Creating Mathematical Models to Estimate the Concentration of Uranium in Children's Teeth Models for the Governorates of Baghdad and Basra Using Neville's and Spline Numerical Methods

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Abstract

The purpose of this research is to develop accurate mathematical models for estimating the concentration of uranium in children's teeth in Baghdad and Basra governorates using numerical methods. To achieve this, the Nivel and Spline methods were employed to estimate the uranium concentration in samples of children's teeth. The results of the assessment showed close agreement with the experimental data. This study is of significant importance as it contributes to the advancement of precise and efficient methods for assessing uranium concentration in human teeth. Such advancements are crucial for effectively monitoring and managing potential health risks associated with uranium exposure.

Keywords: Spline Method, Neville Method, Mathematical Models, Uranium, Basra.

1- Introduction

In this article, we investigate various numerical estimation methods to accurately determine the concentration of uranium in children's teeth. Utilizing the numerical spline method, we develop mathematical models for estimation. Numerical analysis, a critical aspect of mathematics and other sciences, plays a pivotal role in our research.

To achieve accurate estimates, we employ numerical interpolation methods, ensuring that the estimated values closely approximate the real values with minimal error. Notably, recent years have seen a surge in speculative studies, with Stephen Balter et al. exploring techniques to estimate radiation dose on the skin fluoroscopically [1], and Haregeweyn N. and Yohannes F. examining non-agricultural pollution models on watersheds in Ethiopia [2]. Michael G. et al. delve into second-generation computer software for internal dose assessment in nuclear medicine [3]. In 2001, Emily R. Unsworth and her colleagues conducted a study in England, employing ICP mass spectrometry to measure uranium concentrations in seawater and groundwater. Their findings revealed uranium amounts of 2.68 ngml-1 in seawater and 0.3 ngml-1 in groundwater [4]. Hakonson-Hayes et al.

evaluated 238U concentrations in tomato, pumpkin, lettuce, and radish contents irrigated with well water in the Nambe region using ICP-MS in 2002. The measured values were <1 µgl-1, 150 µgl-1, 500 µgl-1, and 1200 µgl-1, respectively [5]. In 2003, Reimann et al. measured uranium activity concentrations in river water, sea water, and tap water as 2.63%, 1.13%, and 1.37%, respectively, using ICPMS in a study conducted by Koji Oshita et al. in Japan. The activation values in drinking water from Eastern Africa were measured as 0.005-48 g/l, 598-45800 g/l, and 0.002-1.59 µg/l, respectively, by Clemens Reimann et al. [6]. Furthermore, in 2017, Arif, G. E. et al. demonstrated the estimation of uranium radiation effects on workers in selected chemical factories, relying on the numerical analysis spline method [7]. Overall, our study delves into the importance of numerical methods for accurately estimating uranium concentration in children's teeth, contributing to the field's understanding and monitoring of potential health risks associated with uranium exposure.

2- Neville's Method: [8-9]

In Lagrange interpolation there is a practical difficulty which is the difficulty of applying the error term, therefore, the required polynomial degree for the wanted accuracy is unknown if calculations are not conducted. A general procedure is to calculate the given results of different polynomials until the achievement of suitable agreement. Thus, the work achieved by the second polynomial in calculating the approximation does not reduce the required work for calculating the third approximation. In addition, it is also not easy to acquire the fourth approximation when the third approximation is known and so on. We can deduce these approximating Polynomials in away by employing previous calculations to a greater benefit.

Theorem1: Let f be defined at $x_0, x_1, \ldots, x_k$, and let x_j and x_i be two distinct numbers in this set.

Then

$$p(x) = \frac{(x - x_j)p_{0,1,\ldots,j-1,j+1,\ldots k}(x) - (x - x_i)p_{0,1,\ldots,i-1,i+1,\ldots k}(x)}{(x_i - x_j)}$$

Using the Nivelle's method to calculate of uranium concentration in child

teeth in Baghdad governorate

$$\text{Ra}=\frac{(U-U_0)R_{a1}-(U-U_1)R_{a0}}{(U_1-U_0)}$$

$$U_0 = 2.18R_{a0} = 0.020$$

$$U_1 = 1.87R_{a1} = 0.017$$

$$\text{Ra}=\frac{(\text{U}-2.18)(0.017)-(\text{U}-1.87)(0.020)}{1.87-2.18}$$

$$\text{Ra}=\frac{0.017U-o.03706\ -0.020U+0.0374}{-0.31}$$

$$\text{Ra}=\frac{-0.003U+0.00034}{-0.31}$$

$$Ra=0.0096U-0.00109$$

Table 1: The resulted values obtained by the mathematical model proposed to calculate of uranium concentration in child teeth in Baghdad governorate by using the Nivelle's method.

