Establishment of Mathematical Models to Estimate the Concentration of Radon in the Soil with Water and Oil in the Qayyarah Refinery

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Abstract

The research aims to build mathematical models for estimating the concentration of Radon in the soil with water and oil using numerical methods such as Neville and Hermite, where we got mathematical models implicitly through the adequate amount of Radon concentration in the soil. We got on miniatures mistake percentage almost non-existent

Keywords: Uranium. Radon. Least Square . Mathematical Models. Neville Method . Hermit Method

1. Introduction

Natural radioactive elements Like Radon and thorium are found in the soil [1]. Radon gas belongs to the series of uranium and is characterized by the ability to transfer in the air from one place to another without impediment. It possibly exposes humans and planets in the oil industry [2, 3]. Natural Radon involves three isotopes, one from each of the three natural radioactive-disintegration series (the series of Uranium, thorium, and actinium). Radon was discovered in 1900 by Friedrich E. Dorn, a German chemist, radon-222 (3.823-day hal–life), the longest-lived isotope, arises in the uranium series [4]. Numerous studies have been published by the International Friedrich E. Dorn Atomic Energy Friedrich E. Dorn Agency on the possibility of the radioactive elements' presence in the oil industry, as well as treatment and safety methods [5].

Researchers conducted A study in Turkey by collecting soil samples, crude oil, sludge and water, measuring the radiation level and determining its danger [6]. Our study aims to estimate Radon concentration in the soil of the Qayyarah refinery with water and oil in Qayyarah refinery using some numerical methods like Neville, Hermite and Least Square methods.

1. Application Field

The study is established based on mathematical modelling by using numerical methods such as Neville to achieve calculations of Radon concentration in the soil. This study was established to estimate the mathematical modelling by accreditation on graph theory methods such as Neville, Hermite and Least Square.

2. Neville's Method

The key idea of this method is to estimate the polynomial value at a specific point without first finding all of the polynomial coefficients. The Neville's method is defined as follows: Let f be a function that has values at the n points $X_0, X_1 \ldots X_n$. Let $\{m_1, m_2, \ldots m_k\}$ be a k distinct integers set from the set $(0, 1.2, \ldots n\}$. Let $Pm_1, m_2 \ldots m_k (x)$ represent the polynomial of Lagrange that agrees with the f function at k points $xm_1, xm_2, \ldots xm_k$, …. i.e. $Pm_1, m_2, \ldots, m_k (xm_1) = f(m_1)$, $Pm_1. M_2, \ldots m_k (xm_2) = f(xm_2), \ldots, Pm_1. M_2 \ldots m_k (xm_k) = f(xm_k)$. Obviously, $Pm_1, m_2, \ldots, m_k (x)$ is the only (k-1) degree polynomial which passes in the k points $(xm_1 . f(xm_1)), \ldots, (xm_k . f(xm_k))$. Neville's method main idea is to recursively use polynomials of Lagrange of lower powers to calculate polynomials of Lagrange of higher power relationships. This concept can be useful; for instance, if we have the polynomial of Lagrange depending on some data set points $(xi, f(xk)), k= 0, 1, \ldots, n$, and we can get a new data point, $(xn+1, f(xn+1))$ [7].

$$AR_n = \frac{\text{[illegible]}}{\text{[illegible]}},$$

$$P_0 = 847 \qquad AR_n = 0.5653$$

$$P_1 = 1970 \qquad AR_{n_1} = 1.3171$$

$$AR_n = \frac{\text{[illegible]}}{1970 - 847}$$

$$= \frac{\text{[illegible]}}{\text{[illegible]}}$$

$$= \frac{\text{[illegible]}}{\text{[illegible]}}$$

$$= 0.0006694568\, p - 0.0017299199 \qquad (1)$$

Table 1: Calculate Radon concentration in the soil using the Neville method

No.	Location	P $Tr.cm^{-2}$	ARn Exp. Bq	ARnCal	E	E^2
1	Qayyarah refinery	847	0.5653	0.5653000	0.000000	0.0000000000
2	Qayyarah refinery	1001	0.6688	0.6683963	0.000404	0.0000001629
3	Qayyarah refinery	1294	0.8655	0.8645472	0.000953	0.0000009079
4	Qayyarah refinery	1372	0.9167	0.9167648	0.000065	0.0000000042
5	Qayyarah refinery	1467	0.9804	0.9803632	0.000037	0.0000000014
6	Qayyarah refinery	1565	1.0452	1.0459700	0.000770	0.0000005929
7	Qayyarah refinery	1742	1.1647	1.1644638	0.000236	0.0000000558
8	Qayyarah refinery	1778	1.1886	1.1885643	0.000036	0.0000000013
9	Qayyarah refinery	1970	1.3171	1.3171000	0.000000	0.0000000000
10	Qayyarah refinery	2308	1.5434	1.5433764	0.000024	0.0000000006
11	Qayyarah refinery	2035	1.3603	1.3606147	0.000315	0.0000000990

