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Reliability Single Stress-Strength Model Estimation Using Exponential T-X (Fréchet)Family Distributions

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Abstract. The reliability of the single and parameter estimate $R_{single} = p[t < u]$ arrangement in the stress-strength model, which has strength (t) according to a stress (u), provided our inspiration for writing this paper. They follow the Exp. T – X Fréchet distribution m(σ,α,β). Numerous estimate methods are reviewed and proposed in this paper. To compare these methods, three metrics Bias, MSE, and MAPE were employed. This paper's conclusions are derived from an analysis of real data collected by Monte Carlo simulation.

Keywords: Exp. T – X distribution, Fréchet distribution, Reliability, Estimation methods, Stress – strength model.

1. INTRODUCTION

In practical domains such as risk management, economic, financial, and actuarial sciences, statistical distributions are widely used in data modeling. However, the presumptive probability model of the phenomenon under study ultimately determines how well the techniques work. Insurance losses in applied areas are typically positive, right-skewed, unimodal, and have substantial tails (see [9]). Actuaries typically search for distributions with a high tail in order to accurately assess the level of business risk involved. The distributions with greater right tail probabilities than exponential ones are known as heavy-tailed distributions [8][9]. Numerous scientific and engineering applications make use of the stress-strength (S.S) example.[14] The Fréchet distribution undergoes transformed through Fréchet distribution with three-parameter used data modeling applications from the fields of engineering and medicine. Here, we present the exponential T-X (ETX) Fréchet family[6], a family of distributions. When modeling heavy-tailed data, the suggested model is incredibly adaptable. Following are some of the statistical properties of (ETX) Fréchet distribution, including the probability density (pdf), cumulative distribution (cdf), reliability (R),the hazard (H), the first raw moment (m_1) functions, respectively, as : (see figure 1and 2) [11][12]

$$m(x,\sigma,\alpha,\beta) = \frac{\sigma(\sigma-1)\alpha\beta^{\alpha}x^{\alpha-1}e^{-(x\beta)^{\alpha}}}{(\sigma-1+e^{-(x\beta)^{\alpha}})^2} \qquad x > 0 \qquad (1)$$

$$M(x,\sigma,\alpha,\beta) = 1 - \left(\frac{\sigma e^{-(x\beta)^{\alpha}}}{\sigma-1+e^{-(x\beta)^{\alpha}}}\right) \qquad x \geq 0 \qquad (2)$$

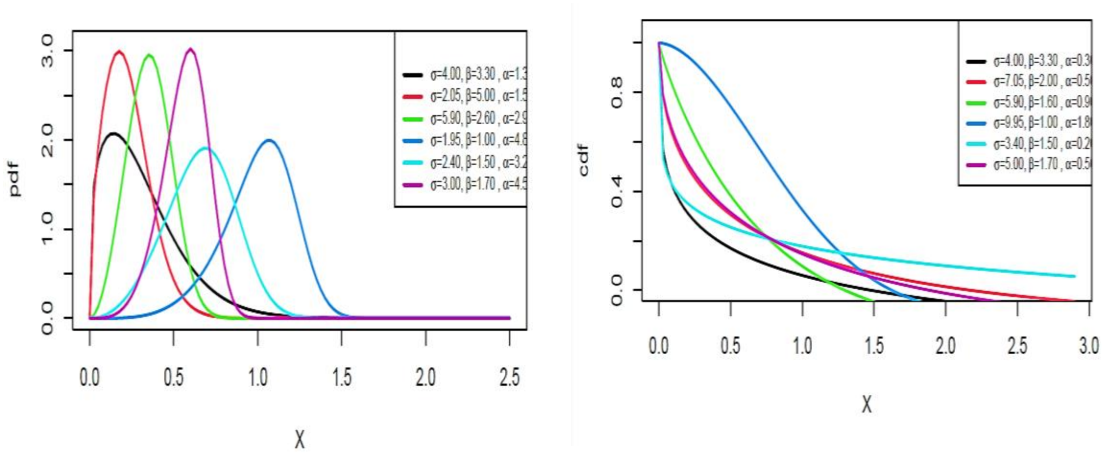

Figure 1: pdf and cdf functions for a ETX (Fréchet) distribution and different value of parameters.

$$R(x) = \frac{\sigma e^{-(x\beta)^{\alpha}}}{\sigma - 1 + e^{-(x\beta)^{\alpha}}} \qquad (3)$$

$$H(x) = \frac{(\sigma - 1)\alpha \beta^{\alpha} x^{\alpha-1}}{\sigma - 1 + e^{-(x\beta)^{\alpha}}} \qquad (4)$$

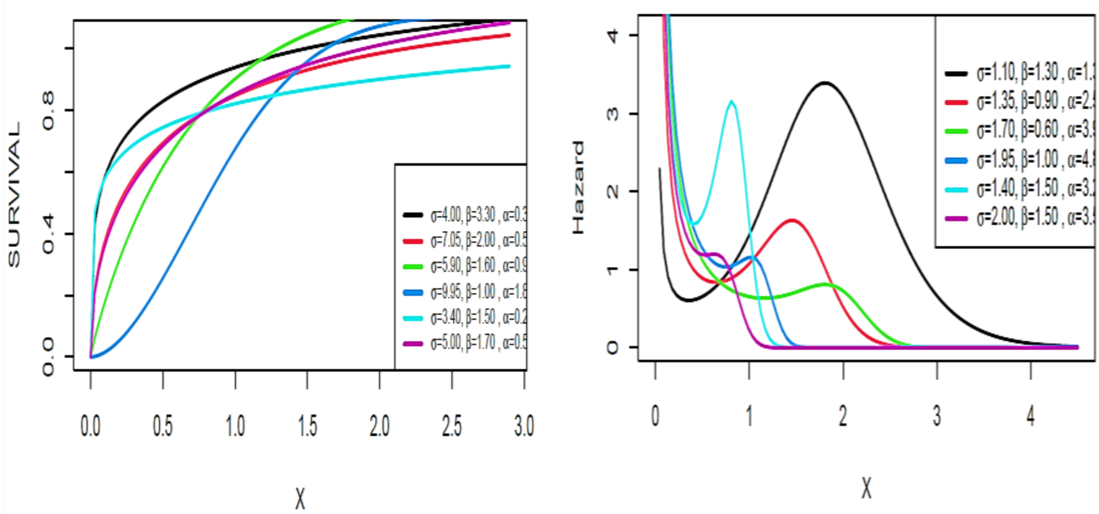

Figure 2: Reliability and hazard functions for a ETX (Fréchet) distribution and different value of parameters.

$$m_1 = \frac{\sigma((\log(4 - 4\sigma) + \log \sigma) + \log(2\sigma - 1))}{\alpha\beta}. \qquad (5)$$

Where σ, α are shape parameters, and β is scale parameter. The paper aims to find a single stress strength model for T-X family statistical distribution. We derived from the family of the exponential distribution and the baseline Fréchet distribution. In addition to conduct simulations for different values of parameters and with different sample sizes. The paper consists of four sections. The second section consisted of finding the stress and strength model for the ETX (Fréchet) distribution. The third section contained estimating the distribution parameters using eight different methods. The fourth section consisted of a Monte Carlo simulation of the distribution. The fourth and the final section contained the most important conclusions that we reached during the paper.

