A NEW TYPE OF NON-ADJACENCY TOPOLOGICAL SPACES ON UNDIRECTED GRAPHS

Zahraa Lateef Hadi [1*] , **Asmhan Flieh Hassan [2]**

[1,2]Department of Mathematics, faculty of education for girls, University of Kufa, Iraq

[*]Corresponding author:

[1]E-mail: zahraal.alshamarti@student.uokufa.edu.iq
[2]E-mail: asmhanf.alzuhairy@uokufa.edu.iq

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ABSTRACT:

In this work, we associate a topology to undirected graph $G = (V, E)$, we named it non-adjacency- $P3$-topology (briefly $NP3$ - topology) of undirected graph G. A sub-basis family will generate the non-adjacency - $P3$ – topology, and it is introduced on the set of vertices V. Then we investigate some properties and discuss it on certain few important types of undirected graphs. Our motivation is to give fundamental steps toward investigation of some properties of simple undirected graphs by their corresponding $NP3$ - topology.

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Key words and phrases*:* undirected graphs, $NP3$ - topology, $NP3$ -topological spaces.

1. Introduction

Numerous applications have utilized the relationship between topology and graph to generate many new types of a topology generated by graph, because of the importance of topological graph theory as It is part of graph theory that has a great role and illustrious history in mathematics. The sources [1-4] include several great introductions to graph theory, topological graph theory, and a few applications.

On the basis of vertices or edges, some topological models are developed or based. In the undirected graphs and directed graphs. In 2013, Jafarian et al. began using a graphic topology for any locally finite without isolated vertices [5]. 2018 [6] saw kilicman and Abdulkalek define a sub-basis family as a set of vertices associated with an Incidence Topology for any simple graph without an isolated vertex sets containing the edge e of each incident vertex. A revised definition of the term "Family of sub-basis" was introduced in 2020. This definition created independent Topology of any un-digraph via vertices that are not adjacent to the Zainab and Asmhan introduce vertex v [7]. Asmhan and Zainab in 2022, give the topology of independent compatible edges [8], which is describted as the topology connected to the group of edges. Asmhan and Iman authored an Independent in 2022. Digraph-based incompatible Edges topology with specific applications [9], the reader can also see [10,11]. In 2023, non-incidence topology was founded by Asmhan and Jafar, in [12]

Accordingly, in our new paper, we define a new kind of topological spaces associated with simple undirected graph G, which we named it non-adjacency - P_3- topological space. In basic kinds of graphs, we will investigate some of the advantages that are achieved with the non-adjacent -path of length two-topology of the undirected graph.

section 2: involve fundamental definitions of graphs and topologies. Definition of a $NP3$ -topological spaces associated with simple undirected graph G, in the section 3: We will discuss some preliminary result in the fourth section of this paper. In the fourth section of our paper: several main conclusions of $NP3$ -topology are introduced.

2. Preliminaries

Basic definitions and introductions to topology and graph theory are covered in the part. Ideas are all often utilized and can be found in books like [1,13,14].

Typically, a graph consists of two sets, $G = (V, E)$, where V is set of vertices and E is set of edges, an edge of the form $(ʒ) = (v, v)$ is loop. Parallel edges are those with the identical end vertices. If a graph contains no parallel edges or loop, it is considered simple. If the vertex v and u are connected by edge then they are adjacent. All of these ideas are well-known and are available in books mentioned above. We use the symbols K_n for the complete graph with n vertices and C_n is cycle graph on n vertices and P_n is path on n edges and $K_{n1,n2}$ is a whole bipartite graph of size partite $n1$ *and* $n2$.

An open family of subsets to the non-empty set X is said to be a topology if the following conditions are hold: $X, \emptyset \in T$, *for every* $\mathbb{E}, \mathbb{D} \in T$, $\mathbb{E} \cap \mathbb{D} \in T$ and

$\cup_{\alpha \in \Delta} A_\alpha \in T$ for every sub − combination A_α of T, then (X,T) is called a topology on space, an open set is sub set of X. Indiscrete topology is defined as T={Ø,X } on X while discrete topology is def. T=P(X) on X.