No	Age of the child	U	Ra.Exp [10]	Ra.cal	Error	$Error^2$
1	9	2.18	0.020	0.021	0.001	0.000001
2	8	2.18	0.020	0.021	0.001	0.000001
3	12	2.81	0.026	0.027	0.001	0.000001
4	11	1.87	0.017	0.018	0.001	0.000001
∑						0.000004

Figure 1 shows the diagram between experimental and estimated values for determination of uranium concentration in child teeth in Baghdad governorate using the Nivelle's method

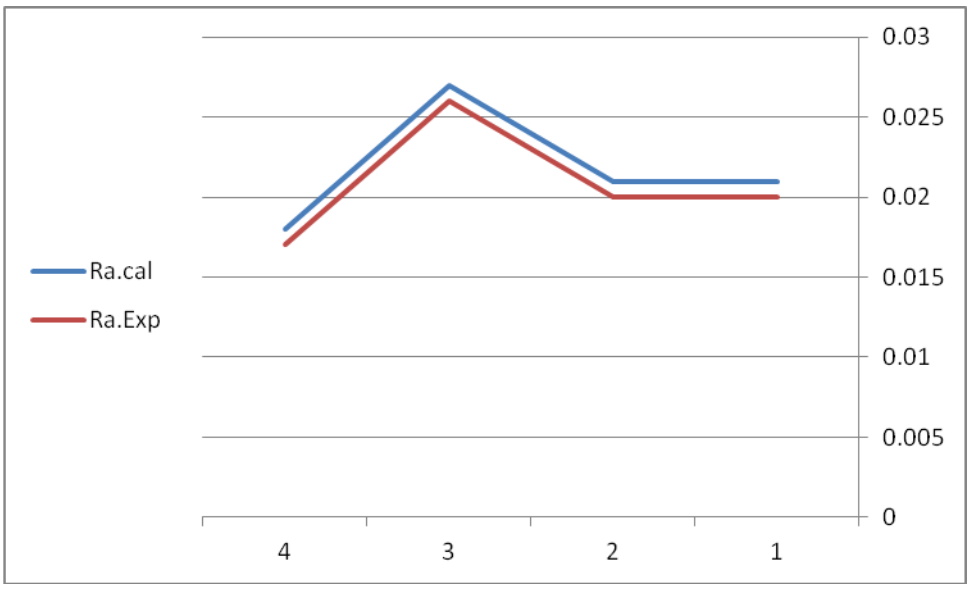


Fig. 1: Cmparison between exp. and estimated values for determination of uranium concentration in child teeth in Baghdad governorate using the Nivelle's method.

Using the Nivelle's method to calculate of uranium concentration in child teeth in Basra governorate

$$Ra=\frac{(U-U_0)R_{a1}-(U-U_1)R_{a0}}{(U_1-U_0)}$$

$$U_0=16.7187R_{a0}=0.15$$

$$U_1=18.125R_{a1}=0.169$$

$$Ra=\frac{(U-16.7187)0.169-(U-18.125)0.15}{18.125-16.7187}$$

$$Ra=\frac{0.169U-2.825-0.15U+2.719}{1.4063}$$

$$Ra=\frac{0.019U-0.106}{1.4063}$$

$$Ra=0.0135U\ -\ 0.0754$$

Table 2: The resulted values obtained by the mathematical model proposed to calculate of uranium concentration in child teeth in Basra governorate by using the Nivelle's method.

No	Age of the child	U	Ra.Exp [10]	Ra.cal	Error	$Error^2$
1	6	16.7187	0.15	0.15	0	0
2	6	16.7187	0.15	0.15	0	0
3	9	18.4375	0.17	0.17	0	0
4	9	19.0625	0.176	0.182	0.006	0.000036
5	10	18.125	0.169	0.169	0	0
∑						0.000036

Figure 2 shows the diagram between experimental and estimated values for determination of uranium concentration in child teeth in Basra governorate using the Nivelle's method

