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Some Grill of Nano Topological Space

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Abstract. In this work, we made a concept game of $\mathfrak{S}$ -nano g-open sets “employing the notion of grill nano topological space, or $G(NT_i.\mathfrak{S})$, where $i = \{0.1.2\}$. The relationships between various kinds of games have been researched with the use of numerous figures and propositions while providing similar examples.

Keywords. $\mathfrak{S}Ng$-closed set, $\mathfrak{S}Ng$-open set, $G(NT_0.\mathfrak{S})$, $G(NT_1.\mathfrak{S})$and $G(NT_2.\mathfrak{S})$.

1. Introduction

Choquet [1] studied grill ($\mathfrak{S}$) on a topological space $(X.\tau)$that has already been explored. In [2] a nano topological space was defined using lower, upper, and boundary conditions. In [3] a game was studied and the concepts of grill Ti-space where $i = \{0.1.2\}$ and denoted by $G\ (Ti.X)$. In [4] introduced grill g-open set on the game of the generalized , grill-g-closed set and insert $\mathfrak{S} - g - Ti$-space with $i = \{0.1.2\}$ were examined, and a game $G(T_i.\mathfrak{S})$ was defined. In [5] introduced the game denoted by (G) between “two “players Å and $\mathcal{B}$, the range of options $J_1.J_2.J_3 \dots J_n$ for every Player. These possibilities are referred to as moves.In [6,7] studied a game is defined as alternating when one of the Players Å chooses one of the options $J_1.J_2.J_3. \dots J_n$. Can be chosen by $\mathcal{B}$ when the choices of Å are Known. In alternating games, the player must determine who starts the game. In this paper provided the sorts of games through a given set. The gaining and losing strategy of any player $\mathcal{P}$ in the game G , if $\mathcal{P}$ has a gaining strategy in G denoted by ($\mathcal{P} \hookrightarrow G$). On the other hand, if $\mathcal{P}$doesn't have a gaining strategy denoted by ($\mathcal{P} \not\hookrightarrow G$). if $\mathcal{P}$ has a losing strategy denoted by ($\mathcal{P} \hookleftarrow G$) and if $\mathcal{P}$ doesn't have a losing strategy denoted by ($\mathcal{P} \not\hookleftarrow G$).

2. Preliminaries

Definition 2.1 [2] Let R be an equivalence relation on U known as the "indiscernibility relation," and let U be a non-empty finite set of objects termed the universe. Then different equivalence classes for U are created. It is argued that elements in the same equivalence class are indistinguishable from one another.

- “The approximation space is referred to as the pair $(U.R)$. Let $X \subseteq U$". The set of all objects that can be categorically identified as X with regard to R is the lower approximation of X with respect to R, and it is denoted by "$L_R(X)$. To put it another way, L_R (X)=_ $\cup_{x \in U}\{R(x) : R(x) \subseteq X\}$, where R(x) stands for the equivalence class established by x U.

- According to $U\_R(X)$, "the set of all objects that can possibly be classified as X with respect to R and" it is the upper approximation of X with respect to R. This is,
  $U_R\ (X) = \cup_{x \in U} \{R(x): R\ (x) \cap X \neq \varphi\ \}$
- The collection of all objects that cannot be classified as either X or not-X with regard to R is known as the boundary region of X with respect to R and is indicated by the symbol $B_R(X)$ thus $B_R(X) = U_R\ (X) - L_R\ (X)$. is defined.

Definition 2.2 [2] The set of all objects that may conceivably be categorized as X with respect to R and it, as denoted by $U\_R(X)$, is the upper approximation of X with regard to R. That is, suppose U is a"universe, R be an equivalence relation on U and" $\tau_R\ (X) = \{\varphi. U. L_R\ (X). U_R\ (X). B_R\ (X)\}$where X agree with the following axioms.

- $U. \varphi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in τ_R(X).

Then $\tau_R(X)$ is called the Nano topology on U with respect to X. The space $(U. \tau_R(X))$ is the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets.