Now, the introducing of non-adjacency -P_3- topological spaces (***NP3*** -Topological Space), which associated with undirected graph G.

3. ***NP3*** -Topological Space

This section consists of the definition of a non-adjacency -P_3-topological space (***NP3*** -Topological Space) associated with simple undirected graph G and some examples on basic undirected graphs.

3.1. Definition: Let $G = (V, E)$ be any undirected graph. The ***NP3***-topological space, briefly the topology named as T_{NP3} , which is a topology relates to the vertices set V of vertices for G , and brought on by sub-basis S_{NP3} whose components are the sets $\omega \subseteq V$, $|\omega| \le 2$ s. t. if $v \in \omega$ and w is non-adjacent with v, then $w \in \omega$, i.e, $w \in \omega$ if w is non-adjacent with v and form with v a two-length path.

3.2. Example: Let a graph $G = (V, E)$ be as in figure (1) below s. t. $V(G) = \{v_1, v_2, v_3, v_4\}$, $E(G) = \{ʒ_1, ʒ_2, ʒ_3\}$, by the definition (3.1) of the ***NP3*** - topology (T_{NP3}) above, we can find our new topology via the following;

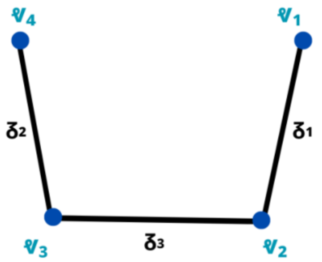

Fig. 3.1.

T_{NP3} has a sub-basis; $S_{NP3} = \{\{v_1, v_3\}, \{v_2, v_4\}\}$. Then by using finite intersection, the following base β_{NP3} is produced $\{\{v_1, v_3\}, \{v_2, v_4\}, \emptyset\}$

Then, utilizing all unions, will generate a topology T_{NP3} as the following:

$\mathrm{T}_{NP3} = \{\emptyset, \mathrm{V}, \{v_1, v_3\}, \{v_2, v_4\}\}$

3.3 Remark: Let T_{NP3} be a topological space on the vertices set V of the un-directed graph C_n s.t. $n = 4$, then T_{NP3} is not discrete topology, but if $n > 4$ then T_{NP3} is discrete, notice the following e.g.

3.4 Example: Consider $G = (\mathrm{V}, E)$ be the cycle C_4 shown in the fig. (2), then:

$\mathrm{V}(G) = \{v_1, v_2, v_3, v_4\}$, $E(G) = \{z_1, z_2, z_3, z_4\}$,

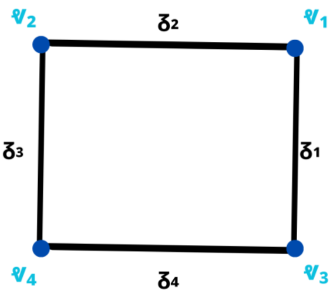

Fig. 3.2.

T_{NP3} has a sub-basis

$S_{NP3} = \{\{v_2, v_3\}, \{v_1, v_4\}\}$. By using finite intersection, the following base β_{NP3} is produced $\{\{v_2, v_3\}, \{v_1, v_4\}, \emptyset\}$

Then, utilizing all union, will generate a topology T_{NP3} as the following:

$\mathrm{T}_{NP3} = \{\emptyset, \mathrm{V}, \{v_2, v_3\}, \{v_1, v_4\}\}$, it is clear that T_{NP3} is not discrete topology.

3.5. Example: Consider $G = (\mathrm{V}, E)$ be the cycle C_5 as shown in the fig. (3), then:

$\mathrm{V}(G) = \{v_1, v_2, v_3, v_4, v_5\}$, $E(G) = \{z_1, z_2, z_3, z_4, z_5\}$,