2. The Proposed model (a single reliability model).

Let u be the stress and t be the strength independent random variables. We used the ETX (Fréchet) distribution to derive the stress strength for single reliability as:

$$R_{single} = p[t < u] = \int_0^{\infty} \int_0^{t} m(u)\, m(t)\, dudt \qquad (6)$$

$$= \int_0^{\infty} \int_0^{t} \frac{\sigma(\sigma-1)\alpha_1 \beta^{\alpha_1} u^{\alpha_1 - 1} e^{-(\beta u)^{\alpha_1}} \sigma(\sigma-1)\alpha_2 \beta^{\alpha_2} t^{\alpha_2 - 1} e^{-(\beta t)^{\alpha_2}}}{(\sigma - 1 + e^{-(\beta u)^{\alpha_1}})^2 (\sigma - 1 + e^{-(\beta t)^{\alpha_2}})^2} dudt$$

$$R_{single} = \frac{\sum_{j=0}^{\infty} \sum_{z=1}^{\infty} \frac{z}{\sigma^z} \binom{z-1}{j} (-1)^j (\sigma-1)\alpha\beta^{\alpha}}{\alpha_1 (i+1) \beta^{\alpha_1}} - \frac{\sum_{j=0}^{\infty} \sum_{z=1}^{\infty} \frac{z}{\sigma^z} \binom{z-1}{j} (-1)^j (\sigma-1)\alpha\beta^{\alpha} \sum_{z=0}^{\infty} \sum_{j=0}^{\infty} \sum_{0}^{\infty} \binom{z}{j} \frac{(-1)^{z+j+k}}{\sigma^z}}{(i+1)\alpha_1 \beta^{\frac{1}{\alpha_1}+1}}$$

$$. \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \Gamma\left(\frac{\alpha_2}{\alpha_1} n + 1\right). \text{ for } \quad z = 1, 2, \ldots \qquad (7)$$

3. Methods of estimation.

In this section, we derived eight estimation methods such as (Maximum likelihood, Exact estimators of moment, Percentile, Approximate least squares, Weighted least squares, and three shrinkage) methods as follows:

3.1.Maximum likelihood Estimator (mle) [3][4] .

The process of estimation in this way can be defined as making parameter values of a function where have a great values as possible. It is characterized by efficiency, adequacy, stability and consistency, in addition to its lack of bias. if $t_1, t_2, \ldots t_n$ are random variables (stress) have probability density function of a distribution ETX (Fréchet). The following formula is the likelihood function, denoted by $L(\Theta, t)$ as follows

$$L(\Theta, t) = \prod_{i=1}^{n} \frac{\sigma(\sigma-1)\alpha\beta^{\alpha} t_i^{\alpha-1} e^{-(\beta t_i)^{\alpha}}}{(\sigma - 1 + e^{-(\beta t_i)^{\alpha}})^2}. \qquad (8)$$

Where Θ are the distribution parameters(σ,α,β), then take the nutural logarthim to the equation (8) as

$$L = n\log(\sigma) + n\log(\sigma - 1) - n\log(\alpha) - n\alpha\log(\beta) + (\alpha - 1)\sum_{i=1}^{n} \log t_i - \sum_{i=1}^{n} (\beta t_i)^{\alpha} - 2\sum_{i=1}^{n} \log\left(\sigma - 1 + e^{-(\beta t_i)^{\alpha}}\right). \qquad (9)$$

To find the (β, σ, and α) estimators by taking the partial derivative of equation (9) with respect to (β, σ, and α) respectively as follows:

$$\frac{\partial L}{\partial \beta} = \frac{n\alpha}{\beta} - \sum_{i=1}^{n} \alpha t_i^{\alpha} \beta^{\alpha-1} + 2\sum_{i=1}^{n} \frac{\alpha e^{-(\beta t_i)^{\alpha}} t_i^{\alpha} \beta^{\alpha-1}}{\sigma - 1 + e^{-(\beta t_i)^{\alpha}}}, \qquad (10)$$

with respect to σ

$$\frac{\partial L}{\partial \sigma}=\frac{n}{\sigma}+\frac{n}{\sigma-1}-2\sum_{i=1}^{n}\frac{1}{\sigma-1+e^{-(\beta t_i)^{\alpha}}} \qquad (11)$$

and with respect to α

$$\frac{\partial L}{\partial \alpha}=\frac{n}{\alpha}-nlog\,(\beta)+\sum_{i=1}^{n}logt_i-\sum_{i=1}^{n}(\beta t_i)^{\alpha}\log(\beta t_i)+2\sum_{i=1}^{n}\frac{e^{-(\beta t_i)^{\alpha}}\log(\beta t_i)(\beta t_i)^{\alpha}}{\sigma-1+e^{-(\beta t_i)^{\alpha}}}. \qquad (12)$$

Similirty , when $u_1, u_2, \ldots, u_m$ be a random sample from the strength u which is distributed as ETX(Fréchet), when σ, β is known and shape parameter α unknown. The method for the mle strength is presented by

$$\frac{\partial L}{\partial \beta}=\frac{n\alpha}{\beta}-\sum_{i=1}^{n}\alpha u_i^{\alpha}\beta^{\alpha-1}+2\sum_{i=1}^{n}\frac{\alpha e^{-(\beta u_i)^{\alpha}}t_i^{\alpha}\beta^{\alpha-1}}{\sigma-1+e^{-(\beta u_i)^{\alpha}}},$$

$$\frac{\partial L}{\partial \sigma}=\frac{n}{\sigma}+\frac{n}{\sigma-1}-2\sum_{i=1}^{n}\frac{1}{\sigma-1+e^{-(\beta u_i)^{\alpha}}},$$

$$\frac{\partial L}{\partial \alpha}=\frac{n}{\alpha}-nlog\,(\beta)+\sum_{i=1}^{n}logu_i-\sum_{i=1}^{n}(\beta u_i)^{\alpha}\log(\beta u_i)\qquad+2\sum_{i=1}^{n}\frac{e^{-(\beta u_i)^{\alpha}}\log(\beta u_i)(\beta u_i)^{\alpha}}{\sigma-1+e^{-(\beta u_i)^{\alpha}}},$$

There is no exact formula to estimate the parameters of EXT(Fréchet) distribution. However, we can estimate them by nonlinear numerical analysis methods.