Definition 2.3 [1,8] A nonempty collection $\mathfrak{S}$ of nonempty subsets of a topological space $\mathfrak{X}$ is named a grill if

- $A\ \in \mathfrak{S}\ and\ A\ \subseteq\ B \subseteq \mathfrak{X}$ then $B \in \mathfrak{S}$.
- $A\ .B \subseteq \mathfrak{X}$ and $A\ \cup\ B \in \mathfrak{S}$ then $A\ \in \mathfrak{S}\ or\ B \in \mathfrak{S}$ [6].

Let $\mathfrak{X}$ be a nonempty set. Then the following families are grills on $\mathfrak{X}$. [1,67]

Definition 2.4 [2] In space $(\mathfrak{X}, T, \mathfrak{S})$, let $D \subseteq \mathfrak{X}$. D is named to be grill-g- closed set denoted by " $\mathfrak{S}$ - g-closed", if (D- U) $\notin$ $\mathfrak{S}$ then, $(cl(D)\ -\ U) \notin \mathfrak{S}$ where, $U \subseteq \mathfrak{X}$ and $U \in t$ Now, D^c is a grill-g- open set denoted by $\mathfrak{S}$ -g-open". The family of all " $\mathfrak{S}$ -g –closed" sets denoted by $\mathfrak{S}gC(\mathfrak{X})$. The family of all " $\mathfrak{S}$ -g-open" sets denoted by $\mathfrak{S}\ g$O $(\mathfrak{X})$

Definition 2.5 [4] The space $(\mathfrak{X}, T, \mathfrak{S})$is a $\mathfrak{S}$ -g-$\mathcal{T}_0$-space shortly $\mathfrak{S}$ -g-$\mathcal{T}_0$-space" if for each $m \neq \mathfrak{o}$ and $m. \mathfrak{o}\ \in X$, there exist $U \in\ \mathfrak{S}\ gO(\mathfrak{X}\ )$ whenever, $m\ \in U$ and $\mathfrak{o} \notin U$ or $m \notin U$ and $\mathfrak{o}\ \in U$.

Definition 2.6 [4] The space $(\mathfrak{X}. T. \mathfrak{S})$is a $\mathfrak{S}$ g $\mathcal{T}_1$,-space shortly $\mathfrak{S}$ g-$\mathcal{T}_1$-space" if for each m, $\mathfrak{o} \in X$ and $m \neq \mathfrak{o}$. Then there are $\mathfrak{S}$ g-open sets U_1, U_2 whenever $m \in U_1$, $\mathfrak{o} \notin U_1$, and $\mathfrak{o} \in U_2$, $m \notin U_2$.

Definition 2.7 [4] The space $(\mathfrak{X}. T. \mathfrak{S})$ is a $\mathfrak{S}g$ $\mathcal{T}_2$-space shortly " $\mathfrak{S}$ g $\mathcal{T}_2$-space" if for each $m \neq \mathfrak{o}$. There are $\mathfrak{S}$ g-open sets U_1, U_2 whenever me $m \in U_1$, $\mathfrak{o} \in U_2$, $U_1 \cap U_2$ =∅

3. Grill Nano g-open –on Game

Definition 3.1 Let $(U,\tau_R(x),\mathfrak{S})$ be grill Nano topological space and $E \subseteq U$,E is called "grill Nano -g-closed set" denoted by $\mathfrak{S}Ng$-closed if $(E-G) \notin \mathfrak{S}$ thans $(CL(E)-G) \notin \mathfrak{S}$ where $G \subseteq U$ and ,$G \in \tau_R(x).E^c$ as "grill Nano -g-open set denoted by $\mathfrak{S}Ng$ –open" . The family of all "grill Nano -g-closed set denoted by $\mathfrak{S}NgC(U)$ .The family of all "grill Nano -g-open set" denoted by $\mathfrak{S}NgO(U)$.