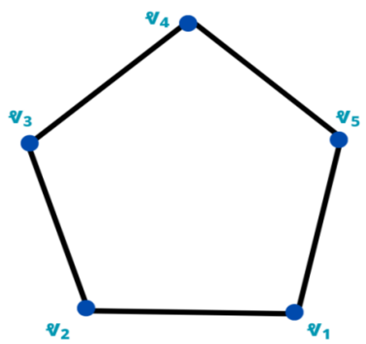

Fig. 3.3

$\mathcal{T}_{NP3}$ has a sub-basis

$S_{NP3} = \{\{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_3, v_5\}\}$. Then using finite intersection, the following base β_{NP3} is produced $\{\emptyset, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_3, v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}\}$

Then, utilizing each unions, will generate a topology $\mathcal{T}_{NP3}$ as the following:

$\mathcal{T}_{NP3} = \{\emptyset, V, \{v_1, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_3, v_5\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\},$ $\{v_1, v_2\}, \{v_2, v_3\}, \{v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_3, v_4\}, \{v_1, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_3\},$

$\{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_2, v_4, v_5\}, \{v_2, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_2, v_3, v_5\},$

$\{v_3, v_4, v_5\}, \{v_1, v_2, v_3, v_4\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_2, v_4, v_5\},$

$\{v_1, v_2, v_3, v_5\}\}$, it is clear that $\mathcal{T}_{NP3}$ is discrete topology.

3.6. Remark: For every undirected path P_n, the topology $\mathcal{T}_{NP3}$ isn't discrete topology as in e.g.3.2.

3.7. Remark: For each undirected tree, we satisfy that $\mathcal{T}_{NP3}$ is discrete topology, notice the following e.g. 3.8.

3.8. Example: Consider $G = (V, E)$ be the undirected tree as shown in the fig.(4), then: $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E(G) = \{z_1, z_2, z_3, z_4, z_5, z_6\}$,

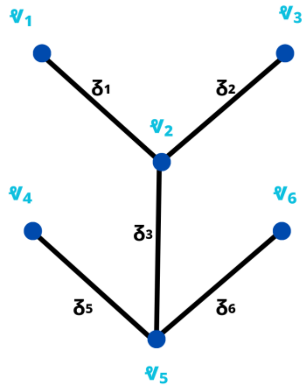

Fig. 3.4.

$\mathcal{T}_{NP3}$ has a sub-basis $S_{NP3} = \{\{v_1, v_3\}, \{v_1, v_5\}, \{v_2, v_6\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_4, v_6\}\}$. Then using finite intersection, the following base β_{NP3} is produced $\{\emptyset, \{v_1, v_3\}, \{v_1, v_5\}, \{v_2, v_6\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_4, v_6\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}\}$

Then, utilizing each unions will generate a topology $\mathcal{T}_{NP3}$ as the following:

$\mathcal{T}_{NP3} = \{\emptyset, V, \{v_1, v_3\}, \{v_1, v_5\}, \{v_2, v_6\}, \{v_2, v_4\}, \{v_3, v_5\}, \{v_4, v_6\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\},$ $\{v_1, v_2\}, \{v_2, v_3\}, \{v_2, v_5\}, \{v_4, v_5\}, \{v_1, v_6\}, \{v_3, v_6\}, \{v_5, v_6\}, \{v_3, v_4\}, \{v_1, v_4\}, \{v_1, v_3, v_4\},$

$\{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_2, v_4, v_5\}, \{v_2, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_2, v_3, v_5\}, \{v_1, v_2, v_3\},$

$\{v_1, v_2, v_6\}, \{v_1, v_3, v_6\}, \{v_2, v_4, v_6\}, \{v_2, v_3, v_6\}, \{v_1, v_4, v_6\}, \{v_2, v_5, v_6\}, \{v_1, v_5, v_6\},$

$\{v_3, v_4, v_6\}, \{v_3, v_5, v_6\}, \{v_1, v_2, v_3, v_6\}, \{v_2, v_3, v_4, v_6\}, \{v_1, v_3, v_4, v_6\}, \{v_1, v_2, v_4, v_6\}$

$\{v_1, v_2, v_5, v_6\}, \{v_2, v_3, v_5, v_6\}, \{v_1, v_3, v_5, v_6\}, \{v_1, v_2, v_5, v_6\}, \{v_3, v_4, v_5, v_6\}$

