21463 a1 = 14.669$

$13\ (1.2035784149 - 1759.4771095571 a_1) + 21463 a_1 = 14.669$

$15.6465193937 - 22873.202424242\ a_1 + 21463 a1 = 14.669$

$-1410.202424242\ a_1 = -\ 0.9775193937$

$a_1 = \frac{0.9775193937}{1410.202424242}$

$a_1 = 0.0006931767$

$a_0 = 1.2035784149 - 1759.4771095571 a_1$

$a_0 = 1.2035784149 - 1759.4771095571(0.0006931767)$

$a_0 = 1.2035784149 - 1.2196285365$

$a_0 = -\ 0.0160501216$

$Cu = a_0 + a_1 * P$

$Cu = -\ 0.0160501216 + 0.0006931767\ P$

Table 3: Calculate uranium concentration in the soil using the Least square method

| No. | Location | P $Tr.cm^{-2}$ | Cu Exp. ppm | P2 | P*Cu | CuCal | E | E^2 |
|---|---|---|---|---|---|---|---|---|
| 1 | Qayyarah refinery | 847 | 0.585 | 717409 | 495.495 | 0.571071 | 0.013929 | 0.0001940 |
| 2 | Qayyarah refinery | 1001 | 0.692 | 1002001 | 692.692 | 0.677820 | 0.014180 | 0.0002011 |
| 3 | Qayyarah refinery | 1294 | 0.870 | 1674436 | 1125.78 | 0.880921 | 0.010921 | 0.0001193 |
| 4 | Qayyarah refinery | 1372 | 0.922 | 1882384 | 1264.984 | 0.934988 | 0.012988 | 0.0001687 |
| 5 | Qayyarah refinery | 1467 | 0.986 | 2152089 | 1446.46 | 1.000840 | 0.014840 | 0.0002202 |
| 6 | Qayyarah refinery | 1565 | 1.072 | 2449225 | 1677.68 | 1.068771 | 0.003229 | 0.0000104 |
| 7 | Qayyarah refinery | 1742 | 1.180 | 3034564 | 2055.56 | 1.191464 | 0.011464 | 0.0001314 |
| 8 | Qayyarah refinery | 1778 | 1.195 | 3161284 | 2124.71 | 1.216418 | 0.021418 | 0.0004587 |
| 9 | Qayyarah refinery | 1970 | 1.371 | 3880900 | 2700.87 | 1.349508 | 0.021492 | 0.0004619 |
| 10 | Qayyarah refinery | 2308 | 1.587 | 5326864 | 3662.796 | 1.583802 | 0.003198 | 0.0000102 |
| 11 | Qayyarah refinery | 2035 | 1.457 | 4141225 | 2964.995 | 1.394564 | 0.062436 | 0.0038982 |
| 12 | Qayyarah refinery | 2029 | 1.383 | 4116841 | 2806.107 | 1.390405 | 0.007405 | 0.0000548 |
| 13 | Qayyarah refinery | 2055 | 1.369 | 4223025 | 2813.295 | 1.408428 | 0.039428 | 0.0015546 |
| | ∑ | 21463 | 14.669 | 37762247 | 25831.426 | | | **0.0074836** |

By using the Least square method, Fig 3 shows the estimated for the URANIUM concentration on the soil below.

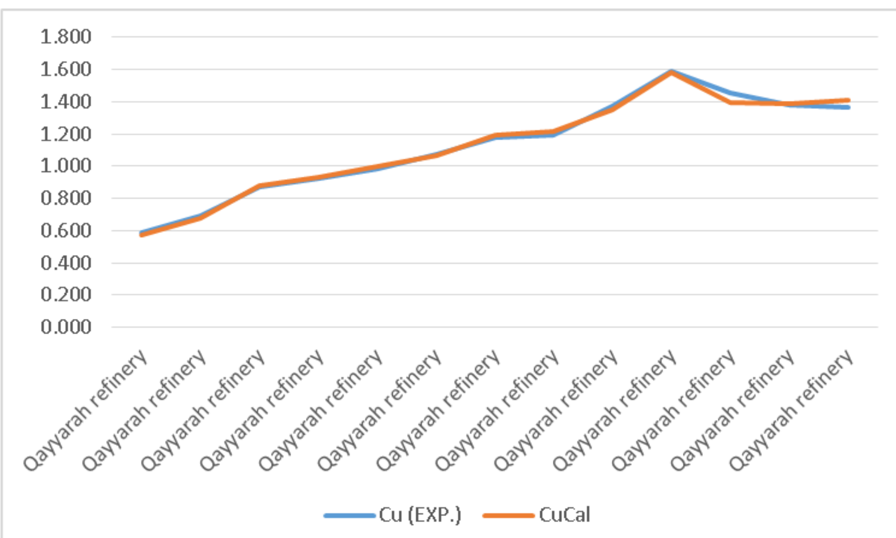


Fig. 3 The graph of estimated values for the concentration of uranium in the soil by using least square method

## 6. Conclusion and results

As shown by a short investigation measuring the Uranium concentration in the soil of Qayyarah City, the research confirmed that mathematical modelling is an effective and accurate method for determining Uranium concentration in the soil. This research used several numerical analytic techniques to demonstrate a relationship between mathematics and healthcare. We do not construct mathematical models that help us solve issues; instead, we gain answers that are compatible with experimental evidence and theoretical values via these models. Previous outcomes: We have employed extrapolation techniques using the Neville, Leas Square, and Hermit methods to validate the mathematical models.

The mathematical models presented in this research are practical and valuable tools. Consequently, the suggested models produced results that were accurate and consistent. The mathematical models derived from the three ways helped us to estimate the uranium concentrations in the soil. The best models were those developed by numerical analysis, which is the approach of the Least square method, which is found to be with the least estimated error.

الملخص

يهدف البحث إلى بناء نماذج رياضية لتقدير تركيز اليورانيوم في التربة بالماء والنفط باستخدام طرق عددية مثل نيفيل وهيرمايت ، حيث حصلنا على نماذج رياضية ضمنية من خلال الكمية المناسبة من تركيز اليورانيوم في التربة. وحصلنا على نسبة خطأ قليلة جدا.

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