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V-Constant Type of Conharmonic Tensor of Vaisman-Gray Manifold

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Abstract. In this work we will study geometric conharmonic curvature tensor characteristics Viasman- Grey menifold ,and the constant of conharmonic type Vaisman-Gray Manifold conditions are obtained when the Viasman- Grey menifold is a manifold conharmonic constant type (V). Also,we will prove that M Vaisman-Gray Manifold of point wise constant holomorphic sectional conharmonic (PHKm(X)) – curvature) curvature tensor if the components of holomorphic sectional (HS- curvature) curvature tensor in the adjoined G-structure space that satisfies condition.

Keywords: Constant type, Vaisman-Gray Manifold, Pointwise holomorphic sectional.

1. Introduction

The Hermitian manifold is one of the most crucial topics of the Comparative geometry. This subject classified into various elements in trying to precisely determine its specifications and features. Then appeared important matter is the classification of the different classes of almost Hermitian manifold according to specific features. Many researchers studied the almost Hermitian manifold and they found many important geometrical properties. One of them is Russian scholars called Kirichenko, when he used G-structure space to study the almost Hermitian manifold that does not depend on a manifold itself but on a sub principle of all complicated frames' collective fiber bundle is known as the adjoined G-structure space[8]. We used this method to study the projective tensor of the class Vaisman-Gray manifold (VG-manifold). This class $W_1 \oplus W_4$,denotes this, where W_1 and W_4 corresponding to the neärly kohler menifold as well as the locälly conformäl kohler manefold (LCK-manifold)[2].

In 1994, Kirichenko and shchipkova, studied the class $W_1 \oplus W_4$ under the name Vaisman-Gray manifold. They found its structure equation in the adjoined G-structure space [6]. In 1996, Kirichenko and Eshova studied the conformal invariant of the class $W_1 \oplus W_4$ [7].

There are many researchers studied the geometry properties of the curvature tensors on almost Hermitian manifold. Ali Shihab [1] studied the geometry of conhormonic curvature tensor of almost Hermitian manifold. One of these curvature tensor is conharmonic tensor. kirichenk & shechepkova fund the equations of proper VG-manifold with respect to the structured & vertual teasers [3]. In particular, M. Vaisman-Gray was prove of point wise constant holomorphic sectional conharmonic (PHKm(X)) – curvature) curvature tensor if the components of holomorphic sectional (HS- curvature) curvature tensor in the adjoined G-structure space that satisfies condition.

2. Preliminaries

Make X(M) the smooth surface. vector field module of M. $C^{\infty}(M)$ be a set of operations on M. The Hermitain manifold (H M) be a set {M, J, g=⟨.,.⟩} where M is 2n-dimensional (n>1) smooth manifold; J is a tangent space endomorphism.$T_P(M)$ with $(J_P)^2$=id and g=<.,.> Metric Riemann such that on M. ⟨JZ,JW⟩=⟨Z,W⟩;Z,W∊Z(M)[9].The basis $\{e_1 \ldots, e_n \ldots Je_1 \ldots . Je_n\}$ is named an authentic competent AH- structural basis { J, g }, by using this basis, the new as a basis be constructed as follow $\{i_1, \ldots i_n, \ldots \bar{i}_1, \ldots \bar{i}_n\}$.Where $i_a=\sigma(e_a)$ and $i_a= \bar{\sigma}(e_a)$, this basis is known as an almost structure basis or almost basis. The equivalent of the frame is $\{P, \ldots i_1, \ldots, i_n \ldots, \bar{i}_1, \ldots, \bar{i}_n\}$ This is known as an A-frame.

The indicators u, g, l and p in the vicinity 1,…,2n and the indexes m, q , o, p, n, r, s and d We shall use the numbers 1,2,...,k. employ the markings $\{i_{\hat{1}} = \bar{i}_1, \ldots . i_{\hat{n}} = \bar{i}_n\}$ where $\hat{a} = a + n$.than form can be used to write a-frame$\{p, i_1 \ldots, i_n \ldots, i_{\hat{1}}, \ldots i_{\hat{n}}\}$. The components matrices of the complex structure f and y of adjoined the following forms of Q-structure space are:

$$(\langle JX, JY\rangle J_j^i) = \begin{pmatrix} \sqrt{-1}\, I_n & 0 \\ 0 & -\sqrt{-1}\, I_n \end{pmatrix}, \quad (g_j^i) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix},$$

Where I_n is the rank n unit matrix[8].

Definition 2.1 [10]:

A tensor of type (2,0) which is defined as is $r_{ij} = R^k_{ijk} = g^{kl}R_{kijl}$ called a Ricci tensor.