$\{v_3, v_4, v_5\}, \{v_1, v_3, v_5\}, \{v_1, v_2, v_3, v_4\}, \{v_2, v_3, v_4, v_5\}, \{v_1, v_3, v_4, v_5\}, \{v_1, v_2, v_4, v_5\},$

$\{v_1, v_2, v_3, v_4, v_6\}, \{v_2, v_3, v_4, v_5, v_6\}, \{v_1, v_3, v_4, v_5, v_6\}, \{v_1, v_2, v_4, v_5, v_6\}, \{v_1, v_2, v_3, v_5, v_6\},$

$\{v_1, v_3, v_4, v_5, v_6\}, \{v_1, v_2, v_4, v_5, v_6\}\{v_1, v_2, v_3, v_4, v_5\}, \{v_2, v_3, v_4, v_5, v_6\}\}$

It is clear that Ʈ_{NP3} is a discrete topology.

3.9. Remark: Ʈ_{NP3} be a $\boldsymbol{NP3}$ -topology space with a set of vertices Ɣ of a complete undirected graph K_n is discrete topology for each $n \geq 3$ **,** Notice the following e.g.

3.10. Example: Consider $\text{Ǵ} = (\text{Ɣ}, \text{Ȩ})$ be a complete un-digraph K_3 **,** as shown in the fig. (5), s. t. $\text{Ɣ}(\text{Ǵ}) = \{v_1, v_2, v_3\}$, $\text{Ȩ}(\text{Ǵ}) = \{ʒ_1, ʒ_2, ʒ_3\}$,

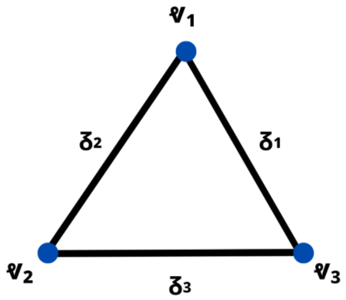

Fig. 3.5

Ʈ_{NP3} has a sub-basis $\text{Ș}_{NP3} = \{\{v_1\}, \{v_2\}, \{v_3\}\}$. Then via using finite intersection, the following base β_{NP3} is produced $\{Ø, \{v_1\}, \{v_2\}, \{v_3\}\}$ **.** Then, utilizing each unions generate a topology Ʈ_{NP3} as the following: $\text{Ʈ}_{NP3} = \{Ø, \text{Ɣ}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_1, v_3\}, \{v_1, v_2\}, \{v_2, v_3\}\}$, clearly a discrete topology.

3.11 Example: Consider $\text{Ǵ} = (\text{Ɣ}, \text{Ȩ})$ be complete un-directed graph K_4 **,** as shown in the fig. (6) s.t. $\text{Ɣ}(\text{Ǵ}) = \{v_1, v_2, v_3, v_4\}$, $\text{Ȩ}(\text{Ǵ}) = \{ʒ_1, ʒ_2, ʒ_3, ʒ_4, ʒ_5, ʒ_6\}$, then;

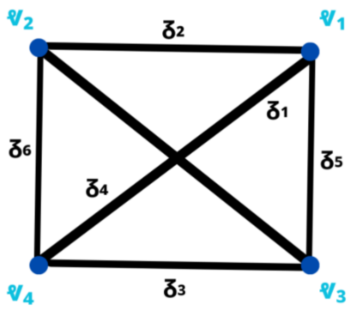

Fig. 3.6

T_{NP3} has a sub-basis $S_{NP3} = \{ \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\} \}$.

Then via using finite intersection, the following base β_{NP3} is produced $\{ \emptyset, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\} \}$.

Then, utilizing each unions will generate a topology T_{NP3} as the following: $\mathrm{T}_{NP3} = \{ \emptyset, V, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_1, v_3\}, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_4\}, \{v_2, v_4\}, \{v_3, v_4\},$

$\{v_1, v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_2, v_3, v_4\} \}$, clearly a discrete topology.

3.13. Remark: T_{NP3} of complete bi-partite undirected graph $K_{n1,n2}$ is discrete topology if $\mathbf{n1} \geq 3$ *and* $\mathbf{n2} \geq \mathbf{3}$, Notice the following example.

