Modified New Iterative Method for Solving Systems of Nonlinear Volterra Integral Equations of the Second Kind

Shangul Mohammed Ghareeb[1], Ghassan E. Arif[2] and Borhan F. Jumaa[3]

[1,2]Department of Mathematics / College of Education for Women / Tikrit University

[3]Computer Science Department, College of Computer Science and Information Technology, University of Kirkuk, Kirkuk, Iraq.

[1]shangullmuhammed@gmail.com

[2]ghasanarif@tu.edu.iq

[3]borhan_nissan@uokirkuk.edu.iq

Abstract

In this paper, we provide an approximate solution approach for evaluating a system of equations of the second sort of nonlinear Volterra integral equations numerically. The algorithm is based on a modified version of the New Iterative Method. Because it is difficult to integrate nonlinear functions, the method being presented here uses the Maclaurin series to turn nonlinear functions into polynomials. The problem may be resolved with the help of the MATLAB 2020b program. In conclusion, many examples are provided to illustrate how accurate and useful this formula may be.

Keywords: Integral equation with nonlinear variables, the Volterra integral equation, and a modified new iterative method.

Introduction:

Integral equations may be found in the scientific literature for a variety of Issues related to mathematical chemistry and physics, such as heat conduction, stereology, the radiation of heat from a semi-infinite substance some examples, and crystal formation, among others(J.R.Rise).

The iterative technique generates fast converging sequential approximate forms of the exact answer if a closed-form version of the solution already exists. The Adomian decomposition method, on the other hand, generates components rather than approximations. The approach may be used for both the same approach is used to linear and nonlinear problems, and there is no need to adhere to any particular limitations, examples include the so-called Adomian polynomials, which are required for nonlinear component analysis (A.M.Wazwaz)

The Modified New Iterative Method is used by Yaseen and Samraiz to solve nonlinear and linear equations. Klein-Gordon Equations, A. K. Jabber Utilize a modified version of the new iterative method when you are trying to solve nonlinear partial differential equations. Lawal and Loyimi. respectively When there are non-local circumstances present, you should solve linear and nonlinear initial boundary value issues by using a novel iterative strategy.

The nonlinear Volterra integral equations that are a component of the system are addressed in detail in the edited volumes written by Wazwaz and Linz which provide several different methods for analytically finding a solution to these equations. The writer of the F.J.Borhan, used the Trapezoidal Predictor-Corrector An approach to the solution about the mathematical structure consisting of two nonlinear Volterra Integral Equations of the Second Kind. In addition to this, the writers in A. M. Dalal, a numerical solution to the System of two nonlinear Volterra integral equations was suggested by the author. In addition, Burhan and Abbas solved systems of two nonlinear Volterra integral equations by using a strategy that did not include the use of Polynomial Splines. Finally, the authors Borhan, and Abbas, techniques of predictor-corrector analysis were used to find a numerical solution to these systems.

As a result, the subject of this study is an investigation into the system of two integral equations of Volterra's second class that must be solved. The following form is used to define the unnamed functions that may be found both within and outside of the integral sign Borhan, andAbbas,

$$\Psi_i(x) = \mathcal{F}_i(x) + \sum_{j=1}^{m} \int_0^x \mathcal{K}_{ij}(x, y, \Psi_j(y))dy \qquad i = 1,2,\dots,m \tag{1}$$

where $\Psi_i(x)$ are unknown functions, the functions $\mathcal{F}_i(x), i = 1,\dots,m$ and kernels $\mathcal{K}_{ij}\left(x, y, \Psi_j(y)\right), 1 \le i,j \le m$ are provided with functions having real values on subsets of $\mathcal{R}^3$ and $\mathcal{R}^1$, respectively.

2 New Iterative Method (NIM) [2]

Take into consideration the generic functional equation that follows:

$$\Psi(\Upsilon) = \varphi(\Upsilon) + \mathcal{N}(\Psi(\Upsilon)) \tag{2}$$

where $\mathcal{N}$ is an operator that is not linear and φ is a function that is already known.