No.	Location	P Tr.cm^{-2}	ARn Exp. Bq	ARnCal	E	E^2
12	Qayyarah refinery	2029	1.3569	1.3565979	0.000302	0.0000000912
13	Qayyarah refinery	2055	1.3739	1.3740038	0.000104	0.0000000108
	∑					**0.0000019279**

Fig.1. shows the estimated and real values for the Radon concentration in the soil using the Neville method as follows:

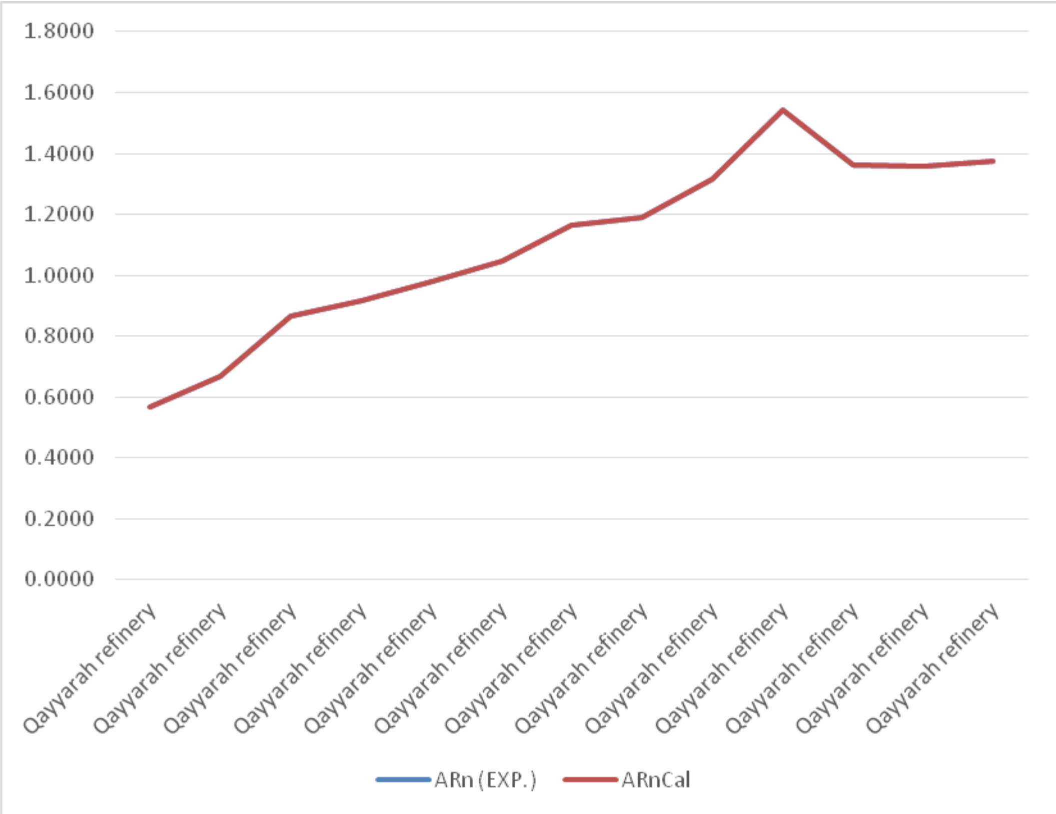

Fig. 1: The estimated values graph for the concentration of Radon on the soil by using Neville's method

3. Hermite Method

The polynomials of Hermite are given in the case of mi=1, for each i=0, 1, …, n. Those polynomials agree with f at x_0, x_1, …, x_n for a given function f. Likewise, they have a similar form as the function at (xi, f(xi)) due to their first derivatives agreeing with these of f function in the sense that the tangent lines are agreed to the polynomial and the function [8].