3.14. Example: Consider $G = (V, E)$ be a complete bi-partite undirected graph $K_{3,3}$, as shown in the figure (7), s.t. $V(G) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$, $E(G) = \{z_1, z_2, z_3, z_4, z_5, z_6\}$,

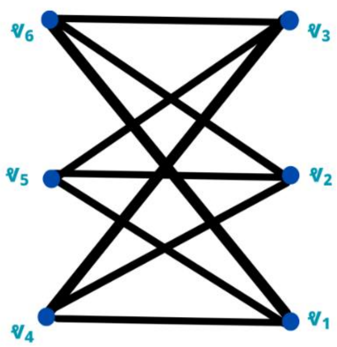

Fig. 3.7

T_{NP3} has a sub-basis

$S_{NP3} = \{ \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}\}$. Then via using finite intersection, the following base β_{NP3} is produced $\{ \emptyset, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\}\}$

Then, utilizing each unions will generate a topology T_{NP3} as the following: $\mathrm{T}_{NP3} = \{ \emptyset, V, \{v_1, v_2\}, \{v_2, v_3\}, \{v_1, v_3\}, \{v_4, v_5\}, \{v_4, v_6\}, \{v_5, v_6\}, \{v_1\}, \{v_2\}, \{v_3\}, \{v_4\}, \{v_5\}, \{v_6\},$ $\{v_1, v_4\}, \{v_2, v_4\}, \{v_2, v_5\}, \{v_2, v_6\}, \{v_3, v_5\}, \{v_3, v_6\}, \{v_3, v_4\}, \{v_1, v_5\}, \{v_1, v_6\},$

$\{v_1, v_3, v_5\}, \{v_1, v_3, v_4\}, \{v_1, v_2, v_3\}, \{v_1, v_2, v_4\}, \{v_1, v_2, v_5\}, \{v_2, v_4, v_5\},$

$\{v_1, v_3, v_6\}, \{v_2, v_3, v_4\}, \{v_1, v_4, v_5\}, \{v_2, v_3, v_5\}, \{v_3, v_4, v_5\}, \{v_4, v_5, v_6\}, \ldots, \ldots$, ... and so on$\}$. Clearly T_{NP3} forms a discrete topology.

4. Preliminary Results:

We show in this section some main properties and results of a new type of topology $\boldsymbol{NP3}$-topology, in addition we show that a $\boldsymbol{NP3}$ -topological space is an Alexandroff space.

4.1. Remark: Let $\acute{G} = (\mathrm{Y}, \mathrm{E})$ be un-directed graph, for any $\mathrm{v} \in \mathrm{Y}$, a set containing all non-adjacent vertices v in construct with v a path of length two is represented by $\tilde{\tilde{A}}_{NP3}(\mathrm{v})$.

4.2. Proposition: Let T_{NP3} be a non-adjacency ($\boldsymbol{P3}$) topological space to undirected graph $\acute{G} = (\mathrm{Y}, \mathrm{E})$. For any $\mathrm{v} \in \mathrm{Y}$, If $|\tilde{\tilde{A}}_{NP3}(\mathrm{v})| \geq 2$. Or $\tilde{\tilde{A}}_{NP3}(\mathrm{v}) = \emptyset$, then $\{\mathrm{v}\} \in \mathrm{T}_{NP3}$.

Proof: Let $|\tilde{\tilde{A}}_{NP3}(\mathrm{v})| = 2$. Then (by using remark 4.1) there are two vertices $\mathrm{v}_1, \mathrm{v}_2 \in \mathrm{Y}$ such that each of them is non-adjacent v and construct with v a two-length path. Hence there exist two open set $\{\mathrm{v}, \mathrm{v}_1\}, \{\mathrm{v}, \mathrm{v}_2\} \in \mathrm{S}_{NP3}$ (by using the definition of T_{NP3}), this signifies $\{\mathrm{v}\}$ is a part of the basis of T_{NP3} . Same way if $|\tilde{\tilde{A}}_{NP3}(\mathrm{v})| > 2$.

