Building Mathematical Models to Estimate the Concentrations of Uranium and Radon Gas in Water

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Abstract

The goal of this research is to create accurate mathematical models using numerical approaches for estimating uranium and radon gases levels in water. For determining the quantities of uranium and radon gas in water, several mathematical approaches were used: Linear regression, linear least squares, and graph method The review of outcomes revealed a high degree of concordance for the experimental data. This study is essential because it helps with the development of accurate and effective methods for evaluating uranium as well as radon gas concentrations in water. These developments are critical to improving the tracking and control of any health risks linked with uranium radiation.

Keywords: Radon gas, Uranium, Experimental data, Mathematical approaches. linear regression

1. Introduction:

In this thesis, we create mathematical models to estimate the concentrations of uranium and radon gas in water using numerical analysis methods. We create mathematical models to obtain accurate results. Numerical analysis plays a major role in our research which is an important aspect in mathematics and other sciences.

We will use numerical analysis methods to achieve accurate estimates, ensuring that the estimated values closely approximate the true values with minimal error. It is worth noting that recent years have witnessed an increase in estimation studies. For example, Steven Balter et al. investigated techniques for fluoroscopic skin radiation dose estimation [1], and Haregeweyn N and Yohannes F examined non-agricultural pollution models on swamps in Ethiopia [2]. Also, Michael J. et al. Using second generation computer software to evaluate internal dose in nuclear medicine [3].

In 2001, Emily R. Unsworth and others in England conducted a study using ICP mass spectrometry to measure uranium concentrations in seawater and groundwater. Their findings showed amounts of 2.68 nanograms of uranium in seawater and 0.3 nanograms in groundwater [4]. In 2002, Håkonsson-Hayes et al evaluated the concentration of 238 units in well-irrigated tomato, pumpkin, lettuce and radish contents in the Nambe region using ICP-MS. The measured values were less than 1 µg-1, 150 µg-1, 500 µg-1, and 1200 µg-1, respectively [5]. In 2003, Reimann and others measured uranium activity concentrations in river water, sea water,

and tap water at 2.63%, 1.13%, and 1.37%, respectively. In a study by Koji Oshita et al using ICPMS in Japan, activation values in drinking water from East Africa were measured as 0.005-48 g/L, 598-45800 g/L, and 0.002-1.59 µg/L, respectively [6]. Moreover, in (2017), Arif et al. presented an estimate of the effects of uranium radiation on workers in selected chemical plants, relying on some numerical analysis methods, Neville and Spline [7].
Overall, our study underscores the importance of numerical methods to accurately estimate uranium concentration and radon gas in water, thus contributing to the understanding and monitoring of potential health risks associated with uranium exposure.

2 Research methodology:

To calculate the concentration of uranium and radon gas in water, the study relied on mathematical modeling based on graph theory and numerical methods such as the Least square method, and the Linear regression method. Consequently, mathematical models were created by relying on the methods of the three aforementioned methods. In this section, three methods are presented for creating mathematical models through which uranium estimate uranium concentration and radon gas in water are calculated.

2.1 Linear regression [8-9]

Linear regression is a statistical procedure for calculating the value of a dependent variable from an independent variable. Linear regression measures the association between two variables. It is a modeling technique where a dependent variable is predicted based on one or more independent variables. Linear regression analysis is the most widely used of all statistical techniques.

Linear regression equation

$$b = \frac{n\sum xiyi - \sum xiyi}{n\sum xi^2 - (\sum xi)^2} \qquad (2.1)$$

$$= \frac{10(296486.145) - (13411)(167.116)}{10(23802995) - (179854921)}$$

$$= \frac{723668.774}{58175029} = 0.0124395086$$

$$a = \frac{\sum yi \sum xi^2 - \sum xi \sum xiyi}{n\sum xi^2 - (\sum xi)^2}$$

$$= \frac{(167.116)(23802995) - (13411)(296486.145)}{10(23802995) - (79854921)}$$

$$= \frac{1685621.825}{58175029} = 0.0289750062$$

$$y = a + bx$$

$$y = 0.0289750062 + 0.012435086x \quad (2.2)$$

We got on Table (2.1) which represents the mathematical model so that we conclude the mathematical model (2.1)

Table 2.1. The calculated uranium concentration in the water using linear regression method.

No.	Grass name	X_i	$R_k Det.$	$R_k Exp.$	Xi^2	$\sum xif(xi)$	E	E^2
1	**Dill**	399	4.99	4.98	159201	1987.02	0.01	0.0001
2	**Chamomile**	525	6.55	6.55	275625	3438.75	0	0
3	**Poot nut**	969	12.08	12.13	938961	11753.97	0.05	0.0025
4	**Slanderous**	1125	14.02	14.051	1265625	15807.373	0.031	0.000961
5	**Ginger**	1153	1437	14.37	1329409	16568.61	0	0
6	**Carway**	1320	16.45	16.45	1742400	21714	0	0
7	**Gray roses**	1349	16.8	16.8	1819801	22663.2	0	0
8	**Rose mary**	1361	16.96	16.96	1852321	23082.56	0	0
9	**Tablet**	1954	24.34	24.27	3818116	47423.58	0.07	0.0049
10	**Snack**	3256	40.532	40.555	10601536	132047.08	0.02	0.0004
	∑	**13411**		**167.116**	**23802995**	**296486.145**		**0.031861**