$\Upsilon = (\gamma_1, \gamma_2, ,\cdots, \gamma_n)$. We are seeking a solution Ψ to the equation (2) that has the form of a series.

$$\Psi(\Upsilon) = \sum_{i=0}^{\infty} \Psi_i(\Upsilon) \tag{3}$$

The nonlinear operator N may be broken down into the following components:

$$\mathcal{N}\left(\sum_{i=0}^{\infty} \Psi_i(\Upsilon)\right) = \mathcal{N}(\Psi_0) + \sum_{i=0}^{\infty}\left\{\mathcal{N}\left(\sum_{j=0}^{i} \Psi_j(\Upsilon)\right) - \mathcal{N}\left(\sum_{j=0}^{i-1} \Psi_j(\Upsilon)\right)\right\} \tag{4}$$

Equation(2), which can be derived from equations (3) and (4), is equal to

$$\sum_{i=0}^{\infty} \Psi_i(\Upsilon) = \varphi(\Upsilon) + \mathcal{N}(\Psi_0) + \sum_{i=0}^{\infty}\left\{\mathcal{N}\left(\sum_{j=0}^{i} \Psi_j(\Upsilon)\right) - \mathcal{N}\left(\sum_{j=0}^{i-1} \Psi_j(\Upsilon)\right)\right\} \tag{5}$$

We define the recurrence relation

$$\left.\begin{array}{c} \Psi_0 = \varphi(\Upsilon) \\ \Psi_1 = \mathcal{N}(\Psi_0) \\ \Psi_{m+1} = \mathcal{N}(\Psi_0 + \Psi_1 + \ \dots, + \Psi_m) - \mathcal{N}(\Psi_0 + \ \Psi_1 + \ \dots + \Psi_{m-1}) \end{array}\right\} \tag{6}$$

Then

$$(\Psi_0 + \Psi_1 + \ \dots, + \Psi_m) = \mathcal{N}(\Psi_0 + \ \Psi_1 + \ \dots + \Psi_m) \tag{7}$$

And

$$\sum_{i=0}^{\infty} \Psi_i(\Upsilon) = \varphi(\Upsilon) + \mathcal{N}\left(\sum_{i=0}^{\infty} \Psi_i(\Upsilon)\right) \tag{8}$$

The k−term approximate solution of (2) is given by

$$\Psi = \Psi_0 + \Psi_1 + \ \dots, + \Psi_{k-1}$$

3 The Modified New Iterative Method (MNIM):[5]

This method depends on splitting the function $\varphi(\Upsilon)$ in equation (2) into two parts. We can set the zeros component $\varphi(\Upsilon)$ in the formula (6) as the sum of two parts $\varphi_1(\Upsilon)$ and $\varphi_2(\Upsilon)$ then we suggested the following modified formula:

$$\left.\begin{array}{c} \Psi_0 = \varphi_1(\Upsilon) \\ \Psi_1 = \varphi_1(\Upsilon) + \mathcal{N}(\Psi_0) \\ \Psi_{m+1} = \mathcal{N}(\Psi_0 + \Psi_1 + \ \dots, + \Psi_m) - \mathcal{N}(\Psi_0 + \ \Psi_1 + \ \dots + \Psi_{m-1}) \end{array}\right\} \tag{9}$$

To solve difficult nonlinear systems of equations of the second kind of Volterra integral equations, this type of modification provides highly accurate results, necessitates a reduced number of calculations in comparison to the iterative method, and it also avoids needless complications in the process of calculating the next term.

4 Numerical Methods

The range $[a, b]$ is utilized as the jumping-off point for the creation of the numerical strategy for the approximation solution of a system of type (1). This is done to make things as straightforward and uncomplicated as is humanly feasible. Accordingly, the $x_j = a + jh, j = 0,1,\dots,N$ and $h = \frac{b-a}{N}$ is used to define a grid that has $N + 1$ points that are uniformly spaced apart

To solve the equation (1)

$$\Psi_i(x) = \mathcal{F}_i(x) + \sum_{j=1}^{m} \int_0^x \mathcal{K}_{ij}(x, y, \Psi_j(y))dy \qquad i = 1,2,\dots,m \tag{10}$$

we apply the Maclaurin series to the known function $\mathcal{F}_i(x)$ to convert it to a polynomial:

$$\mathcal{F}_i(x) = \sum_{n=0}^{\infty} \frac{f^{(k)}(0)}{n!} x^k \tag{11}$$

Suppose that

$$\mathcal{F}_i(x) = \mathcal{F}_{i0}(x) + \mathcal{F}_{i1}(x) \tag{12}$$

$$\Psi_{i0} = \mathcal{F}_{i0} \tag{13}$$

$$\Psi_{i1} = \mathcal{F}_{i1} + \sum_{j=1}^{m} \int_0^x \mathcal{K}_{ij}(x, y, \Psi_{j0}(y))dy \qquad i = 1,2,\dots,m \tag{14}$$

$$\Psi_{i,p+1} = \sum_{j=1}^{m} \int_0^x \mathcal{K}_{ij}(x, y, \sum_{r=0}^{p} \Psi_{jr}(y))dy - \sum_{j=1}^{m} \int_0^x \mathcal{K}_{ij}(x, y, \sum_{r=0}^{p-1} \Psi_{jr}(y))dy \tag{15}$$