If $\tilde{\tilde{A}}_{NP3}(\mathrm{v}) = \emptyset$, afterward, there is an open set $\{\mathrm{v}\} \in \mathrm{S}_{NP3}$ (by using the definition of T_{NP3}), then $\{\mathrm{v}\} \in \mathrm{T}_{NP3}$.

4.3. corollary: Let $\acute{G} = (\mathrm{Y}, \mathrm{E})$ be any un-directed graph, If $|\tilde{\tilde{A}}_{NP3}(\mathrm{v})| \geq 2$ Or $\tilde{\tilde{A}}_{NP3}(\mathrm{v}) = \emptyset$,$\forall\, \mathrm{v} \in \mathrm{Y}$, then T_{NP3} is a discrete topology.

Proof: Assume $|\tilde{\tilde{A}}_{NP3}(\mathrm{v})| \geq 2$, then $\{\mathrm{v}\} \in \mathrm{T}_{NP3}$, $\forall\, \mathrm{v} \in \mathrm{Y}$ (by using prop. 4.2).

Now, if $\tilde{\tilde{A}}_{NP3}(\mathrm{v}) = \emptyset$, then $\{\mathrm{v}\} \in \mathrm{T}_{NP3}$, $\forall\, \mathrm{v} \in \mathrm{Y}$ (by using proposition 4.2).

therefor $\mathrm{T}_{NP3} = P(\mathrm{Y})$ (a power set for Y). Thereupon T_{NP3} is discrete topology (def. of discrete topology).

4.4. Definition [15]: Alexandroff spaces are topological space that contain any arbitrary or random intersection of open sets.

4.5. Proposition: Let $\acute{G} = (\mathrm{Y}, \mathrm{E})$ be a graph. A $\boldsymbol{NP3}$ - topological space of $\acute{G}$ is an Alexandroff space.

Proof: We must to prove that $(\mathrm{Y}, \mathrm{T}_{NP3})$ is a space of Alexandroff, it is enough to prove that a random intersection for each elements of S_{NP3} is open, because $|\omega| \leq 2$, $\forall\, \omega \in \mathrm{S}_{NP3}$, hence either $\bigcap_{j=1}^{\infty} \omega j = \emptyset$ for every $j \geq 2$ is open set, or $\bigcap_{j=1}^{\infty} \omega_j = \omega$ s. t. $\omega_j = \omega$,$\forall j$ a set is open since $\omega \in \mathrm{S}_{NP3}$. Alternatively, $\bigcap_{j=1}^{\infty} \omega j = \{\mathrm{v}\}$ s. t. $\mathrm{v} \in \omega_j$, $\forall\, j \geq 2$, because of $\{\mathrm{v}\} \in \mathrm{T}_{NP3}$ (by proposition 4.2). Then $\{\mathrm{v}\}$ is open set. Accordingly, $(\mathrm{Y}, \mathrm{T}_{NP3})$ is an Alexandroff space.

4.6. Remark [15]: Because $(\mathrm{Y}, \mathrm{T}_{NP3})$ is a space of Alexandroff in any un-directed graph $\acute{G} = (\mathrm{Y}, \mathrm{E})$. For all $\mathrm{v} \in \mathrm{Y}$, the smallest set that is both open and contains v and intersects all open sets that contains v, is represented by $\mathrm{U}_{NP3}(\mathrm{v})$. Furthermore, the collection $\mathrm{D}_{NP3} = \{\mathrm{U}_{NP3}(\mathrm{v}) \mid \mathrm{v} \in \mathrm{Y}\}$, represent the minimal (smallest) basis of $(\mathrm{Y}, \mathrm{T}_{NP3})$.

4.7. Proposition: In any undirected graph $\acute{G} = (\mathrm{Y}, \mathrm{E})$, $\mathrm{U}_{NP3}(\mathrm{v}) = \bigcap_{\omega_i \in \mathrm{S}_{NP3}} \omega_i$ such that $\mathrm{v} \in \omega_i$, $\forall\, i \geq 1$.