3.2. The Precise Estimators of Moments Method (PEMM) . [1] [15]

We suggest the expected value $E(r)$, the variance $var(r)$ and coefficient of variation $cv(r)$ for a ETX (Fréchet) distribution's as follows:

$$E(r)=\frac{\varphi\Gamma\left(\frac{1}{\alpha}+1\right)}{\alpha(i+1)^{\frac{1}{\alpha}+1}\beta^{1+\alpha}}, \qquad \text{for } \alpha,\beta>0 \qquad (13)$$

$$var(r)=\frac{\varphi\Gamma\left(\frac{2}{\alpha}+1\right)}{\alpha(i+1)^{\frac{2}{\alpha}+1}\beta^{2+\alpha}}-\left(\frac{\varphi\Gamma\left(\frac{1}{\alpha}+1\right)}{\alpha(i+1)^{\frac{1}{\alpha}+1}\beta^{1+\alpha}}\right)^2 \qquad (14)$$

$$cv(r)=\frac{\sqrt{var}}{\mu_1}=\frac{\sqrt{\frac{\varphi\Gamma\left(\frac{2}{\alpha}+1\right)}{\alpha(i+1)^{\frac{2}{\alpha}+1}\beta^{2+\alpha}}-\left(\frac{\varphi\Gamma\left(\frac{1}{\alpha}+1\right)}{\alpha(i+1)^{\frac{1}{\alpha}+1}\beta^{1+\alpha}}\right)^2}}{\frac{\varphi\Gamma\left(\frac{1}{\alpha}+1\right)}{\alpha(i+1)^{\frac{1}{\alpha}+1}\beta^{1+\alpha}}}. \qquad (15)$$

Then the PEMM is given below:

$$\alpha_{1PEMM} = \frac{\sqrt{\frac{\varphi\Gamma\left(\frac{2}{\alpha_{01}}+1\right)}{\alpha_{01}(i+1)^{\frac{2}{\alpha_{01}}+1}\beta^{2+\alpha_{01}}} - \left(\frac{\varphi\Gamma\left(\frac{1}{\alpha_{01}}+1\right)}{\alpha_{01}(i+1)^{\frac{1}{\alpha_{01}}+1}\beta^{1+\alpha_{01}}}\right)^2}}{\frac{\varphi\Gamma\left(\frac{1}{\alpha_{01}}+1\right)}{\alpha_{01}(i+1)^{\frac{1}{\alpha_{01}}+1}\beta^{1+\alpha_{01}}}} \quad (16)$$

$$\alpha_{2PEMM} = \frac{\sqrt{\frac{\varphi\Gamma\left(\frac{2}{\alpha_{02}}+1\right)}{\alpha_{02}(i+1)^{\frac{2}{\alpha_{02}}+1}\beta^{2+\alpha_{02}}} - \left(\frac{\varphi\Gamma\left(\frac{1}{\alpha_{02}}+1\right)}{\alpha_{02}(i+1)^{\frac{1}{\alpha_{02}}+1}\beta^{1+\alpha_{02}}}\right)^2}}{\frac{\varphi\Gamma\left(\frac{1}{\alpha_{02}}+1\right)}{\alpha_{02}(i+1)^{\frac{1}{\alpha_{02}}+1}\beta^{1+\alpha_{02}}}} \quad (17)$$

3.3.The Percentile Estimator. (prec) [16]

Using the graphical approximation to the best linear unbiased estimators, Kao (1959) first investigated this strategy, by a straight line between the theoretical points derived from the distribution function and the sample percentile points, one may obtain the estimators. When dealing with a ETX (Fréchet) distribution, one might apply the same. idea to get the estimators of and based on percentiles due to the distribution function's structure.[10]

$var_1 = \sum_{i=1}^{n}(M(t_i) - q_i)^2$ and $var_2 = \sum_{j=1}^{m}\left(M(u_j) - q_j\right)^2$,where $q_i = \frac{i}{n+i}, q_j = \frac{j}{m+j}$ then,

$$\hat{\alpha}_{1\text{Pres}} = \sum_{i=1}^{n} \frac{\ln[-\ln[\sigma - \sigma(1-q_i) + (1-q_i)] - \ln(1-q_i+\sigma)]}{2\ln\beta t_i} \quad (18)$$

$$\hat{\alpha}_{2\text{Pres}} = \sum_{i=1}^{n} \frac{\ln[-\ln[\sigma - \sigma(1-q_i) + (1-q_i)] - \ln(1-q_i+\sigma)]}{2\ln\beta u_i} \quad (19)$$

3.4.The Approximate Least Squares Estimator (alst)

Approximated least squares technique estimators can be made by reducing the sum of square error between the value and its expected value. Among the most efficient and well-liked techniques, the LS approach is widely used to fit models and solve mathematical and engineering problems, particularly in linear and non-linear situations. [7].

$$Q_1(\Theta, t) = \sum_{i=1}^{n}\left(M(t_i) - \frac{i}{n+1}\right)^2 , \qquad \forall\, i = 1,2,\dots,n$$

$$\frac{\partial Q_1}{\partial \sigma} = \sum_{i=1}^{n}\left(1 - \frac{\sigma e^{-(t\beta)^\alpha}}{(\sigma - 1 + e^{-(t\beta)^\alpha})} - \frac{i}{n+1}\right)\frac{e^{-(t\beta)^\alpha}\left(1 - e^{-(t\beta)^\alpha}\right)}{(\sigma - 1 + e^{-(t\beta)^\alpha})^2}). \quad (20)$$

$$\frac{\partial Q_1}{\partial \beta} = \sum_{i=1}^{n}(1 - \frac{\sigma e^{-(t\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})} - \frac{i}{n+1}) * (\frac{(\alpha\sigma t e^{-(t\beta)^\alpha})(t\beta)^{\alpha-1}(\sigma-1)}{(\sigma-1+e^{-(t\beta)^\alpha})^2}. \quad (21)$$

$$\frac{\partial Q_1}{\partial \alpha} = \sum_{i=1}^{n}(1 - \frac{\sigma e^{-(t\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})} - \frac{i}{n+1}) \frac{(1-e^{-(t\beta)^\alpha})(\sigma-1)((t\beta)^\alpha(\ln t\beta)e^{-(x\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})^2}). \quad (22)$$

when $u_1, u_2, \dots, u_m$ be a random sample from the strength u which is distributed as ETX(Fréchet), when σ, β is known and shape parameter α is unknown . The alst method for the strength u is presented by

$$Q_2(\theta, u) = \sum_{j=1}^{m}(M(u_i) - \frac{j}{n+1})^2 , \qquad \forall\, j = 1,2,\dots,m$$

$$\frac{\partial Q_2}{\partial \sigma} = \sum_{j=1}^{m}(1 - \frac{\sigma e^{-(u\beta)^\alpha}}{(\sigma-1+e^{-(u\beta)^\alpha})} - \frac{j}{n+1}) \frac{e^{-(u\beta)^\alpha}(1-e^{-(u\beta)^\alpha})}{(\sigma-1+e^{-(u\beta)^\alpha})^2}). \quad (23)$$

$$\frac{\partial Q_2}{\partial \beta} = \sum_{i=1}^{m}(1 - \frac{\sigma e^{-(u\beta)^\alpha}}{(\sigma-1+e^{-(u\beta)^\alpha})} - \frac{j}{n+1}) * (\frac{(\alpha\sigma u)(u\beta)^{\alpha-1}(\sigma-1)}{(\sigma-1+e^{-(u\beta)^\alpha})^2}. \quad (24)$$

$$\frac{\partial Q_2}{\partial \alpha} = \sum_{j=1}^{m}(1 - \frac{\sigma e^{-(u\beta)^\alpha}}{(\sigma-1+e^{-(u\beta)^\alpha})} - \frac{j}{n+1}) * (\frac{(1-e^{-(u\beta)^\alpha})(\sigma-1)((u\beta)^\alpha(\ln u\beta)e^{-(u\beta)^\alpha}}{(\sigma-1+e^{-(u\beta)^\alpha})^2}). \quad (25)$$