Example 3.2 Let (U,$\tau_R(x).\mathfrak{S}$) be grill Nano topological space
$U=\{a_1 . a_2 . a_3\}$. U /R= $\{\{a_1\}. \{a_1 . a_3\}\}$ $X=\{a_1 . a_3\} \subseteq U$.
$\tau_R(x)=\{\emptyset . U. \{a_1\}. \{a_2\} \quad \{a_1 . a_2\}\}$
$\tau_R(x)-closed \ =\{\emptyset . U. \{a_2 . a_3\}. \quad \{a_1 . a_3\}. \{a_3\}\}$
$\mathfrak{S}=\{U. \{a_1\}. \{a_1 . a_2\}. \{a_1 , a_3\}\}$
Then $\mathfrak{S}NgC(U)$)= P(X) / $\{\emptyset\}$ and $\mathfrak{S}NgC(U)$ is $(\mathfrak{S}NgO(U))^c$

Remark 3.3 For any $(U,\tau_R(x).\mathfrak{S})$ then

- Each Nano closed set is a $\mathfrak{S}Ng$- closed set
- Each Nano open set is a $\mathfrak{S}Ng$-open set.

Convers above Remark is not true. Shows from exam 3.2

- $\{a_1\}$ is $\mathfrak{S}NgC$ but $\{a_1\}$ is not Nano closed set.
- $\{a_1 . a_3\}$ is $\mathfrak{S}NgO$ set but $\{a_1 . a_3\}$ is not Nano open.

Definition 3.4 Let $(U,\tau_R(x).\mathfrak{S})$ be grill Nano $g-T_0$space denoted by $\mathfrak{S}Ng-T_0$-space if for every i≠j and i≠j∈ $U . \exists\, G \in \ \mathfrak{S}NgO(U)$ whenever $i \in G and\ j \ \notin G\ or\ i \notin G and\ j \ \in\ G$.

Definition 3.5 Let $(U,\tau_R(x).\mathfrak{S})$be grill Nano $g-T_1$space denoted by $\mathfrak{S}Ng-T_1$-space if for every i≠j and i.j∈ $U . \exists\, \mathfrak{S}NgO\ sets\ G_1 . G_2$ whenever $i \in G_1 and\ j \ \notin G_1\ and\ i \notin G_2 . j \ \in\ G_2$

Definition3.6 Let $(U,\tau_R(x).\mathfrak{S})$ be grill Nano $g-T_2$ space denoted by $\mathfrak{S}Ng-T_2$-space if for every i≠j then are $\mathfrak{S}NgO\ sets\ G_1 . G_2$ whenever $i \in G_1 and\ j \ \in G_2\ and\ G_1 \cap G_2=\emptyset$.

Definition 3.7 Let $(U,\tau_R(x),\mathfrak{S})$ be a grill Nano topological space, $G(NT_0.\mathfrak{S})$ is a game that is defined as follows :In the m-th inning, the two players A and B will play an inning for each natural number., the prime race, A will select $a_m \neq b_m$, whenever a_m, b_m belong to U. Next B choose NG_m belong to $\mathfrak{S}NgO(U)$ such that a_m belong to N$G_m$and b_m, not belong to NG_m, B get in the game, whenever $\mathcal{B}$ ={ NG_1, NG_2, ..., NG_m...} satisfies that for all $a_m \neq b_m$in U ∃ NG_m belong to P such that a_m belong to NG_m and $b_m \notin$ NG_m. Other hand A gets.

Example 3.8 Let $G(NT_0.\mathfrak{S})$ be a game $U=\{a_1 . a_2 . a_3\}$ and $U/R=\{\{a_2\}. \{a_1 . a_3\}\}$
$X=\{a_1 . a_2\} \subseteq U$ then $\tau_R(x)=\{\emptyset . U, \{a_2\}. \{a_1 . a_3\}\}$
$\tau_R(x)$ –closed= $\{\emptyset . U, . \{a_1 . a_3\}. \{a_2\}\}$, $\mathfrak{S}=\{U, \{a_1\}. \{a_1 . a_2\}. \{a_1 . a_3\}\}$

- Then $\mathfrak{S}NgC(U)=\{U, \emptyset . \{a_1\}. \{a_2\}. \{a_1 , a_2\}. \{a_1 . a_3\}\}$
- $\mathfrak{S}NgO(U)=\{\emptyset . U, \{a_2 . a_3\}. \{a_1 . a_3\}. \{a_3\}. \{a_2\}\}$