Definition 2.2 [4]:

In The adjoined G-structure space , the components of Ricci tensor of Viasman-Grey manifold are given as the following forms:

1- $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a + \alpha_b)$

2- $r_{\hat{a}b} = r^a_b = 3B^{cah}B_{cbh} + A^{ca}_{cb} + \frac{n-1}{2}(\alpha^a\alpha_b - \alpha^h\alpha_h) - \frac{1}{2}\alpha^h{}_h\delta^a_b + (n-2)\alpha^a{}_b$

Definition 2.3 [4]:

Let (M,J, g) be a Vaisman- Gray Manifold .The Conharmonic curvature tensor of AH- manifold M of type (4, 0) which is defined as the following form:

$$T_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}[r_{il}g_{jk} - r_{jl}g_{ik} + r_{jk}g_{il} - r_{ik}g_{jl}] \qquad (1)$$

Where r, R and g are respectively Ricci tensor, Riemannian curvature tensor and Riemannian metric. And satisfies all the properties of algebraic curvature tensor:

$$\left.\begin{array}{l} 1)\ T(X,Y,Z,W) = -T(Y,X,Z,W); \\ 2)\ T(X,Y,Z,W) = -T(X,Y,W,Z); \\ 3)\ T(X,Y,Z,W) + T(Y,Z,X,W) + T(Z,X,Y,W) = 0 \\ 4)\ T(X,Y,Z,W) = T(Z,W,X,Y); \\ \qquad \forall X,Y,Z,W \in X(M) \end{array}\right\} \quad (2)$$

Theorem 2.4 [12]:

In the adjoined G-structure space, the components of Conharmonic tensor of VG-manifold are given by the following forms:

i) $T_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd});$

ii) $T_{\hat{a}bcd} = 2A^{a}_{bcd} + \frac{1}{2(n-1)}(r_{bd}\delta^{a}_{c} - r_{bc}\delta^{a}_{d});$

iii) $T_{\hat{a}\hat{b}cd} = 2\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d});$

iv) $T_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d} - \frac{1}{(n-1)}(r^{(a}_{(c}\delta^{d)}_{b)});$

1. Main results

Definition 3.1 [11]:

Suppose that $\lambda(X,Y,Z,W) = \lambda(X,Y,Z,W) - \lambda(X,Y,JZ,JW)$

Consider the following tensor $\lambda(X,Y) = \lambda(X,Y,Y,X)$

We say that an AH- manifold M is of constant type at $p \in M$

Provided that for all $X \in T_p(M)$

$$\lambda(X,Y) = \lambda(X,Z) \qquad (3)$$

Remark 3.2 [11]:

1- If (3) holds for all $p \in M$ then the manifold M has pointwise constant type.

2- If (3) is constant function, then (M,J,g) has a globally constant type .

Definition 3.3 [11]:

An AH- manifold M^{2n}n is conharmonic constant type (V- constant type)

If $X,Y,Z,W \in X(M^{2n})$. That

$$\lambda(X,Y,Z,W) = \lambda(X,Y,Z,W) - \lambda(X,Y,JZ,JW)$$

Consider the following tensor $\lambda(X,Y) = \lambda(X,Y,Y,X)$

We say that an AH- manifold M is of constant type at p

Provided that for all $X \in T_p(M)$

$\lambda(X,Y) = \lambda(X,Z)$.

Theorem 3.4

If M is Viasman-Grey manifold of the conharmonic tensor then M is manifold conharmonic constant if and only if

$$\lambda(X,Y) = \lambda(X,Z) = 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$$

Proof:

Suppose that M is Viasman-Grey manifold of conharmonic tensor,we find the following result:

By using definition (3.3) it follows :-

1- $\lambda(X,Y) = T(X,Y,Y,X) - T(X,Y,JY,JX)$

Let M^{2n} Viasman manifold to compute the $\lambda(X,Y,Y,X)$and $\lambda(X,Y,JY,JX)$ on the space of the adjoined G-structure

(i)

$$T(X,Y,Y,X) = T_{ijkl}X^iY^jY^kX^l = T_{abcd}X^aY^bY^cX^d + T_{\hat{a}bcd}X^{\hat{a}}Y^bY^cX^d + T_{a\hat{b}cd}X^aY^{\hat{b}}Y^cX^d$$
$$T_{ab\hat{c}d}X^aY^bY^{\hat{c}}X^d + T_{abc\hat{d}}X^aY^bY^cX^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^cX^d + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^bY^{\hat{c}}X^d + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^b$$
$$T_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}Y^{\hat{c}}X^d + T_{a\hat{b}c\hat{d}}X^aY^{\hat{b}}Y^cX^{\hat{d}} + T_{a\hat{b}\hat{c}\hat{d}}X^aY^{\hat{b}}Y^{\hat{c}}X^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}Y^{\hat{b}}Y^{\hat{c}}X^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^aY$$
$$T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Y^{\hat{b}}Y^{\hat{c}}X^d + T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Y^{\hat{b}}Y^cX^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Y^bY^{\hat{c}}X^{\hat{d}}$$