3.5. The weighted least squares estimator (wls).

The weighted least squares estimator of ETX (Fréchet) distribution These can be acquired by reducing as follows:

$$W_1(\theta, t) = \sum_{i=1}^{n}\frac{(n+1)^2(n+2)}{i(n-i+1)}(M(t_i) - \frac{i}{n+1})^2 . \quad (26)$$

$$\frac{\partial W_1}{\partial \sigma} = \sum_{i=1}^{n}(1 - \frac{\sigma e^{-(t\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})} - \frac{i}{n+1}). \frac{(n+1)^2(n+2)}{i(n-i+1)}. (\frac{e^{-(t\beta)^\alpha}(1-e^{-(t\beta)^\alpha})}{(\sigma-1+e^{-(t\beta)^\alpha})^2}), \quad (27)$$

$$\frac{\partial W_1}{\partial \beta} = \sum_{i=1}^{n}(1 - \frac{\sigma e^{-(t\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})} - \frac{i}{n+1}). \frac{(n+1)^2(n+2)}{i(n-i+1)})(\frac{(\alpha\sigma t e^{-(t\beta)^\alpha})(t\beta)^{\alpha-1}(\sigma-1)}{(\sigma-1+e^{-(t\beta)^\alpha})^2}. \quad (28)$$

$$\frac{\partial W_1}{\partial \alpha} = \sum_{i=1}^{n}(1 - \frac{\sigma e^{-(t\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})} - \frac{i}{n+1}). \frac{(n+1)^2(n+2)}{i(n-i+1)} (\frac{(1-e^{-(t\beta)^\alpha})(\sigma-1)((t\beta)^\alpha(\ln t\beta)e^{-(t\beta)^\alpha}}{(\sigma-1+e^{-(t\beta)^\alpha})^2}). \quad (29)$$

In the same way, the wls method for the strength is presented by

$$W_2(\theta, u) = \sum_{j=1}^{m}\frac{(m+1)^2(m+2)}{j(m-j+1)}(M(u_j) - \frac{j}{m+1})^2 . \quad (30)$$

$$\frac{\partial W_2}{\partial \sigma} = \sum_{j=1}^{m}(1 - \frac{\sigma e^{-(u\beta)^\alpha}}{(\sigma-1+e^{-(u\beta)^\alpha})} - \frac{j}{m+1}). \frac{(m+1)^2(m+2)}{j(m-i+1)}. (\frac{e^{-(u\beta)^\alpha}(1-e^{-(u\beta)^\alpha})}{(\sigma-1+e^{-(u\beta)^\alpha})^2}) \quad (31)$$

$$\frac{\partial W_2}{\partial \beta} = \sum_{j=1}^{m}(1 - \frac{\sigma e^{-(u\beta)^\alpha}}{(\sigma-1+e^{-(u\beta)^\alpha})} - \frac{j}{m+1})(\frac{(m+1)^2(m+2)}{j(m-j+1)}). (\frac{(\alpha\sigma u e^{-(u\beta)^\alpha})(u\beta)^{\alpha-1}(\sigma-1)}{(\sigma-1+e^{-(u\beta)^\alpha})^2}. \quad (32)$$

$$\frac{\partial W_2}{\partial \alpha} = \sum_{j=1}^{m}(1 - \frac{\sigma e^{-(u\beta)^\alpha}}{(\sigma - 1 + e^{-(u\beta)^\alpha})} - \frac{j}{m+1}). \frac{(m+1)^2(m+2)}{j(m-j+1)} * (\frac{(1 - e^{-(u\beta)^\alpha})(\sigma - 1)((u\beta)^\alpha(\ln u\beta)e^{-(u\beta)^\alpha}}{(\sigma - 1 + e^{-(u\beta)^\alpha})^2}). \quad (33)$$

3.6. The Method for Estimating Shrinkage (shr) [5] [17]

The shrinkage estimation method can be conceptualized as a Bayesian approach that depends on prior information. Thompson in (1968) has suggested the problem of shrink an unbiased estimator and he presented the primary reasons for using earlier estimates [4]. The parameter was used as an initial value α_0, where $[\alpha_0 = \alpha_0 \pm \epsilon]$, $\epsilon = 0.001$, in the shrinkage estimation approach. The normal estimator ($\hat{\alpha}_{MLE}$) was then applied to them using a shrinkage weight factor $\Omega(\alpha)$, $0 < \Omega(\alpha) \leq 1$, which may be expressed as:

$$\widehat{\alpha_1}_{sh} = \psi(\hat{\alpha})\hat{\alpha}_{MLE} + (1 - \psi(\hat{\alpha}))\hat{\alpha}_0 . \qquad (34)$$

3.6.1. The Weight-Based Shrinkage Function (shr1).[13]

We shall examine the weight reduction function. The function form in this subsection is represented by $\Omega(\hat{\alpha}) = \left[\frac{\tan n}{n}\right]$, where $\hat{\alpha}$ represents the sample size and n represents the number of participants. The value of $\Omega(\hat{\alpha})$ is between 0 and 1. Using the given expressions $\Omega(\hat{\alpha}_1) = \left[\frac{\tan n}{n}\right]$ and $\Omega(\hat{\alpha}_2) = \left[\frac{\tan m}{m}\right]$, where n and m represent number of particioants (the sample sizes of u and t) respectively, the shrinkage estimator utilizes the $\hat{\alpha}_1$ and $\hat{\alpha}_2$ shrinkage weight functions described in equation (34).

$$\widehat{\alpha_1}_{shr1} = \left|\frac{\tan n}{n}\right| \hat{\alpha}_{1_{MLE}} + \left(1 - \left|\frac{\tan n}{n}\right|\right) \hat{\alpha}_{1_0} \qquad (35)$$

$$\widehat{\alpha_2}_{shr1} = \left|\frac{\tan m}{m}\right| \hat{\alpha}_{2_{MLE}} + \left(1 - \left|\frac{\tan m}{m}\right|\right) \hat{\alpha}_{2_0} \qquad (36)$$

Now, from equations (35), and (36) were used to get the single stress strength relaibility formula as follows

$$\hat{R}_{Shr1} = 1 - \sum_{f=0}^{\infty} \frac{(-1)^f}{f!} \Gamma\left(\frac{\widehat{\alpha_1}_{shr1}}{\widehat{\alpha_2}_{shr1}} f + 1\right) \qquad (37)$$

3.6.2 The Constant Shrinkage Estimate(shr2).[13].

In the scenario, there is a constant shrinkage factor, we can suppose that $\Omega(\hat{\alpha}) = \left|\frac{\tan n}{n}\right| = 0.001$, and has a value between 0 and 1 where , n refer to the sample size.