Then in the first race A shall choose $a_1 \neq a_2$ whenever $a_1 . a_2 \in U$ following B choose $\{a_2 . a_3\} \in \mathfrak{S}NgO(U)$ such that $a_2 \in \{a_2 . a_3\}$ and $a_1 \notin \{a_2 . a_3\}$in the other race A shall choose $a_1 \neq a_3$whenever $a_1 . a_3 \in U$ following B choose $\{a_2 . a_3\} \in \mathfrak{S}NgO(U)$such that $a_2 \in \{a_2 . a_3\}$ and $a_1 \notin \{a_2 . a_3\}$in the tertiary race , A shall choose $a_2 \neq a_3$whenever $a_2 . a_3 \in U$ following B choose $\{a_1 . a_3\} \in \mathfrak{S}NgO(U)$ such that $a_3 \in \{a_1 . a_3\}$and $a_2 \notin \{a_1 . a_3\}$ B get in the game ,whenever $\mathcal{B} = \{\{a_2 . a_3\} . \{a_2 . a_3\}\}$ satisfies that for all $a_m \neq b_m$ in U $\exists\, NG_m \in \mathcal{B}$ such that $a_m \in NG_m$ and $b_m \notin NG_m$ whenever $NG_m \in \mathfrak{S}NgO(U)$ so B is the getter of the game .

Theorem 3.9 Let $(U,\tau_R(x),\mathfrak{S})$ be $\mathfrak{S}NgT_0$- space if and only if B $\hookrightarrow G(NT_0 . \mathfrak{S})$
Proof. since $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_0$- space, then any choice for the primary player A in the m-th inning $a_m \neq b_m$ whenever $a_m . a_m \in U$. The other it is possible to locate player B. $NG_m \in \mathfrak{S}NgO(U)$ so $\mathcal{B} =\{ NG_1 . NG_2 \ldots\ldots NG_m \ldots\}$is the gaining strategy for B . Contrary to lucid .

Theorem 3.10 The space$(U,\tau_R(x),\mathfrak{S})$ is a $GNgT_0$- space if and only if. there is a $\mathfrak{S}Ng - closed$ set containing only one of the items $a \neq b$.
Proof. Suppose that two points are a and b. belong to U with $a \neq b$ since U is $\mathfrak{S}NgT_0$- space $.\exists G\ is\ a\ \mathfrak{S}Ng - open$ set contain only one of them ,therefore $(U - G)$ is $\mathfrak{S}Ng - closed$ set contain the other one .Contrary to Suppose that a and b are two points belong to U with $a \neq b$ $.\exists H\ is\ a\ \mathfrak{S}Ng - closed$ set contain only one of them ,therefore $(U - H)$is $\mathfrak{S}Ng - open$ set contain the other one .

Corollary 3.11 Let $(U,\tau_R(x).\mathfrak{S})$ be a grill Nano topological space, B $\hookrightarrow G(NT_0 . \mathfrak{S})$ if and only if, for each $a \neq b$ of U $\exists H \in \mathfrak{S}NgO(U)$ such that $a \in H\ and\ b \notin H.$
Proof. Suppose that $a \neq b$ with $a . b \in U$.since B $\hookrightarrow G(NT_0 . \mathfrak{S})$ then by Theorem 3.9 the space$(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_0$- space therefore theorem 3.10 is applicable. Contrary to: by theorem 3.10 the grill Nano topological-space $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_0$- space, therefore Theorem 3.9 is applicable.

Theorem 3.12 $(U,\tau_R(x),\mathfrak{S})$ be not a $\mathfrak{S}NgT_0$- space iff $A \hookrightarrow G(NT_0, \mathfrak{S})$.
Proof. Of the m-th race A of $G(NT_0 . \mathfrak{S})$ choose $a_m \neq b_m$ whenever $a_m . b_m \in U$. B of $G(NT_0 . \mathfrak{S})$ cannot be founder $G_m\ is\ a\ \mathfrak{S}Ng - open\ set$ contain only one point of them, because $(U,\tau_R(x).\mathfrak{S})$ be not a $\mathfrak{S}NgT_0$- space then $A \hookrightarrow G(NT_0 . \mathfrak{S})$ Contrary to lucid .