Figure below shows the estimated and experimental values of uranium concentration effect on the water by using the linear regression method

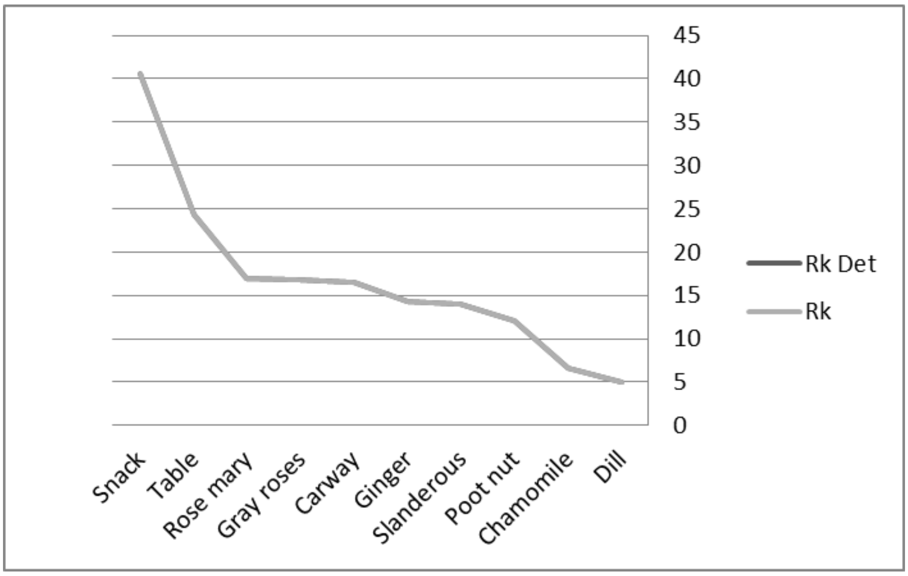

Fig. 2.1. The experimental and estimated values of the uranium Concentration in the water using the linear regression method.

2.2 Least square method [10-11]

The least squares approach is the technique of determining the best-fitted lines for any collection of data represented by a formula. This method includes decreasing the total of the total square of the remaining sections of the locations from a curve or line, and the overall pattern of results is determined statistically. The least square approach is used to fit curves during the analysis of regression.

Least square method can be stated as follows:

To establish the least squares method step by step, let's start from the beginning and go through each step:

Step 1: Begin with the given equations (2.3) and (2.4):

$$ma_o + a_I \sum_{i=1}^{m} N_i = \sum_{i=1}^{m} f(N_i) \qquad (2.3)$$

$$a_o \sum_{i=1}^{m} N_i + a_I \sum_{i=1}^{m} (N_i)^2 = \sum_{i=1}^{m} N_i f(N_i) \qquad (2.4)$$

Step 2: Solve equations (2.5) and (2.6):

$$Ioa_o + I3411a_I = 167.116 \qquad (2.5)$$

$$I3411a_o + 23802995a_I = 296486.145 \qquad (2.6)$$

Step 3: Multiply equation (2.5) by -1341.1:

$$-13411a_o - 17985492.1a_1 = -224119.2676 \qquad (2.7)$$

Step 4: Solve the resulting system of equations (2.6) and (2.7):

$$-13411a_o - 17985492.1a_1 = -224119.2676 \qquad (2.8)$$

$$13411a_o + 23802995a_I = 296486.145 \qquad (2.9)$$

Step 5: Solve for a_I by dividing equation (2.9) by 5817502.9: a_I= 0.012439508

Step 6: Substitute the value of a_I into equation (2.6) to solve for a_0:

$$13411a_0 + 23802995(0.012439508) = 296486.145$$

$$13411a_0 + 296097.5467 = 296486.145$$

$$13411a_0 = 296486.145 - 296097.5467$$

$$a_0 = 388.5983 / 13411 \approx 0.028976086$$

Step 7: Finally, substitute the values of a_0 and a_I into equation (2.5) to obtain the expression for R_k:

$$R_k = 0.028976086 + 0.012439508\,N \qquad (2.10)$$

We got on table (2.10) which represents the mathematical model so that we conclude the mathematical model (2.10).

Table 2.2 The calculated uranium concentration in the water using the last square method.