In order to obtain the constant shrinkage estimators, we substitute $\hat{\alpha}_1$ and $\hat{\alpha}_2$ into equation (34) by taking the forms below as $\Omega(\hat{\alpha}_1) = \left|\frac{\tan n}{n}\right|$, and $\Omega(\hat{\alpha}_2) = \left|\frac{\tan m}{m}\right|$, $0 \leq \Omega(\hat{\alpha}_i) \leq 1$. as shown:

$$\widehat{\alpha_1}_{shr1} = \Omega(\hat{\alpha}_1)\hat{\alpha}_{1_{MLE}} + (1 - \Omega(\hat{\alpha}_1))\hat{\alpha}_{1_0} \qquad (38)$$

and

$$\widehat{\alpha_2}_{shr1} = \Omega(\hat{\alpha}_2)\hat{\alpha}_{2_{MLE}} + (1 - \Omega(\hat{\alpha}_2))\hat{\alpha}_{2_0} \qquad (39).$$

Here, we substuted by $\widehat{\alpha_1}_{shr1} = 0.001\hat{\alpha}_{1_{MLE}} + 0.999\hat{\alpha}_{1_0}$ and $\widehat{\alpha_2}_{shr1} = 0.001\hat{\alpha}_{2_{MLE}} + 0.999\hat{\alpha}_{2_0}$. Therefore, from equations (38), and (39) to get the single relability as follows:

$$\hat{R}_{Shr2} = 1 - \sum_{f=0}^{\infty} \frac{(-1)^f}{f!} \Gamma\left(\frac{\widehat{\alpha_1}_{shr2}}{\widehat{\alpha_2}_{shr2}} f + 1\right) \qquad (40)$$

3.6.3 The Shrinkage function (shr3).[4]

In this part, the shrinkage weight factor is determined by the sizes g and h. Specifically, we define $\Omega(\hat{\alpha}_1)$ as e^{-g} and $\Omega(\hat{\alpha}_2)$ as e^{-g}, where $\Omega(\hat{\alpha})$ is a function that satisfies $0 \leq \Omega(\hat{\alpha}) < 1$. Hence, the shrinkage estimator employs the shrinkage function of $\hat{\alpha}_1$ and $\hat{\alpha}_2$, as defined in equation (34) below:

$$\widehat{\alpha_1}_{shr3} = e^{-g}\hat{\alpha}_{1_{MLE}} + (1 - e^{-g})\hat{\alpha}_{1_0} \qquad (41)$$

and

$$\widehat{\alpha_2}_{shr3} = e^{-h}\hat{\alpha}_{2_{MLE}} + (1 - e^{-h})\hat{\alpha}_{2_0} \qquad (42)$$

Now, in which equations (41), and (42) were used to get

$$\hat{R}_{Shr3} = 1 - \sum_{f=0}^{\infty} \frac{(-1)^f}{f!} \Gamma\left(\frac{\widehat{\alpha_1}_{shr3}}{\widehat{\alpha_2}_{shr3}} f + 1\right) \qquad (43)$$

4. The simulation.

The Monte Carlo simulation was employed to validate the effectiveness of the proposed estimate approach for assessing the reliability of a single component system. [2] The eight proposed estimate methods are implemented using various sample sizes (25, 50, 75, 100). The statistical results for each sample are determined using bias, mean absolute percentage error, and mean squared error criteria, with 1000 repeats. Hence, the subsequent procedures elucidate the Monte Carlo simulations for each model.

Step 1: To determine the performance, begin by initializing and generating random samples that adhere to a continuous uniform distribution within the range of 0 to 1. The distribution U is a uniform distribution with a range from 0 to 1.

Step 2: Convert the given uniform random sample into a random sample of the power Fréchet distribution by utilising the cumulative distribution function.

Step 3: Calculate the estimated parameters of the 8 method mentioned as in section 2.

Step 4: The estimated reliability of stress-strength models using various estimating methods, such as $\hat{R}_{MLE}, \hat{R}_{Pres}, \hat{R}_{alst}, \hat{R}_{wls}, \hat{R}_{Shr1}, \hat{R}_{Shr2}, \hat{R}_{Shr3}$, and $\hat{R}_{PEMM}$, have been calculated in section 2. The reliability of estimation is explained by the findings in Tables 1, 3, and 5. On the other hand, the results in Tables 2, 4, and 6 demonstrate the comparison between these approaches when biassed MSE and MAPE criteria are utilised. Nevertheless, all estimators rely on the values of the sample size.

Table 1 : $R = 0.9973695, \sigma = 0.2 ; \beta = 0.7 ; \alpha_1 = 2; \ \alpha_2 = 1$

n ,m	$\hat{R}_{MLE}$	$\hat{R}_{Prec}$	$\hat{R}_{alst}$	$\hat{R}_{wls}$	$\hat{R}_{Shr1}$	$\hat{R}_{Shr2}$	$\hat{R}_{Shr3}$	$\hat{R}_{PEMM^*}$
25,25	0.9954028	0.9884392	0.9886431	0.9884258	0.9883402	0.9883279	0.9883297	0.9898492
25,50	0.9955645	0.98843528	0.9887909	0.9884218	0.9883475	0.9883318	0.9883297	0.9903102
25,75	0.9955544	0.98833736	0.9890803	0.9883400	0.9883400	0.9883255	0.9883297	0.9902962
25,100	0.9954077	0.9883614	0.9891485	0.9883478	0.9883959	0.9883392	0.9883297	0.9902216
50,25	0.9942659	0.9884850	0.9884850	0.9884718	0.9883313	0.9883298	0.9883297	0.9887191
50,50	0.9923691	0.9882344	0.9886239	0.9882205	0.9883323	0.9883302	0.9883297	0.9885987
50,75	0.9951661	0.9883673	0.9886011	0.9883537	0.9883414	0.9883284	0.9883297	0.9893366
50,100	0.9926940	0.9881839	0.9887758	0.9881699	0.9883417	0.9883300	0.9883297	0.9889866
75,25	-5.447361	0.9883762	0.9880236	0.9883627	0.9882569	0.9883163	0.9883297	0.9881923
75,50	0.9950735	0.9883237	0.9883594	0.9883100	0.9883782	0.9883350	0.9883297	0.9885378
75,75	0.9902873	0.9883806	0.9883417	0.9883671	0.9883321	0.9883302	0.9883297	0.9886158
75,100	0.9901972	0.9883844	0.9883743	0.9883709	0.9883329	0.9883303	0.9883297	0.9887106
100,25	0.9851628	0.9883954	0.9878708	0.9883819	0.9883151	0.9883270	0.9883297	0.9876542
100,50	0.9872024	0.9883604	0.9882654	0.9883468	0.9883129	0.9883265	0.9883297	0.9881602
100,75	0.9886783	0.9883304	0.9883209	0.9883168	0.9883249	0.9883288	0.9883297	0.9883116
100,100	0.9899873	0.9883544	0.9884088	0.9883408	0.9883308	0.9883299	0.9883297	0.9885465