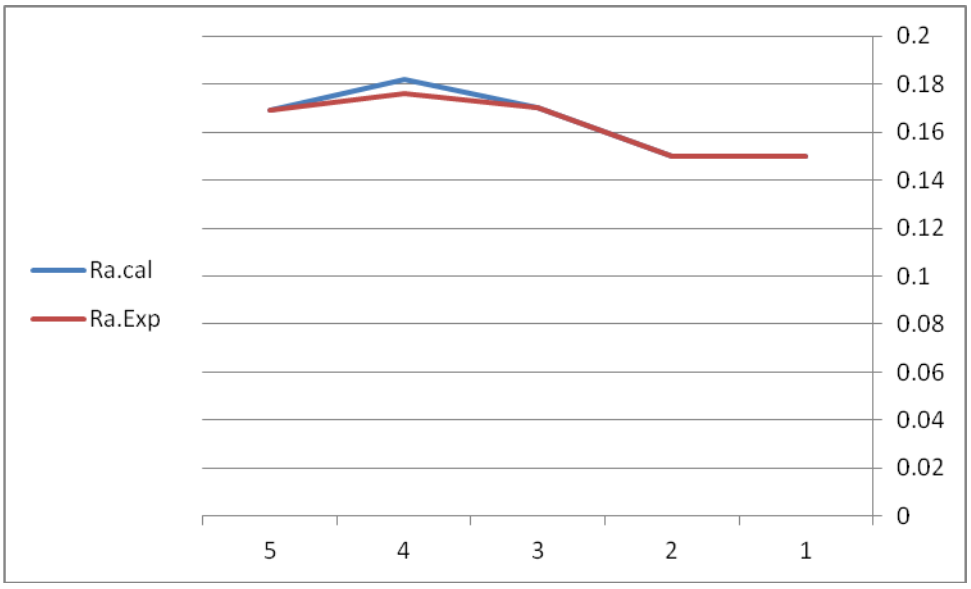


Fig. 2: Comparison between exp. and estimated values for determination of uranium concentration in child teeth in Basra governorate using the Nivelle's method

3- Spline Method: [11]

Let the given data point be (x_j,y_j) $a = x_0 < x_1 < x_2 < .... < x_n = b$, where $j = 1,2,.....n$, $h_j = x_j - x_{j-1}$, and let be the spline of degree one defined in the interval $[x_{j-1},x_j]$. Further, let S_j obviously, $S_j(x)$ represents a straight line joining the points (x_{j-1},y_{j-1}) and (x_j,y_j). Hence, we write

$$S_j(x) = y_{j-1} + m_j(x - x_{j-1})$$

$$m = \frac{y - y_{j-1}}{x - x_{j-1}},$$

Using the spline method to calculate of uranium concentration in child teeth in Baghdad governorate

$$R_a = R_{a0} + \frac{R_{a1} - R_{a0}}{U_1 - U_0}(U - U_0)$$

$$U_0 = 2.18R_{a0} = 0.020$$

$$U_1 = 1.87R_{a1} = 0.017$$

$$R_a = 0.020 + \frac{0.017 - 0.020}{1.87 - 2.18}(U - 2.18)$$

$$R_a = 0.020 + \frac{-0.003}{-0.31}(U - 2.18)$$

$$R_a = 0.020 + 0.0096(U - 2.18)$$

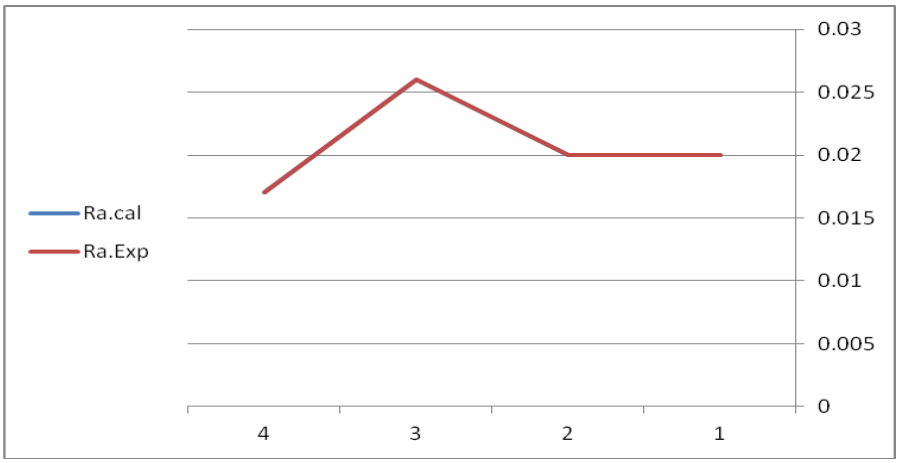
$$R_a = 0.020 + 0.0096U - 0.020$$

$$R_a = 0.0096U$$

Table 3: The resulted values obtained by the mathematical model proposed to calculate of uranium concentration in child teeth in Baghdad governorate by using the Spline method.