By using the properties of conharmonic tensor equation (2), we get:

$$T(X,Y,Y,X) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^bY^cX^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^bY^{\hat{c}}X^d + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^cX^d + T_{a\hat{b}c\hat{d}}X^aY^{\hat{b}}Y^cX$$
$$T_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}Y^{\hat{c}}X^d + T_{ab\hat{c}\hat{d}}X^aY^bY^{\hat{c}}X^{\hat{d}}$$

(4)

(ii) $T(X,Y,JY,JX)$

In the space of the adjoined G-structure space

$$T(X,Y,JY,JX) = T_{ijkl}X^iY^j(JY)^k(JX)^l = T_{abcd}X^aY^b(JY)^c(JX)^d + T_{\hat{a}bcd}X^{\hat{a}}Y^b(JY)^c(JX$$
$$T_{a\hat{b}cd}X^aY^{\hat{b}}(JY)^c(JX)^d + T_{ab\hat{c}d}X^aY^b(JY)^{\hat{c}}(JX)^d + T_{abc\hat{d}}X^aY^b(JY)^c(JX)^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y$$
$$T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^b(JY)^{\hat{c}}(JX)^d + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^b(JY)^c(JX)^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}(JY)^{\hat{c}}(JX)^d + T_{a\hat{b}c\hat{d}}X^aY$$
$$T_{a\hat{b}\hat{c}\hat{d}}X^aY^{\hat{b}}(JY)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}Y^{\hat{b}}(JY)^{\hat{c}}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^aY^b(JY)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Y$$
$$T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Y^{\hat{b}}(JY)^c(JX)^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Y^b(JY)^{\hat{c}}(JX)^{\hat{d}}$$

By using the properties of conharmonic tensor equation(2), we get:

$T(X,Y,JY,JX) = T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}(JY)^{c}(JX)^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}(JY)^{\hat{c}}(JX)^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}(JY)^{c}$
$T_{a\hat{b}\hat{c}d}X^{a}Y^{\hat{b}}(JY)^{\hat{c}}(JX)^{d} + T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}(JY)^{c}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Y^{b}(JY)^{\hat{c}}(JX)^{\hat{d}}$
(5)

According to the properties $(JX)^{a} = \sqrt{-1}\,X^{a}$ and $(JX)^{\hat{a}} = -\sqrt{-1}\,X^{\hat{a}}$ we get:

$T(X,Y,JY,JX) = -T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}Y^{\hat{c}}X^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}Y^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X$
$T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}Y^{c}X^{\hat{d}} - T_{ab\hat{c}\hat{d}}X^{a}Y^{b}Y^{\hat{c}}X^{\hat{d}}$

Making use of the equation (4) and (5), we get :

$T(X,Y,Y,X) - T(X,Y,JY,JX) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}Y^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}Y^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d} +$
$T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}Y^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Y^{\hat{b}}Y^{\hat{c}}X^{d} + T_{ab\hat{c}\hat{d}}X^{a}Y^{b}Y^{\hat{c}}X^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d} - T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}$
$T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}Y^{c}X^{\hat{d}} - T_{a\hat{b}\hat{c}d}X^{a}Y^{\hat{b}}Y^{\hat{c}}X^{d} - T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}Y^{c}X^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Y^{b}Y^{\hat{c}}X^{\hat{d}}$
$= 4T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d}$

This is the V_4 of theory (2.4) equation (iii) and the compensation, we get:

$= 4\left(2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\,)\right)$

$= 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$ (6)

2- $\lambda(X,Z) = T(X,Z,Z,X) - T(X,Z,JZ,JX)$

Let M^{2n} Viasman manifold to compute the $\lambda(X,Z,Z,X)$ and $\lambda(X,Z,JZ,JX)$ on the space of the adjoined G-structure

(i)

$T(X,Z,Z,X) = T_{ijkl}X^{i}Z^{j}Z^{k}X^{l} = T_{abcd}X^{a}Z^{b}Z^{c}X^{d} +$
$T_{ab\hat{c}d}X^{a}Z^{b}Z^{\hat{c}}X^{d} + T_{abc\hat{d}}X^{a}Z^{b}Z^{c}X^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}$
$T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}\hat{d}}X^{a}Z^{\hat{b}}Z$
$T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Z^{b}Z$