Proof: Because $U_{NP3}(v)$ is the smallest set of all open sets with intersections that contain v (using def. of $U_{NP3}(v)$ and S_{NP3} is a sub-basis of T_{NP3} , then

$U_{NP3}(v) \subseteq \bigcap_{\omega_i \in S_{NP3}} \omega_i$ s. t. $i \geq 1$ -------------(1).

Then, $v \in \omega_i$, $\forall\, i$, so $v \in \bigcap_{\omega_i \in S_{NP3}} \omega_i$, $\forall\, i \geq 1$.

Hence $\bigcap_{\omega_i \in S_{NP3}} \omega_i \subseteq U_{NP3}(v)$ ----------(2)

By (1) and (2) $U_{NP3}(v) = \bigcap_{\omega_i \in S_{NP3}} \omega_i$, $\forall i \geq 1$.

4.8. Remark: Consider $G = (V, E)$ be any undirected graph. For any $v \in \omega$,

1. If $|\tilde{\tilde{A}}_{NP3}(v)| \geq 2$. Or $\tilde{\tilde{A}}_{NP3}(v) = \emptyset$, then $U_{NP3}(v) = v$
   Because if $|\tilde{\tilde{A}}_{NP3}(v)| \geq 2$, then $\bigcap_{\omega_i \in S_{NP3}} \omega_i = \{v\}$ s.t. $v \in \omega_i$, $\forall\, i \geq 2$
   Since $|\omega_i| \leq 2$, $\forall\, i$. Hence $U_{NP3}(v) = \{v\}$ (by using prop. 4.7)

Now, if $\tilde{\tilde{A}}_{NP3}(v) = \emptyset$ then, there exists open set $\omega \in S_{NP3}$ s.t. $\omega = \{v\}$. Since ω of the family is the only open set in S_{NP3} which contains v , so $U_{NP3}(v) = \{v\}$ (by prop. 4.7)

2. If $|\tilde{\tilde{A}}_{NP3}(v)| = 1$, then $\exists$ an open set $\omega = \{v, w\}$ in S_{NP3} s.t. $\tilde{\tilde{A}}_{NP3}(v) = \{w\}$, and because ω is only open set in the family S_{NP3} which include v , (by using prop.4.7) $U_{NP3}(v) = \{v, w\}$.

4.9. Corollary: Let $G = (V, E)$ to be the graph, then $\forall\, v, w \in V$ in G , we have $w \in U_{NP3}(v) \Leftrightarrow \tilde{\tilde{A}}_{NP3}(v) = \{w\}$

Proof: Using remark 4.8 makes it evident.

4.10. Corollary: For any $v \in V$ in the graph $G = (V, E)$, $U_{NP3}(v) \subseteq \omega_i$ and so $\overline{U_{NP3}(v)} \subseteq \overline{\omega_i}$, $\forall\, i$, s. t. $v \in \omega_i$, $\forall\, i$.

Proof: since $U_{NP3}(v) = \bigcap_{\omega_i \in S_{NP3}} \omega_i$ (by using prop. 4.7) s. t. $v \in \omega_i$, $\forall\, i \geq 1$. So, $U_{NP3}(v) \subseteq \omega_i$, $\forall i$.

Now, to prove $\overline{U_{NP3}(v)} \subseteq \overline{\omega_i}$, $\forall\, i$. Let $w \in \overline{U_{NP3}(v)} \to \mathcal{M} \cap U_{NP3}(v) \neq \emptyset, \forall\, \mathcal{M} \in T_{NP3}$, s. t. $w \in \mathcal{M}$. Since $U_{NP3}(v) \subseteq \omega_i \to \mathcal{M} \cap \omega_i \neq \emptyset$ $\forall\, \mathcal{M} \in T_{NP3}$ s. t. $w \in \mathcal{M}$,

This implies $w \in \overline{\omega_i}$ and so, $\overline{U_{NP3}(v)} \subseteq \overline{\omega_i}$, $\forall i$.

4.11. Corollary: Let $G = (V, E)$ to be the graph $v \in V$, $\overline{\{v\}} \subseteq \overline{U_{NP3}(v)} \subseteq \overline{\omega_i}$, $\forall\, i$ $s.t.$ $v \in \omega_i$, $\forall\, i$.