No	Age of the child	U	Ra.Exp [10]	Ra.cal	Error	$Error^2$
1	9	2.18	0.020	0.020	0	0
2	8	2.18	0.020	0.020	0	0
3	12	2.81	0.026	0.026	0	0
4	11	1.87	0.017	0.017	0	0
∑						0

Figure 3 shows the diagram between experimental and estimated values for determination of uranium concentration in child teeth in Baghdad governorate using the Spline



Method

Fig. 3: Comparison between exp. and estimated values determination of uranium concentration in child teeth in Baghdad governorate using the Spline method.

Using the spline method to calculate of uranium concentration in child teeth in Basra governorate

$$R_a = R_{a0} + \frac{R_{a1} - R_{a0}}{U_1 - U_0}(U - U_0)$$

$$U_0 = 16.7187 R_{a0} = 0.15$$

$$U_1 = 18.125 R_{a1} = 0.169$$

9

$$R_a = 0.15 + \frac{0.169 - 0.15}{18.125 - 16.7187}(U - 16.7187)$$

$$R_a = 0.15 + \frac{0.19}{1.4063}(U - 16.7187)$$

$$R_a = 0.15 + 0.135(U - 16.7187)$$

$$R_a = 0.15 + 0.135U - 2.257$$

Table 4: The resulted values obtained by the mathematical model proposed to calculate of uranium concentration in child teeth in Basra governorate by using the Spline method.

No	Age of the child	U	Ra.Exp [10]	Ra.cal	Error	$Error^2$
1	6	16.7187	0.15	0.15	0	0
2	6	16.7187	0.15	0.15	0	0
3	9	18.4375	0.17	0.38	0.21	0.0441
4	9	19.0625	0.176	0.466	0.29	0.0841
5	10	18.125	0.169	0.339	0.17	0.0289
$\sum$						*0.1571*

Figure 4 shows the diagram between experimental and estimated values for determination of uranium concentration in child teeth in Basra governorate using the Spline method

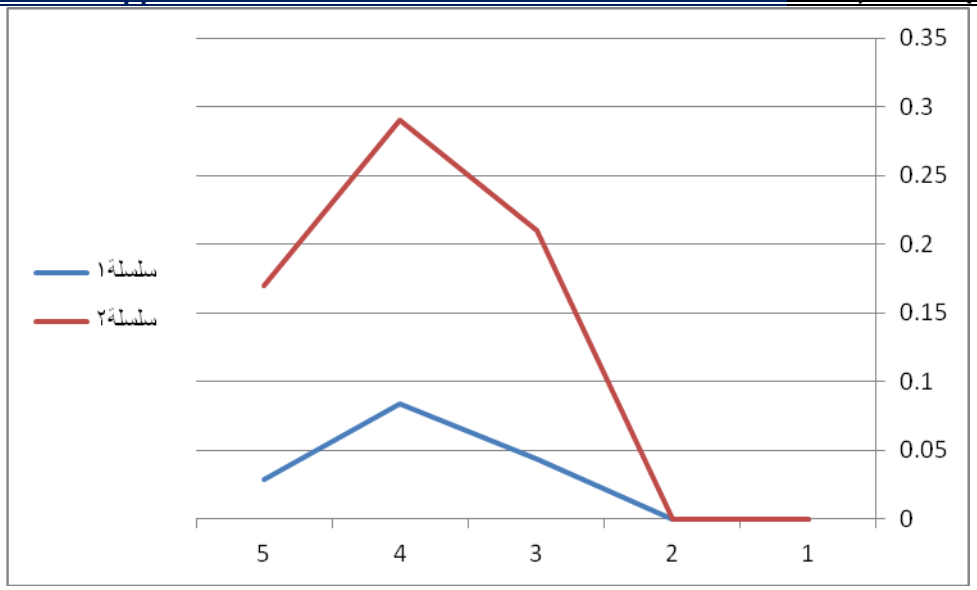


Fig. 4: Comparison between exp. and estimated values for determination of uranium concentration in child teeth in Basra governorate by using the Spline method.

4- Conclusions

This project utilized the Nivel and Spline numerical methods to estimate the concentration of uranium in children's teeth in Baghdad and Basra governorates. The results obtained from both methods were compared and analyzed, and it was concluded that the Spline method provided better results for estimating uranium concentrations in samples from Baghdad governorate, as shown in Table (1). On the other hand, Neville's method was found to be more effective than the Spline method for estimating uranium concentrations in samples from Basra Governorate, as indicated in Table (2). These findings demonstrate the importance of selecting appropriate numerical methods when developing models for estimating uranium concentrations in human teeth, as different methods may be more suitable for different regions or sample types.

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