Where $p = 1,2,\dots,N$

$$\Psi_i(x) = \sum_{j=0}^{p+1} \Psi_{ij}(x) \tag{16}$$

5 Numerical Example:

In this part, we provide a variety of examples to exemplify the methodologies discussed in the previous section (section 3).

5.1 Example (1)

The following is a definition of a system consisting of two nonlinear Volterra integral equations of the second kind:

$$\Psi_1(x) = \sec(x) - x + \int_0^x \left(\Psi_1{}^2(y) - \Psi_2{}^2(y)\right) dy$$

$$\Psi_2(x) = 3\tan(x) - x - \int_0^x \left(\Psi_1{}^2(y) + \Psi_2{}^2(y)\right) dy$$

which has the exact solution: $\left(\Psi_1(x), \Psi_2(x)\right) = (\sec(x), \tan(x))$

let $\mathcal{F}_1(x) = \sec(x) - x$ and $\mathcal{F}_2(x) = 3\tan(x) - x$

apply the Maclaurin series on $\mathcal{F}_1(x)$ and $\mathcal{F}_2(x)$ we get

$$\mathcal{F}_1(x) = 1 + x^2 + \frac{5}{24}x^4 - x$$

$$\mathcal{F}_2(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{2}{3}x^3 - x$$

Suppose that

$$\mathcal{F}_{10} = 1 + x^2$$

$$\mathcal{F}_{20} = x + \frac{1}{3}x^3$$

Apply the MNIM we obtain

$$\Psi_1 = \Psi_{10} + \Psi_{11} + \Psi_{12}$$

$$\Psi_1 = 1 + \frac{1}{2}x^2 + \frac{5}{24}x^4 - \frac{1}{36}x^6 - \frac{121}{2520}x^7 - \frac{29}{2016}x^8 - \frac{1397}{181440}x^9 - \frac{17}{3360}x^{10} - \frac{139}{51975}x^{11} - \frac{5}{9072}x^{12} - \frac{1}{1365}x^{13}$$

$$\Psi_2 = \Psi_{20} + \Psi_{21} + \Psi_{22}$$

$$\Psi_2 = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{1}{36}x^6 - \frac{271}{2520}x^7 + \frac{29}{2016}x^8 - \frac{1049}{60480}x^9 + \frac{17}{3360}x^{10} - \frac{1637}{415800}x^{11} + \frac{5}{9072}x^{12} + \frac{4}{12285}x^{13} - \frac{2}{59535}x^{15}$$

x	**Exact solution**	**MNIM**	**Error**
0.0	1.000000000000000	1.000000000000000	0.000000000000000
0.1	1.005020918400456	1.005020800601886	0.000000117798569
0.2	1.020338844941193	1.020330899610309	0.000007945330884
0.3	1.046751601538086	1.046655618794738	0.000095982743348
0.4	1.085704428383239	1.085128793635235	0.000575634748003
0.5	1.139493927324549	1.137134160425380	0.002359766899170

L.S.E.	5.909131010704959e-006

Table (1): describe a comparison between the precise and the numerical using a modified new iterative technique for $\Psi_1(x)$ for example1. The comparison should be based on the least square error with h set to 0.1

x	Exact solution	MNIM	Error
0.0	0.000000000000000	0.000000000000000	0.000000000000000
0.1	0.100334672085451	0.100334683817448	0.000000011731997
0.2	0.202710035508673	0.202709762988137	0.000000272520535
0.3	0.309336249609623	0.309321356718823	0.000014892890800
0.4	0.422793218738162	0.422641508749376	0.000151709988785
0.5	0.546302489843790	0.545452714172594	0.000849775671197
L.E.S.	7.453564846565573e-007		

Table(2): describe a comparison between the precise and the numerical using a modified new iterative technique for $\Psi_2(x)$ for example1. The comparison should be based on the least square error with h set to 0.1.