Table2 : $R = 0.997369, \sigma = 0.2 ; \beta = 0.7 ; \alpha_1 = 2; \ \alpha_2 = 1$

n ,m	Criteria	$\hat{R}_{MLE}$	$\hat{R}_{Pres}$	$\hat{R}_{alst}$	$\hat{R}_{wls}$	$\hat{R}_{Shr1}$	$\hat{R}_{Shr2}$	$\hat{R}_{Shr3}$	$\hat{R}_{PEMM^*}$	Finest
25,25	Baise	2.0318909	2.0318859	2.0320898	2.0318725	2.0317870	2.0317747	2.0317765	2.0332959	$\hat{R}_{Shr2}$
	MSES	4.1569083	4.1285606	4.1293891	4.1285062	4.1281584	4.1281085	4.1281157	4.1342924	$\hat{R}_{Shr2}$
	Map	1.9539565	1.9472828	1.9474782	1.9472700	1.9471880	1.9471762	1.9471779	1.9486341	$\hat{R}_{Shr2}$
25,50	Baise	2.0390113	2.0318820	2.0322377	2.0322377	2.0318686	2.0317785	2.0317765	2.0337569	$\hat{R}_{Shr3}$
	MSES	4.1575677	4.1285445	4.1299901	4.1299901	4.1284900	4.1281240	4.1281157	4.1361675	$\hat{R}_{Shr3}$
	Map	1.9541115	1.9472790	1.9476199	1.9476199	1.9472662	1.9471799	1.9471779	1.9490759	$\hat{R}_{Shr3}$

25,75	Baise	2.0390011	2.0317841	2.0325271	2.0317704	2.0317868	2.0317722	2.0317765	2.0337430	$\hat{R}_{wls}$
	MSES	4.1575269	4.1281466	4.1311665	4.1280912	4.1281577	4.1280985	4.1281157	4.1361107	$\hat{R}_{wls}$
	Map	1.9541017	1.9471852	1.9478973	1.9471721	1.9471878	1.9471739	1.9471779	1.9490625	$\hat{R}_{wls}$
25,100	Baise	2.0388544	2.0318081	2.0325952	2.0317946	2.0318426	2.0317860	2.0317765	2.0336684	$\hat{R}_{Shr3}$
	MSES	4.1569291	4.1282445	4.1314435	4.1281893	4.1283847	4.1281545	4.1281157	4.1358072	$\hat{R}_{Shr3}$
	Map	1.9539612	1.9472083	1.9479626	1.9471953	1.9472414	1.9471871	1.9471779	1.9489910	$\hat{R}_{Shr3}$
50,25	Baise	2.0377126	2.0319318	2.0314562	2.0319185	2.0317780	2.0317766	2.0317765	2.0321658	$\hat{R}_{alst}$
	MSES	4.1522741	4.1287470	1.9468710	4.1286929	4.1281222	4.1281162	4.1281157	4.1296981	$\hat{R}_{Shr3}$
	Map	1.9528669	1.9473268	1.9468710	1.9473140	1.9471794	1.9471780	1.9471779	1.9475511	$\hat{R}_{Shr3}$
50,50	Baise	2.0358158	2.0316812	2.0320706	2.0316673	2.0317791	2.0317769	2.0317765	2.0320454	$\hat{R}_{wls}$
	MSES	4.1445482	4.1277285	4.1293112	4.1276721	4.1281263	4.1281175	4.1281157	1.9474357	$\hat{R}_{Shr3}$
	Map	1.9510491	1.9470866	1.9474598	1.9470733	1.9471804	1.9471783	1.9471779	1.9474357	$\hat{R}_{Shr3}$
50,75	Baise	2.0386128	2.0318140	2.0320478	2.0318005	2.0317881	2.0317751	2.0317765	2.0327833	$\hat{R}_{Shr2}$
	MSES	4.1559454	4.1282685	4.1292185	4.1282133	4.1281630	4.1281102	4.1281157	4.1322083	$\hat{R}_{Shr2}$
	Map	1.9537296	1.9472139	1.9474380	1.9472009	1.9471891	1.9471766	1.9471779	1.9481429	$\hat{R}_{Shr2}$
50,100	Baise	2.0361408	2.0316307	2.0322226	2.0316167	2.0317884	2.0317768	2.0317765	2.0324333	$\hat{R}_{wls}$
	MSES	4.1458760	4.1275233	4.1299287	4.1274664	4.1281643	4.1281170	4.1281157	4.1307856	$\hat{R}_{wls}$
	Map	1.9513605	1.9470382	1.9476054	1.9470248	1.9471894	1.9471782	1.9471779	1.9478074	$\hat{R}_{wls}$
75,25	Baise	0.0440391	2.0318230	2.0314703	2.0318094	2.0317037	2.0317630	2.0317765	2.0316391	$\hat{R}_{alst}$
	MSES	2.0984883	4.1283048	4.1268719	4.1282497	4.1278199	4.1280612	4.1281157	4.1275575	$\hat{R}_{MLE}$
	Map	0.0717377	1.9472225	1.9468845	1.9472095	1.9471081	1.9471651	1.9471779	1.9470463	$\hat{R}_{MLE}$
75,50	Baise	2.0385202	2.0317704	2.0318061	2.0317568	2.0318249	2.0317817	2.0317765	2.0319845	$\hat{R}_{wls}$
	MSES	4.1555687	4.1280913	4.1282363	4.1280358	4.1283126	4.1281372	4.1281157	4.1289614	$\hat{R}_{wls}$
	Map	1.9536409	1.9471722	1.9472063	1.9471591	1.9472244	1.9471830	1.9471779	1.9473773	$\hat{R}_{wls}$
75,75	Baise	2.0337340	2.0318274	2.0317885	2.0318138	2.0317789	2.0317769	2.0317765	2.0320625	$\hat{R}_{Shr3}$
	MSES	4.1360749	4.1283226	4.1281645	4.1282676	4.1281256	4.1281176	4.1281157	4.1292782	$\hat{R}_{Shr3}$
	Map	1.9490540	1.9472267	1.9471894	1.9472137	1.9471803	1.9471784	1.9471779	1.9474521	$\hat{R}_{Shr3}$
75,1	Baise	2.0336439	2.0318312	2.0318211	2.0318177	2.0317796	2.0317770	2.0317765	2.0321574	$\hat{R}_{Shr3}$