Definition 3.13 Let $(U,\tau_R(x).\mathfrak{S})$ be a grill Nano "topological space", and describe the game $G(NT_1 . \mathfrak{S})$ as follows: the two players A and B compete in a race for all natural numbers, with the m-th race, the prime round, being the most difficult. A shill picks $a_m \neq b_m$, whenever $a_m . b_m$. belong to U. Therefore B choose $G_m\ and\ H_m$, belong to $\mathfrak{S}NgO(U)$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$,B get in the game whenever $\mathcal{B} =\{\{G_1 - H_1\}, \{G_2 - H_2\},\ldots, \{G_m - H_m\},\ldots\}$ satisfies that for all $a_m \neq b_m$ of U $\exists \{G_m . H_m\} \in \mathcal{B}$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$, other hand A get .

Example 3.14 From Example 3.8
$\mathfrak{S}NgC(U) = \{U, \emptyset . \{a_1\}, \{a_2\}, \{a_1 . a_2\}, \{a_1 . a_3\}\}$

$\mathfrak{S}NgO(U) = \{ \emptyset, U, \{a_2 . a_3\}, \{a_1 . a_3\}, \{a_3\}, \{a_2\}\}$

Then in the prime race A shall choose $a_1 \neq a_2$ whenever a_1 and $a_2 \in U$ therfore B choose $\{a_2 . a_3\}$and$\{a_1 . a_3\} \in \mathfrak{S}NgO(U))$ such that $a_1 \in (\{a_1 . a_3\}$-$\{a_2 . a_3\})$ and $a_2 \in (\{a_2 . a_3\}$-$\{a_1 . a_3\})$in the other race A shall choose $a_1 \neq a_3$whenever a_1 and $a_3 \in U$ therfore B can't find $G_m . H_m \in \mathfrak{S}NgO(U)$, such that $a_1 \in (G_m - H_m)$ and $a_3 \in (H_m - G_m)$ then A get in the game .

Theorem 3.15 $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space if and only if $B \hookrightarrow G(NT_0.\mathfrak{S})$.

Proof. Suppose that $(U,\tau_R(x).\mathfrak{S})$ be a grill Nano topological space in the prime run A shall select $a_1 \neq b_1$ whenever a_1 and $b_1 \in U$,therefore ,since $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space B can be founder $G_1 . H_1 \in \mathfrak{S}NgO(U)$ such that $a_1 \in (G_1 - H_1)$ and $b_1 \in (H_1 - G_1)$ in the other race A shall choose $a_2 \neq b_2$whenever a_2 and $b_2 \in U$ therefore can be founder $G_2 . H_2 \in \mathfrak{S}NgO(U)$ such that $a_2 \in (G_2 - H_2)$ and $b_2 \in (H_2 - G_2)$ in the m-th race , A shall choose $a_m \neq b_m$whenever a_m and $b_m \in U$ therefore B can be founder $G_m . H_m \in \mathfrak{S}NgO(U)$ such that $a_m \in (G_m - H_m)$ and $b_m \in (H_m - G_m)$. So $\mathcal{B}$ =$\{\{G_1 . H_1\}$, $\{G_2 . H_2\}$,..., $\{G_m . H_m\}$,...} is the gaining strategy for B. Contrary to lucid

Theorem 3.16 $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space if and only if for every point $a \neq b$ $\exists$ two $\mathfrak{S}Ng - closed$ sets $K_1 and$ K_2 such that $a \in (K_1 - K_2)$ and $b \in (K_2 - K_1)$.