5.2 Example (2):

The following is a definition of a system consisting of two nonlinear Volterra integral equations of the second kind:

$$\Psi_1(x) = \frac{1}{4} - \frac{1}{4}e^{2x} + \int_0^x (x-y)\Psi_2{}^2(y)dy$$

$$\Psi_2(x) = -xe^x + 2e^x - 1 + \int_0^x ye^{-2\Psi_1(y)}dy$$

which has the exact solution: $\left(\Psi_1(x), \Psi_2(x)\right) = \left(-\frac{1}{2}x\,,\quad e^x\right)$

let $\mathcal{F}_1(x) = \frac{1}{4} - \frac{1}{4}e^{2x}$ and $\mathcal{F}_2(x) = -xe^x + 2e^x - 1$

apply the Maclaurin series on $\mathcal{F}_1(x)$ and $\mathcal{F}_2(x)$ we get

$$\mathcal{F}_1(x) = -\frac{1}{2}x - \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{1}{6}x^4 - \frac{1}{15}x^5$$

$$\mathcal{F}_2(x) = 1 + x - \frac{1}{6}x^3 - \frac{1}{12}x^4 - \frac{1}{40}x^5$$

Suppose that

$$\mathcal{F}_{10} = -\frac{1}{2}x$$

$$\mathcal{F}_{20} = 1 + x$$

Apply the MNIM we obtain

$$\Psi_1 = \Psi_{10} + \Psi_{11} + \Psi_{12}$$

$$\Psi_1 = -\frac{1}{2}x + \frac{1}{45}x^6 - \frac{2}{315}x^7 + \frac{1}{560}x^8 + \frac{97}{181440}x^9 + \frac{31}{201600}x^{10} + \frac{127}{3326400}x^{11} + \frac{79}{9979200}x^{12} + \frac{1}{725760}x^{13} + \frac{19}{50803200}x^{14} + \frac{1}{12700800}x^{15} + \frac{1}{169344000}x^{16}$$

$$\Psi_2 = \Psi_{20} + \Psi_{21} + \Psi_{22}$$

$$\Psi_2 = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{5}{144}x^6 + \frac{37}{840}x^7$$

x	Exact solution	MNIM	Error
0.0	0.00000000000000	0.000000000000000	0.000000000000000
0.1	-0.050000000000000	-0.049999977124450	0.000000022875550
0.2	-0.100000000000000	-0.099998491646225	0.000001508353775
0.3	-0.150000000000000	-0.149982282765014	0.000017717234986
0.4	-0.200000000000000	-0.199897246938604	0.000102753061396
0.5	-0.250000000000000	-0.249594984058298	0.000405015941702
L.S.E.	1.749122807290687e-007		

Table (3): describe a comparison between the precise and the numerical using a modified new iterative technique for $\Psi_1(x)$ for example 2. The comparison should be based on the least square error with h set to 0.1.

x	Exact solution	MNIM	Errer

0.0	1.000000000000000	1.000000000000000	0.000000000000000
0.1	1.105170918075648	1.105170955793651	0.000000037718003
0.2	1.221402758160170	1.221405452698413	0.000002694538243
0.3	1.349858807576003	1.349892695714286	0.000033888138283
0.4	1.491824697641270	1.492033056507937	0.000208358866666
0.5	1.648721270700128	1.649584573412698	0.000863302712570
L.S.E.	7.898606587246861e-007		

Table(4): describe a comparison between the precise and the numerical using a modified new iterative technique for $\Psi_2(x)$ for example 2. The comparison should be based on the least square error with h set to 0.1.

6 Conclusion:

The purpose of this article is to provide a suggestion for an effective modification of the iterative approach described in [2] for numerically locating the precise solutions to a system consisting of two nonlinear Volterra integral equations of the second sort. The alteration that has been suggested is user-friendly and speedy to get the best solution in comparison with the new iterative method. The results showed that our suggested approaches were preferable because they produced more accurate solutions to the systems.

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تأثير المستخلص (الكحولي) لبعض الحشرات الطبية
(ذبابة البطل الزرقاء والزنبور الاحمر الشرقي) في تثبط فطري
(*Trichophyton rubrum* , *Candida Spp*)

اسماء الباحثين : عدنان سعد مزاحم ,سعيد ماهر لفته ,سهى ماهر عبدالرشيد
جامعة تكريت / كلية العلوم /قسم علوم الحياة
Adnan.s.muzahem.bio2022235@st.tu.edu.iq/
saeed.m.lafta@tu.edu.iq
dr.suhamaher@tu.edu.iq