00	MSES	4.1357079	4.1283381	4.1282971	4.1282832	4.1281287	4.1281181	4.1281157	4.1296638	$\hat{R}_{Shr3}$
	Map	1.9489676	1.9472304	1.9472207	1.9472174	1.9471810	1.9471785	1.9471779	1.9475430	$\hat{R}_{Shr3}$
100, 25	Baise	2.0286095	2.0318422	2.0313175	2.0318287	2.0317619	2.0317738	2.0317765	2.0311009	$\ddot{R}_{MLE}$
	MSE	4.1152671	4.1283827	4.1262511	4.1283279	4.1280565	4.1281047	4.1281157	4.1253713	$\ddot{R}_{MLE}$
	Mape	1.9441429	1.9472409	1.9467381	1.9472279	1.9471640	1.9471753	1.9471779	1.9465305	$\ddot{R}_{MLE}$
100, 50	Baise	2.0306491	2.0318072	2.0317122	2.0317936	2.0317596	2.0317733	2.0317765	2.0316070	$\ddot{R}_{MLE}$
	MSE	4.1235373	4.1282405	4.1278545	4.1281853	4.1280473	4.1281028	4.1281157	4.1274272	$\ddot{R}_{MLE}$
	Map	1.9460975	1.9472074	1.9471163	1.9471943	1.9471618	1.9471749	1.9471779	1.9470155	$\ddot{R}_{MLE}$
100, 75	Baise	2.0321250	2.0317772	2.0317677	2.0317635	2.0317716	2.0317755	2.0317765	2.0317583	$\ddot{R}_{PEMM}$
	MSES	4.1295325	4.1281186	4.1280801	4.1280631	4.1280961	4.1281120	4.1281157	4.1280420	$\ddot{R}_{PEMM}$
	Map	1.9475120	1.9471786	1.9471695	1.9471655	1.9471733	1.9471770	1.9471779	1.9471605	$\ddot{R}_{PEMM}$
100, 100	Baise	2.0334341	2.0318011	2.0318556	2.0317875	2.0317775	2.0317767	2.0317765	2.0319933	$\hat{R}_{Shr3}$
	MSES	4.1348543	4.1282160	4.1284372	4.1281607	4.1281199	4.1281165	4.1281157	4.1289968	$\hat{R}_{Shr3}$
	Map	1.9487665	1.9472016	1.9472537	1.9471885	1.9471789	1.9471781	1.9471779	1.9473857	$\hat{R}_{Shr3}$

Table 3: $R = 4.6092809$, $\sigma = 4$; $\beta = 2$; $\alpha_1 = 2$; $\alpha_2 = 3$

n ,m	$\ddot{R}_{max}$	$\ddot{R}_{Prec}$	$\ddot{R}_{alst}$	$\ddot{R}_{wlse}$	$\ddot{R}_{Shr1}$	$\ddot{R}_{Shr2}$	$\ddot{R}_{Shr3}$	$\ddot{R}_{Em}$
25,25	0.6355840	0.9608324	0.9608149	0.9608321	0.9608350	0.9608332	0.9608338	0.9605842
25,50	0.9134389	0.9608309	0.9608147	0.9608314	0.9607615	0.9608343	0.9608338	0.9602425
25,75	0.9125204	0.9608359	0.9607999	0.9608361	0.9606346	0.9608335	0.9608338	0.9604520
25,100	0.9178498	0.9608334	0.9607775	0.9608339	0.9608106	0.9608337	0.9608338	0.9603778
50,25	0.9159595	0.9608279	0.9608187	0.9608270	0.9608374	0.9608348	0.9608338	0.9608113
50,50	0.9273577	0.9608356	0.9608369	0.9608351	0.9608319	0.9608333	0.9608338	0.9608116
50,75	0.9246708	0.9608369	0.9608362	0.9608368	0.9608328	0.9608344	0.9608338	0.9608165
50,100	0.9170091	0.9608374	0.9608330	0.9608373	0.9596853	0.9608346	0.9608338	0.9607846
75,25	0.9455679	0.9608241	0.9607747	0.9608231	0.9608366	0.9608344	0.9608338	0.9607029
75,50	0.9608354	0.9608345	0.9608349	0.9608006	0.9608006	0.9608343	0.9608338	0.9608208
75,75	0.9211503	0.9608356	0.9608365	0.9608352	0.9608363	0.9608336	0.9608338	0.9608311
75,100	0.9189571	0.9608357	0.9608363	0.9608353	0.9608341	0.9608335	0.9608338	0.9608315

100,25	0.9096935	0.9608241	0.9607970	0.9608230	0.9608205	0.9608358	0.9608338	0.9606625
100,50	0.9553279	0.9608324	0.9608313	0.9608317	0.9608336	0.9608338	0.9608338	0.9608213
100,75	0.9209811	0.9608330	0.9608322	0.9608324	0.9608369	0.9608348	0.9608338	0.9608351
100,100	0.9273271	0.9608325	0.9608331	0.9608318	0.9608353	0.9608340	0.9608338	0.9608222

Table 4: $R = 4.6092809$, $\sigma = 4$; $\beta = 2$; $\alpha_1 = 2$; $\alpha_2 = 3$

n ,m	Criteria	$\bar{\bar{R}}_{MLE}$	$\bar{\bar{R}}_{Prec}$	$\bar{\bar{R}}_{alst}$	$\bar{\bar{R}}_{wls}$	$\bar{\bar{R}}_{Shr1}$	$\bar{\bar{R}}_{Shr2}$	$\bar{\bar{R}}_{Shr3}$	$\bar{\bar{R}}_{PEMM}$	Finest
25,25	Baise	3.7236968-	3.3984489-	3.3984659	3.3984487-	-3.3984458	-3.3984476	-3.3984470	-3.3986966	$\bar{\bar{R}}_{MLE}$
	MSES	98.643629	11.549452	11.549570	11.549453	11.549434	11.549446	11.549442	11.551139	$\bar{\bar{R}}_{PEMM}$
	Map	0.8541998	0.7795892	0.7795932	0.7795892	0.7795886	0.7795890	0.7795889	0.7796461	$\bar{\bar{R}}_{Shr1}$
25,50	Baise	3.4458419-	3.3984499-	3.3984661-	3.3984494-	3.3985193-	3.3984465-	3.3984470-	3.3990383-	$\bar{\bar{R}}_{MLE}$
	MSES	12.533945	11.549462	11.549572	11.549458	11.549933	11.549439	11.549442	11.553615	$\bar{\bar{R}}_{Shr2}$
	Map	0.7993959	0.7795895	0.7795932	0.7795894	0.7796054	0.7795888	0.7795889	0.7797245	$\bar{\bar{R}}_{Shr2}$
25,75	Baise	-3.4467604	-3.3984449	-3.3984809	3.3984447-	-3.3986462	-3.3984474	-3.3984470	-3.3988288	$\bar{\bar{R}}_{MLE}$
	MSES	11.903523	11.549427	11.549672	11.549426	11.550796	11.549444	11.549442	11.552037	$\bar{\bar{R}}_{Shr1}$
	Map	0.790671	0.7795884	0.7795966	0.7795883	0.7796346	0.7795889	0.7795889	0.7796764	$\bar{\bar{R}}_{wls}$
25,100	Baise	-3.4414310	-3.3984474	-3.3985033	-3.3984469	-3.3984702	-3.3984471	-3.3984470	-3.3989030	$\bar{\bar{R}}_{MLE}$
	MSES	11.8434812	11.5494452	11.5498250	11.5494417	11.549600	11.5494431	11.5494424	11.5525422	$\bar{\bar{R}}_{wls}$
	Map	0.7894492	0.7795890	0.7796018	0.7795888	0.7795942	0.7795889	0.7795889	0.7796935	$\bar{\bar{R}}_{wls}$
50,25	Baise	-3.4433213	-3.3984529	-3.3984621	-3.3984538	-3.3984434	-3.3984460	-3.3984470	-3.3984695	$\bar{\bar{R}}_{MLE}$
	MSES	11.8566433	11.5494826	11.5495450	11.5494887	11.549417	11.5494354	11.5494424	11.5495955	$\bar{\bar{R}}_{Shr1}$
	Map	0.78988288	0.77959027	0.77959237	0.77959047	0.7795880	0.77958867	0.77958891	0.77959408	$\bar{\bar{R}}_{Shr1}$
50,50	Baise	-3.4319231	3.3984452-	-3.3984439	3.3984457-	-3.3984489	-3.3984475	-3.3984470	3.3984692	$\bar{\bar{R}}_{MLE}$
	MSES	11.7796620	11.5494303	11.5494210	11.5494333	11.549455 2	11.5494456	11.5494424	11.5495932	$\bar{\bar{R}}_{alst}$
	Map	0.7872681	0.7795885	0.7795881	0.7795886	0.7795893	0.7795890	0.7795889	0.7795940	$\bar{\bar{R}}_{alst}$
50,75	Baise	-3.4346100	3.3984439-	3.3984446-	3.3984440-	3.3984480-	3.3984464-	3.3984470-	3.3984643-	$\bar{\bar{R}}_{MLE}$
	MSES	11.7966173	11.5494213	11.5494260	11.5494216	11.549449 1	11.549438	11.5494424	11.5495599	$\bar{\bar{R}}_{Pres}$