Proof Suppose that a and b are two points of U with $a \neq b$ since U is a $\mathfrak{S}NgT_1$- space then $\exists$ $G_1 and$ G_2 are $\mathfrak{S}Ng - open$ sets such that $a \in (G_1 - G_2)$ and $b \in (G_2 - G_1)$.then $\exists$ $\mathfrak{S}Ng - closed$ sets $(U - G_1)$ and $(U - G_2)$, such that $a \in \{(U - G_2) -(U - G_1)\}$ and $b \in \{(U - G_1) -(U - G_2)\}$ whenever $(U - G_2) = K_1$ and $(U - G_1) = K_2$. Then $\exists$ two $\mathfrak{S}Ng - closed$ sets $(K_1 and$ $K_2)$ satisfy $a \in (K_1 \cap K_2^c)$ and $b \in (K_2 \cap K_1^c)$ then $a \in (K_1 - K_2)$ and $b \in (K_2 - K_1)$. contrary to suppose that a and b are two points of U with $a \neq b$ $\exists$ two $\mathfrak{S}Ng - closed$ sets $K_1 and$ K_2 satisfy $a \in (K_1 \cap K_2^c)$ and $b \in (K_2 \cap K_1^c)$ then $\exists \mathfrak{S}NgO(U - K_1)$ and $(U - K_2)$ whenever a $\in \{(U - K_2) -(U - K_1)\}$ and $b \in \{(U - K_1) -(U - K_2)\}$ whenever $(U - K_2) = G_1$and $(U - K_1) = G_2$.

Corollary3.17 Let $(U,\tau_R(x).\mathfrak{S})$ be space, B $\hookrightarrow G(NT_1.\mathfrak{S})$ if and only if for each $a_1 \neq a_2$ of U $K_2, K_1 \in \mathfrak{S}NgO(U)$, such that $a_1 \in (K_1 - K_2)$ and $a_2 \in (K_2 - K_1)$.

Proof. suppose that $a_1 \neq b_1$ with $a_1 . b_1 \in U$ since B $\hookrightarrow G(NT_1.\mathfrak{S})$ so by-theorem 3.15 the space $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space. So, Theorem 3.16 is, applicable. contrary to Theorem 3.16 the grill Nano topological-space $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space so theorem 3.15 is, applicable.

Definition 3.18 Let $(U,\tau_R(x).\mathfrak{S})$ be a grill Nano topological space, $G(NT_2.\mathfrak{S})$ is a game defined as follows: In the m-th race, the prime round, the two players A and B compete in a race for each natural number. A shill picks $a_m \neq b_m$, whenever $a_m . b_m$.belong to U. Therefore B choose disjoin G_m and H_m.$\boldsymbol{i.e}G_m \cap H_m = \varphi$ belong to $\mathfrak{S}NgO(U)$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$,B get in the game whenever $\mathcal{B}$ =$\{\{G_1 - H_1\}$, $\{G_2 - H_2\}$,..., $\{G_m - H_m\}$,...} satisfies that for all $a_m \neq b_m$ of U $\exists\{G_m . H_m\} \in \mathcal{B}$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$, other hand A get .

Example 3.19 From Example 3.8

$\mathfrak{S}NgC(U) = \{U, \emptyset, \{a_1\}, \{a_2\}, \{a_1 . a_2\}, \{a_1 . a_3\}\}$

$\mathfrak{S}NgO(U) = \{ \emptyset. U, \{a_{2\,.}\, a_3 \}, \{a_{1\,.}\, a_3 \}, \{a_3 \}, \{ a_2 \}\}$

Then in the prime race A shall choose $a_1 \neq a_2$ whenever a_1 and $a_2 \in U$ therfore B can't find two disjoin $\mathfrak{S}NgO(U)$ with $a \in G_m$, $b \in H_m$ i.e., $G_m \cap H_m = \emptyset$ then A is get in the game.

Theorem 3.20 A space $(U,\tau_R(x).\mathfrak{S})$ is $\mathfrak{S}NgT_2$- space if and only if B $\hookrightarrow G(NT_2.\mathfrak{S})$.
Proof. Suppose that $(U,\tau_R(x).\mathfrak{S})$ a grill Nano topological space in the prime race A shall choose $a_1 \neq b_1$ whenever a_1 and $b_1 \in U$ therefore .Since$(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_2$- space then B can be found G_1 and $K_1 \in \mathfrak{S}NgO(U)$ such that $a_1 \in G_1$ and $b_1 \in K_1$.$G_1 \cap K_1 = \emptyset$ in the other race A shall choose $a_2 \neq b_2$,whenever $a_2$$and$ $b_2 \in U$ .Therefore B choose G_2 , $K_2 \in \mathfrak{S}NgO(U)$ such that $a_2 \in G_2$ and $b_2 \in K_2$, $G_2 \cap K_2 = \emptyset$ in the m-th race A shall choose $a_m \neq b_m$ whenever a_m and $b_m \in U$, therefore B choose G_m , $K_m \in \mathfrak{S}NgO(U)$ such that $a_m \in G_m$ and $b_m \in K_m$, $G_m \cap K_m = \emptyset$. So $\mathcal{B} = \{\{G_1 , K_1\},\{G_2 , K_2\},\dots\dots\dots.\{G_m , K_m\}\dots\dots\}$. Is the winning strategy for B. Contrary to is Lucid.