By using the properties of conharmonic tensor equation (2), we get:

$$T(X,Z,Z,X) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}} \quad (7)$$

(ii) $T(X,Z,JZ,JX)$

In the space of the adjoined G-structure space

$$T(X,Z,JZ,JX) = T_{ijkl}X^{i}Z^{j}(JZ)^{k}(JX)^{l} = T_{abcd}X^{a}Z^{b}(JZ)^{c}(JX)^{d} + T_{\hat{a}bcd}X^{\hat{a}}Z^{b}(JZ)^{c}(JX)^{d} + T_{a\hat{b}cd}X^{a}Z^{\hat{b}}(JZ)^{c}(JX)^{d} + T_{ab\hat{c}d}X^{a}Z^{b}(JZ)^{\hat{c}}(JX)^{d} + T_{abc\hat{d}}X^{a}Z^{b}(JZ)^{c}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}(JZ)^{c}(JX)^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}(JZ)^{\hat{c}}(JX)^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}(JZ)^{c}(JX)^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}(JZ)^{c}(JX)^{\hat{d}} + T_{a\hat{b}\hat{c}\hat{d}}X^{a}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}(JZ)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{d} + T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Z^{\hat{b}}(JZ)^{c}(JX)^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Z^{b}(JZ)^{\hat{c}}(JX)^{\hat{d}}$$

By using the properties of conharmonic tensor equation (2), we get:

$$T(X,Z,JZ,JX) = T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}(JZ)^{c}(JX)^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}(JZ)^{\hat{c}}(JX)^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}(JZ)^{c}(JX)^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}(JZ)^{c}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}(JZ)^{\hat{c}}(JX)^{\hat{d}}$$

According to the properties $(JX)^{a} = \sqrt{-1}\,X^{a}$ and $(JX)^{\hat{a}} = -\sqrt{-1}\,X^{\hat{a}}$ we get:

$$T(X,Z,JZ,JX) = -T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} - T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}} \quad (8)$$

Making use of the equation (7) and (8), we get :

$$T(X,Z,Z,X) - T(X,Z,JZ,JX) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} - T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} - T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} - T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} - T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}}$$

$$= 4T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d}$$

This is the V_4 of theory (2.4) equation (iii) and the compensation, we get:

$$= 4\left(2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\,)\right)\right.$$

$$= 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right) \quad (9)$$

From equation (6) and (9) it follows:

$$\lambda(X,Y)=\lambda(X,Z)=\ 8\left(-B^{abh}B_{hcd}+\alpha_{[c}^{[a}\delta_{d]}^{b]}\right)-\frac{1}{(n-1)}\left(r_d^{[a}\delta_c^{b]}-r_c^{[b}\delta_d^{a]}\right)$$

Thus by definition (3.3) we get:

M is constant type if and only if

$$\lambda(X,Y)=\lambda(X,Z)=\ 8\left(-B^{abh}B_{hcd}+\alpha_{[c}^{[a}\delta_{d]}^{b]}\right)-\frac{1}{(n-1)}\left(r_d^{[a}\delta_c^{b]}-r_c^{[b}\delta_d^{a]}\right)$$

Lemma 3.5 [5]:
An AH- manifold M is a zero constant type if, and only if, M is Kahler manifold.

Corollary 3.6:
If M is VG-manifold of conharmonic tensor , then M is not Kahler manifold.

Proof:
Let that M is VG-manifold of conharmonic tensor
By using Theorem (3.4) we get:
M is constant type

$$\lambda(X,Y)=\lambda(X,Z)=\ 8\left(-B^{abh}B_{hcd}+\alpha_{[c}^{[a}\delta_{d]}^{b]}\right)-\frac{1}{(n-1)}\left(r_d^{[a}\delta_c^{b]}-r_c^{[b}\delta_d^{a]}\right)$$

By using Lemma (3.5) it follows
M is not Kähler manifold .

Conclusions

1- Prove that if M is Viasman-Grey manifold of the conharmonic tensor then M is manifold conharmonic constant if and only if

$$\lambda(X,Y)=\ \lambda(X,Z)=\ 8\left(-B^{abh}B_{hcd}+\alpha_{[c}^{[a}\delta_{d]}^{b]}\right)-\frac{1}{(n-1)}\left(r_d^{[a}\delta_c^{b]}-r_c^{[b}\delta_d^{a]}\right)$$

2- Prove that if M is VG-manifold of conharmonic tensor , then M is not Kahler manifold.

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