Using the Hermite method, we will estimate the concentration of Radon in the soil of Qayyarah City with water and oil in the Qayyarah refinery. By using the method of Hermite, the formula can be expressed as follows [9]:

$P_0 = 847$, $F(P_0) = 0.5653$

$P_1 = 1001$, $F(P_1) = 0.6688$

$P_2 = 1294$, $F(P_2) = 0.8655$

$P_3 = 1372$, $F(P_3) = 0.9167$

$F[P_0, P_1] = \frac{F(P1) - F(P0)}{P1 - P0}$

$= \frac{0.6688 - 0.5653}{1001 - 847} = \frac{0.1035}{154} = 0.0006$

$F[P_1, P_2] = \frac{F(P2) - F(P1)}{P2 - P1}$

$= \frac{0.8655 - 0.6688}{1294 - 1001}$

$= \frac{0.1967}{293} = 0.0006$

$F[P_2, P_3] = \frac{F(P3) - F(P2)}{P3 - P2}$

$= \frac{0.9167 - 0.87655}{1372 - 1294}$

$= \frac{0.0512}{78} = 0.0006$

$F[P_0, P_1, P_2] = \frac{F[P1,P2] - F[P0,P1]}{P2 - P0}$

$= \frac{0.0006 - 0.0006}{1294 - 847} = \frac{0}{447} = 0$

$F[P_1, P_2, P_3] = \frac{F[P2,P3] - F[P1,P2]}{P3 - P1}$

$= \frac{0.0006 - 0.0006}{1372 - 1001} = \frac{0}{371} = 0$

$F[P_0, P_1, P_2, P_3] = \frac{F[P1,P2,P3] - F[P0,P1,P2]}{P3 - P0}$

$= \frac{0 - 0}{1372 - 847} = \frac{0}{525} = 0$

$ARn = F[P0] + F[P_0, P_1](P - P_0) + F[P_0, P_1, P_2](P - P_0)(P - P_1) + F[P_0, P_1, P_2, P_3]$
$(P - P_0)(P - P_1)(P - P_2)$

$= 0.5653 + 0.0006720779\,(P-847) + 0\,(P - 847)(P - 1001) + 0\,(P - 847)(P - 1001)(P - 1294)$

$= 0.5653 + 0.0006720779P - 0.5692499813 + 0 + 0$

$= 0.0006720779P - 0.0039499813 \quad (2)$

Table 2: Calculate Radon concentration in the soil using the Hermit method

No.	Location	P $Tr.cm^{-2}$	ARn Exp. ppm	ARnCal	E	E^2
1	Qayyarah refinery	847	0.5653	0.5653000	0.000000	0.0000000000
2	Qayyarah refinery	1001	0.6688	0.6688000	0.000000	0.0000000000
3	Qayyarah refinery	1294	0.8655	0.8657188	0.000219	0.0000000479
4	Qayyarah refinery	1372	0.9167	0.9181409	0.001441	0.0000020762
5	Qayyarah refinery	1467	0.9804	0.9819883	0.001588	0.0000025227
6	Qayyarah refinery	1565	1.0452	1.0478519	0.002652	0.0000070327
7	Qayyarah refinery	1742	1.1647	1.1668097	0.002110	0.0000044509
8	Qayyarah refinery	1778	1.1886	1.1910045	0.002405	0.0000057817
9	Qayyarah refinery	1970	1.3171	1.3200435	0.002943	0.0000086641
10	Qayyarah refinery	2308	1.5434	1.5472058	0.003806	0.0000144842
11	Qayyarah refinery	2035	1.3603	1.3637285	0.003429	0.0000117549
12	Qayyarah refinery	2029	1.3569	1.3596961	0.002796	0.0000078181
13	Qayyarah refinery	2055	1.3739	1.3771701	0.003270	0.0000106936
	$\sum$					**0.0000753**

Using Hermite's method, the estimated values for the concentration of Radon in the soil are shown below:

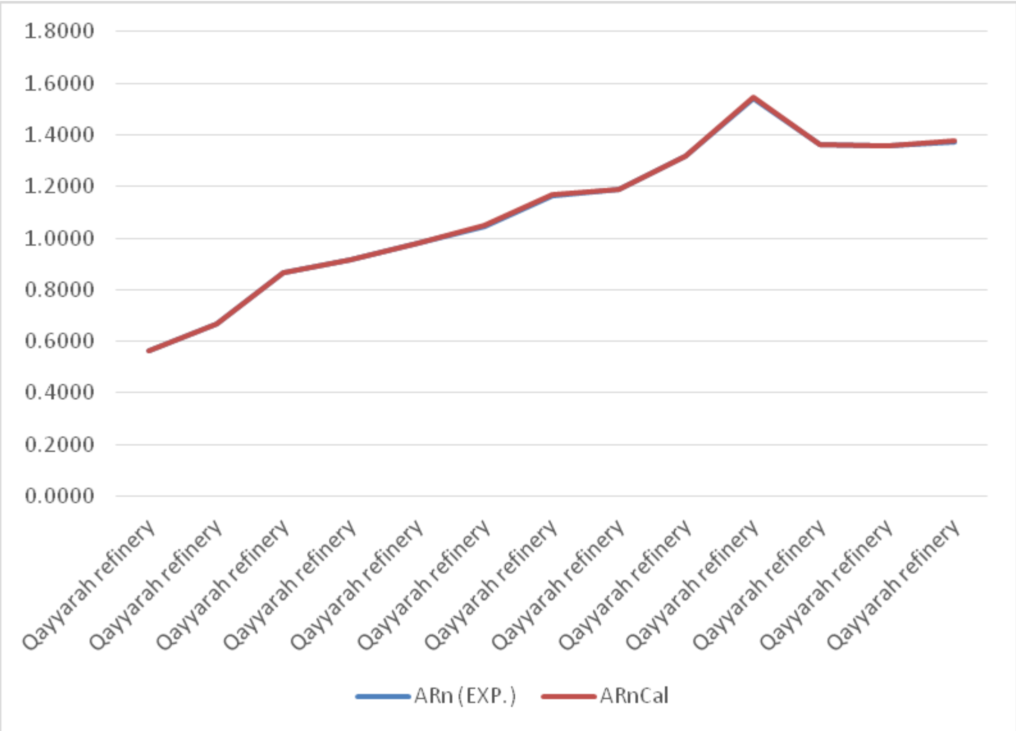

Fig. 2: The estimated values graph for the concentration of Radon on the soil by using Hermite's method.

4. Least Square Method

It is an essential statistical method, mainly for the analysis of regression. It estimates the trendline that reduces the overall significant mistakes or deviations in the line of analysis of observed points. It is a typical estimation approach that is used to solve equations systems where the number of equations exceeds the number of variables to obtain a solution that attempts to reduce the value of the squares mean the of the error that is produced from approximating each equation [10].

We used the Least squares method to estimate Radon's concentration in the Qayyarah refinery's soil. The rule of the Least squares method is formed as shown below [11]:

$N * a_0 + a_1 \sum P = \sum ARn$

$13\, a_0 + 21463 a_1 = 14.3468$

$a_0 * \sum P + a_1 \sum p^2 = \sum P * ARn$

$21463\, a_0 + 37762247 a_1 = 25243.7496$

$\pm 13\, a_0 \pm 21463 a_1 = \pm 14.3468$

$21450\, a_0 + 37740784\, a_1 = 25229.4028$

$21450\, a_0 = 25229.4028 - 37740784\, a_1$

$$a_0 = \frac{25229.4028 - 37740784\, a1}{21450}$$

$a_0 = 1.1761959347 - 1759.4771095571 a_1$

$13\, a_0 + 21463 a1 = 14.3468$

$13\,(1.1761959347 - 1759.4771095571 a_1) + 21463 a_1 = 14.3468$

$15.2905471511 - 22873.202424242\, a_1 + 21463 a1 = 14.3468$

$-1410.202424242\, a_1 = -\,0.9437471511$

$$a_1 = \frac{0.9437471511}{1410.202424242}$$

$a_1 = 0.0006692281$

$a_0 = 1.1761959347 - 1759.4771095571 a_1$

$a_0 = 1.1761959347 - 1759.4771095571(0.0006692281)$

$a_0 = 1.1761959347 - 1.177491523$

$a_0 = -\,0.0012955883$

$ARn = a_0 + a_1 * P$

$ARn = -\,0.0012955883 + 0.0006692281\, P$

Table 3: Calculate Radon concentration in the soil using the Least Square method

No.	Location	P $_{Tr.cm^{-2}}$	ARn Exp. Bq	P^2	P*ARn	ARnCal	E	E^2
1	Qayyarah refinery	847	0.5653	717409	478.8091	0.565540612	0.000241	0.0000000579
2	Qayyarah refinery	1001	0.6688	1002001	669.4688	0.668601740	0.000198	0.0000000393
3	Qayyarah refinery	1294	0.8655	1674436	1119.9570	0.864685573	0.000814	0.0000006633
4	Qayyarah refinery	1372	0.9167	1882384	1257.7124	0.916885365	0.000185	0.0000000344
5	Qayyarah refinery	1467	0.9804	2152089	1438.2468	0.980462034	0.000062	0.0000000038
6	Qayyarah refinery	1565	1.0452	2449225	1635.7380	1.046046388	0.000846	0.0000007164
7	Qayyarah refinery	1742	1.1647	3034564	2028.9074	1.164499762	0.000200	0.0000000401
8	Qayyarah refinery	1778	1.1886	3161284	2113.3308	1.188591974	0.000008	0.0000000001
9	Qayyarah refinery	1970	1.3171	3880900	2594.6870	1.317083769	0.000016	0.0000000003
10	Qayyarah refinery	2308	1.5434	5326864	3562.1672	1.543282867	0.000117	0.0000000137
11	Qayyarah refinery	2035	1.3603	4141225	2768.2105	1.360583595	0.000284	0.0000000804
12	Qayyarah refinery	2029	1.3569	4116841	2753.1501	1.356568227	0.000332	0.0000001101
13	Qayyarah refinery	2055	1.3739	4223025	2823.3645	1.373968157	0.000068	0.0000000046
	Σ	21463	14.3468	37762247	25243.7496			**0.0000017644**

By using the Least square method, the estimated values for the Radon concentration on the soil is shown below.

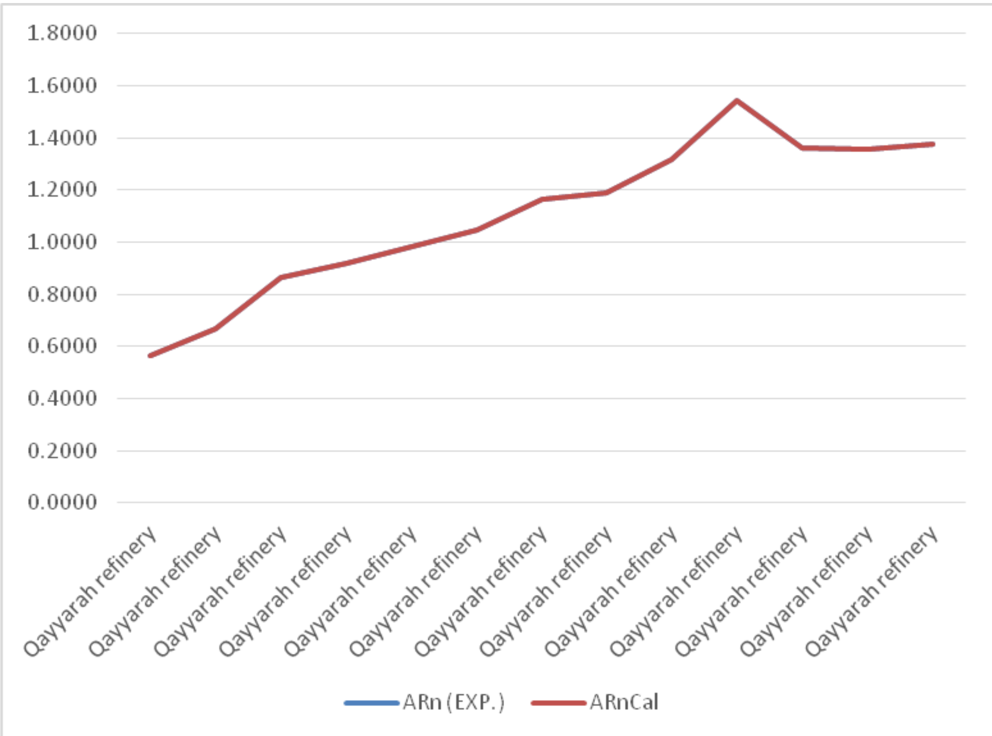

Fig. 3: The estimated values graph for the concentration of Radon on soil by using Least Square's method

5. Conclusion and Result

As explained by a short investigation that measured the concentration of Radon in the soil of Qayyarah City, the research confirmed that mathematical modelling is an effective and accurate method for identifying Radon concentration in the soil. This research used several numerical analytic techniques to demonstrate a relationship between mathematics and healthcare. We do not create mathematical models that aid us in solving issues; alternatively, we obtain answers compatible with theoretical values and experimental evidence through these models. We have utilized techniques of extrapolation which are the Neville, Leas Square, and Hermit methods, for validating the mathematical models.

The mathematical models that have been presented in this research are practical and valuable tools. Thus, the recommended models gave results that were consistent and accurate. The derived mathematical models helped us estimate the concentrations of Radon in the soil. The best models were these developed by numerical analysis is the approach of the Least square method, which is found to be with the least estimated error.

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