Corollary 3.21 A space $(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_2$- space if and only if A $\nrightarrow G(NT_2.\mathfrak{S})$.
Proof. From theorem3.20 the proof is lucid.

Theorem3.22 A space $(U,\tau_R(x).\mathfrak{S})$ is not a $\mathfrak{S}NgT_2$- space iff A $\hookrightarrow G(NT_2.\mathfrak{S})$.
Proof: by corollary3.21 the proof is lucid

Theorem 3.23 A space $(U,\tau_R(x),\mathfrak{S})$ is not a $\mathfrak{S}NgT_2$- space if and only if B $\nrightarrow G(NT_2.\mathfrak{S})$.
Proof. by theorem3.23 the proof is lucid

Remark 3.24 For any space $(U,\tau_R(x).\mathfrak{S})$:

- If B $\hookrightarrow G(NT_{i+1}.\mathfrak{S})$ then B $\hookrightarrow G(NT_i.\mathfrak{S})$ where $i = 0.1$
- If B $\hookrightarrow G(NT_i\,.\mathfrak{S})$ then B $\hookrightarrow G(NT_i,\mathfrak{S})$ where $i = 0.1$

The relationships described in the Remark 3. 34 are made clearer by **Figure 1** that follows.

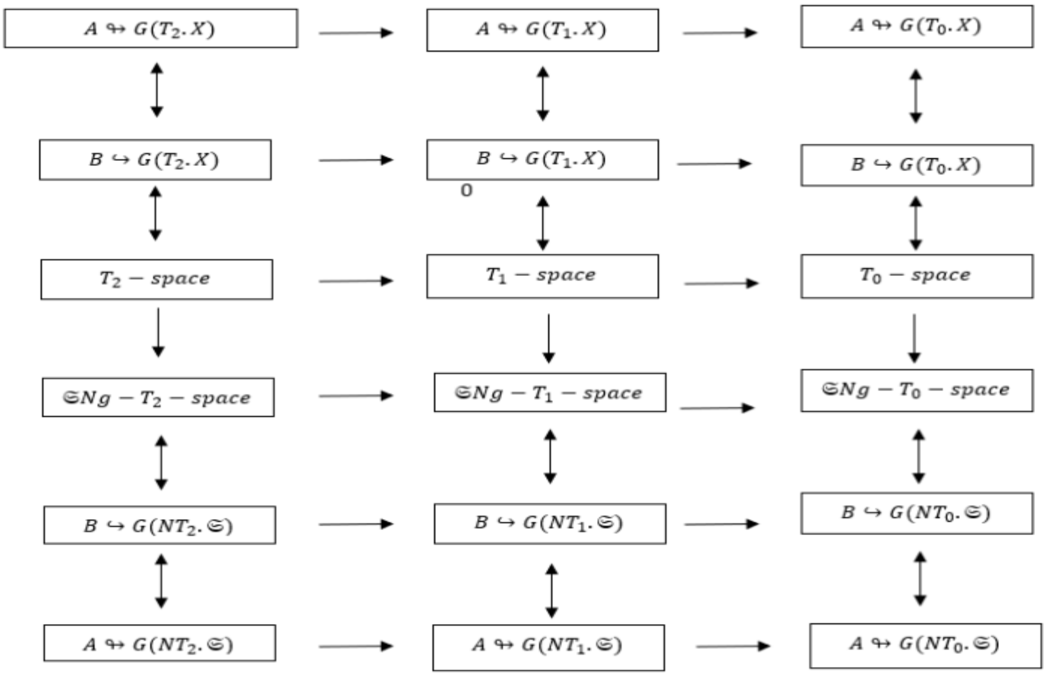

Figure 1. The relationships described in the Remark 3. 24

Any player's winning and losing tactics in in $G(T_i.X)$ and $G(T_i.\mathfrak{S})$.