Proof: Let $w \in \overline{\{v\}} \to \mathcal{M} \cap \{v\} \neq \emptyset, \forall\, \mathcal{M} \in T_{NAV}(G)$ s.t. $w \in \mathcal{M}$, But $\{v\} \in U_{NP3}(v)$, *so* $\mathcal{M} \cap U_{NP3}(v) \neq \emptyset, \forall\, \mathcal{M} \in T_{NP3}$ s. t. $w \in \mathcal{M}$. Hence $w \in \overline{U_{NP3}(v)}$.

Since $\overline{U_{NP3}(v)} \subseteq \overline{\omega_i}$ (by using corollary 4.10) , then , $\overline{\{v\}} \subseteq \overline{U_{NP3}(v)} \subseteq \overline{\omega_i}$, $\forall i$.

4.12. Corollary: Let $\acute{G} = (\mathrm{Y}, E)$ be any undirected graph. For any $w, v \in \mathrm{Y}$, We have $w \in \overline{\{v\}} \Leftrightarrow \tilde{\tilde{A}}_{NP3}(w) = \{v\}$.

Proof : ⇒) If $w \in \overline{\{v\}}$, then $U_{NP3} \cap \{v\} \neq \emptyset, \ \forall \ U_{NP3} \in T_{NP3}$ s. t. $w \in U_{NP3}$ this means $w \in U_{NP3}$. Hence $\tilde{\tilde{A}}_{NP3}(w) = \{v\}$ (by using corollary 4.9)

⇐) suppose $\tilde{\tilde{A}}_{NP3}(w) = \{v\}$ this implies $v \in U_{NP3}(w)$ (by using corollary 4.9)

Then, $U_{NP3} \cap \{v\} \neq \emptyset, \forall \ U_{NP3} \in T_{NP3}$ s. t. $w \in T_{NP3}$.

Hence $w \in \overline{\{v\}}$.

4.13. Remark [15]: Every Alexandroff topological space (X, T) be the T_1- space if and only if $U_{NP3}(x) = \{x\}$. Consequently, because X is discrete, the topological space (Y, T_{NP3}) is T_1- space $\Leftrightarrow$ it is discrete. Now, if (Y, T_{NP3}) is a space of Alexandroff, then it is T_0-space if and only if $U_{NP3}(v) = U_{NP3}(w)$ implies $v = w$, i.e. $U_{NP3}(v) \neq U_{NP3}(w)$, $\forall$ distinct vertices $v, w \in \mathrm{Y}$. Also, a non-adjacency (path of length tow) topological space T_{NP3} is T_0-space if and only if $\tilde{\tilde{A}}_{NP3}(v) \neq \{w\}$ or $\tilde{\tilde{A}}_{NP3}(w) \neq \{v\}$, $\forall\, w, v \in \mathrm{Y}$ s. t. $w \neq v$ (by using corollary 4.9) .

4.14. Proposition: Let $\acute{G} = (\mathrm{Y}, E)$ be any un-directed graph, Afterward (Y, T_{NP3}) is T_2-space $\Leftrightarrow$ (Y, T_{NP3}) is T_1-space.

Proof:⟹) intelligible.

⟸) Assume (Y, T_{NP3}) is T_1-space, then (Y, T_{NP3}) is discrete (using remark (4.13)).

This implies $\forall\, v \in \mathrm{Y}, \{v\} \in T_{NP3}$. So $\forall\, v, w \in \mathrm{Y}$, s.t. $v \neq w$, $\exists \{v\}, \{w\} \in T_{NP3}$ s.t. $w \in \{w\}$ and $v \in \{v\}$ and $\{w\} \cap \{v\} = \emptyset$.

Hence (Y, T_{NP3}) is T_2-space.

6. conclusions

We introduced a new type of non-adjacency topology on undirected graph which called a ***NP3*-** topology, that relates the topology for a set of vertices. This space is the Alexandroff space, which indicated that some topological features examined in some fundamental graphs. This topology can solve a variety of undirected graph dependent problems in Global Positioning System GPS. We may consider the applications of using it in other research in the future.

7. References

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