	Map	0.7878845	0.7795882	0.7795883	0.7795882	0.7795891	0.7795887	0.7795889	0.779592	$\bar{R}_{Pres}$
50,100	Baise	3.4422717-	3.3984434-	3.3984478-	3.3984435-	3.3995955-	3.3984462-	3.3984470-	3.3984962-	$\bar{R}_{MLE}$
	MSES	11.8492474	11.5494176	11.5494479	11.5494182	11.5572552	11.5494368	11.5494424	11.5497767	$\bar{R}_{Pres}$
	Map	0.7896421	0.7795880	0.7795891	0.7795880	0.7798523	0.7795887	0.7795889	0.7796001	$\bar{R}_{Pres}$
75,25	Baise	11.5495079	11.5498440	11.5495149	11.5494231	11.5494379	11.5494424	11.5503319	11.5503319	$\bar{R}_{wls}$
	MSES	0.7795911	0.7796024	0.7795913	0.7795882	0.7795887	0.7795889	0.7796189	0.7796189	$\bar{R}_{wls}$
	Map	3.4405100-	3.3984454-	3.3984468-	3.3984475-	3.3986033-	3.3984446-	3.3984470-	3.3984526-	$\bar{R}_{MLE}$
75,50	Baise	11.8371533	11.5494316	11.5494413	11.5494455	11.5505047	11.5494258	11.5494424	11.5494802	$\bar{R}_{Shr2}$
	MSES	0.7892379	0.7795885	0.7795888	0.7795890	0.7796247	0.7795883	0.7795889	0.7795901	$\bar{R}_{Shr2}$
	Map	3.4381305-	3.3984452-	3.3984443-	3.3984456-	3.3984445-	3.3984472-	3.3984470-	3.3984497-	$\bar{R}_{MLE}$
75,75	Baise	11.8207942	11.5494301	11.5494237	11.5494330	11.5494256	11.5494440	11.5494424	11.5494608	$\bar{R}_{aLst}$
	MSES	0.7886921	0.7795885	0.7795882	0.7795885	0.7795883	0.7795889	0.7795889	0.7795895	$\bar{R}_{aLst}$
	Map	-3.4403237	-3.3984451	-3.3984445	-3.3984455	-3.3984467	-3.3984473	-3.3984470	-3.3984493	$\bar{R}_{MLE}$
75,100	Baise	11.8358614	11.5494295	11.5494250	11.5494323	11.5494406	11.5494441	11.5494424	11.5494580	$\bar{R}_{aLst}$
	MSES	0.7891952	0.7795884	0.7795883	0.7795885	0.7795888	0.7795889	0.7795889	0.7795894	$\bar{R}_{aLst}$
	Map	-3.4134258	3.3984621-	-3.3984977	-3.3984634	-3.3984638	-3.3984490	-3.3984470	-3.3985290	$\bar{R}_{MLE}$
100,25	Bais	11.6517511	11.5495448	11.5497866	11.5495537	11.5495567	11.5494559	11.5494424	11.5499997	$\bar{R}_{Shr3}$
	MSE	0.7830249	0.7795923	0.7796005	0.7795926	0.7795927	0.7795893	0.7795889	0.7796077	$\bar{R}_{Shr3}$
	Mape	-3.4039529	3.3984484-	-3.3984495	-3.3984491	-3.3984472	-3.3984470	-3.39844706688121	-3.3984595	$\bar{R}_{MLE}$
100,50	Baise	11.5882587	11.5494519	11.5494596	11.5494565	11.5494436	11.5494426	11.5494424	11.5495277	$\bar{R}_{Shr3}$
	MSE	0.7808519	0.7795892	0.7795894	0.7795893	0.7795889	0.7795889	0.7795889	0.7795917	$\bar{R}_{Shr3}$
	Map	-3.4382997	-3.3984478	-3.3984486	-3.3984484	-3.3984439	-3.3984460	-3.3984470	-3.3984457	$\bar{R}_{MLE}$
100,75	Bais	11.8220771	11.5494474	11.5494533	11.5494519	11.5494214	11.5494352	11.5494424	11.5494336	$\bar{R}_{Shr1}$

	MSES	0.7887309	0.7795890	0.7795892	0.7795892	0.7795882	0.7795886	0.7795889	0.7795886	$\ddot{R}_{Shr1}$
	Map	-3.4319537	-3.3984483	-3.3984477	-3.3984490	-3.3984455	-3.3984468	-3.3984470	-3.3984586	$\ddot{R}_{MLE}$
100,100	Baise	11.7784328	11.5494513	11.5494468	11.5494559	11.5494319	11.5494410	11.5494424	11.5495212	$\ddot{R}_{Shr2}$
	MSES	0.7872751	0.7795892	0.7795890	0.7795893	0.7795885	0.7795888	0.7795889	0.7795915	$\ddot{R}_{Shr1}$
	Map	11.5495079	11.5498440	11.5495149	11.5494231	11.5494379	11.5494424	11.5503319	11.5503319	$\ddot{R}_{wls}$

5. The Conclusions

In this work, simulation were conducted for different values of parameters , and in Tables (1-4) for random data, two cases were set for the values of α_1 and α_2 when $\alpha_1$is greater than α_2 and vice versa, we conclude the following:

1- The MSE value decreases with increasing sample size (n, m) for all the factors mentioned below.

2- When the value of α decreases, the estimated stability value decreases.

3- When $\alpha_1 > \alpha_2$, the (Sh3) method is the best.

4- When $\alpha_1 < \alpha_2$, the estimation methods alternate with each other depending on the sample

Size.but the best methods are (Sh3, MLE, LSE, WLS), respectively.

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