Remark 3.25 For any space $(X.t.\mathfrak{S})$:

- If $A \hookrightarrow G(T_i.\mathfrak{S})$then $A \hookrightarrow G(T_{1+i}.\mathfrak{S})$, whenever $i = \{0.1\}$
- If $B \looparrowright G(T_i.\mathfrak{S})$ then $B \looparrowright G(T_{1+i}.\mathfrak{S})$, whenever $i = \{0.1\}$.
- If $A \hookrightarrow G(T_i.\mathfrak{S})$then $A \hookrightarrow G(T_i.X)$, whenever $i = \{0.1.2\}$.

The relationships described in the Remark 3. 25 are made clearer by **Figure 2** that follows.

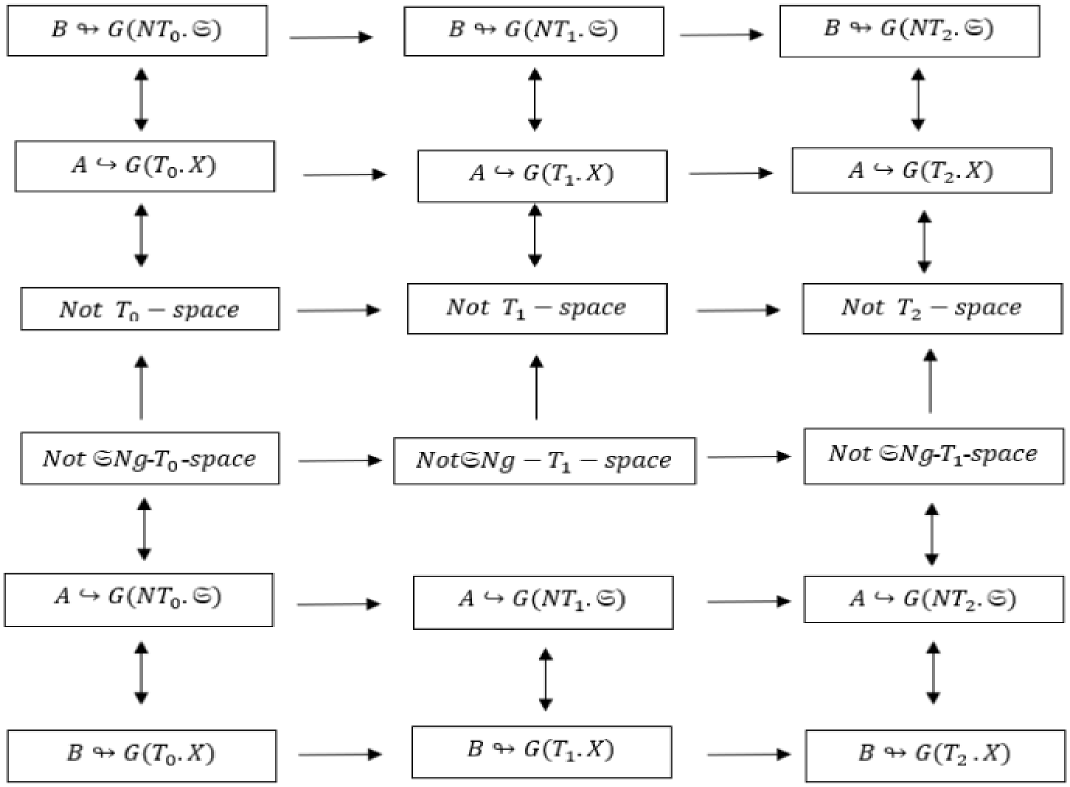

Figure 2. The relationships described in the Remark 3. 25

The winning and losing strategy whenever X is not $\mathfrak{S}Ng - T_i - space$ and not $T_i - space$

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