No.	Grass name	Ni	RkDet.	Rk Exp.	$\sum Nif(Ni)$	E	E^2
1	Dill	399	4,99	4,98	1987,02	0,01	0,0001
2	Chamomile	525	6,55	6,55	3438,75	0	0
3	Poot nut	969	12,08	12,13	11753,97	0,05	0,0025
4	Slanderous	1125	14,02	14,051	15,807,373	0,03	0,000961
5	Ginger	1153	1437	14,37	16568,61	0	0
6	Carway	1320	16,45	16,45	21714	0	0

7	Gray roses	1349	16,8	16,8	22663,2	0	0
8	Rose mary	1361	16,96	16,96	23082,56	0	0
9	Tablet	1954	24,34	24,27	47423,58	0,07	0,0049
10	Snack	3256	40,53	40,555	132047,08	0,02	0,0004
	∑	13411		167,116	296,486,145		0.009086

Below Figure shows the estimated and experimental values of uranium concentration effect on the water by using the last square method

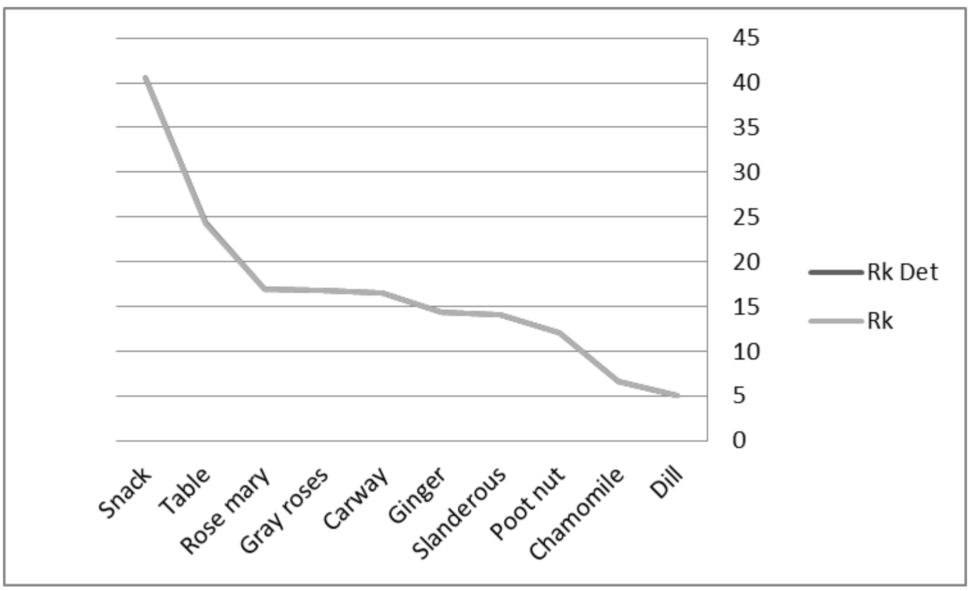

Fig 2. 2 The experimental and estimated values of the uranium Concentration in the water using the last square method.

2.3 Graph Theory [12-13]

Graph Theory is a branch of mathematics that studies the relationships and interactions between different objects, utilizing graphical representations to visualize these relationships. It is considered a fundamental field in pure mathematics, with applications extending to various domains such as computer science, networks, social relationships, recommendation systems, transportation, and many others**.** Graph Theory focuses on studying objects known as vertices or nodes and the links or edges that connect them. These nodes and edges are represented in graphical diagrams, where nodes are represented by points and edges are represented by lines or arcs connecting the nodes. Graph Theory provides mathematical tools and concepts to analyze and understand the structural properties and general characteristics of complex systems Graph Theory is a mathematical discipline that examines the connections and interactions among distinct entities using visual representations. It serves as a foundational area of pure mathematics, finding applications in computer science, network analysis, social dynamics, recommendation systems, transportation, and more. Graph Theory centers on the examination of nodes (or vertices) and the edges (or links) that link them, represented visually as points and lines/arcs respectively. By employing mathematical tools and concepts, Graph Theory facilitates the analysis and comprehension of the structural properties and general characteristics of intricate systems depicted through graphical diagrams. Key concepts in Graph Theory encompass degree, path, cycle, connectivity, trees, shortest paths, and other theoretical constructs that aid in data analysis and understanding the relationships between objects**.**

The applications of Graph Theory are vast and diverse, encompassing social network analysis, data organization, computer network design, artificial intelligence applications, ecological system analysis, recommendation systems, operational planning, and numerous other domains**.**

A graph, which is a fundamental mathematical structure, comprises a collection of vertices (or nodes) and a series of edges (or arcs/links) that establish connections between pairs of vertices. The edges signify relationships or associations among the vertices. Graphs offer a means to model real-world scenarios and problems, enabling the examination of relationships and interactions between different entities**.**

The generation of interpolating polynomials can be carried out recursively, as illustrated in the subsequent examples.

Using the graph method to calculate of uranium and radon gases levels in water as follows:

$$N1 = 399 \qquad Rk1 = 4.98$$

$$N9 = 1954 \qquad Rk10 = 40.555$$

$$R_k = \frac{(N-N_1)Rk_{10}(N-N_9)Rk_1}{N9\_N1}$$

$$R_k = \frac{(N-399)(40.555)-(N-1954)4.98}{1954\_399}$$

$$= \frac{40.555N - 16181.445 - 4.98N + 9730.92}{1555}$$

$$= \frac{35.575N - 6450.525}{1555}$$

$$R_k = 0.0228778135N\_4.1482475884 \qquad (2.11)$$

Table 2.3. The calculated uranium concentration in the water using linear regression method. by using the graph method.

Grass name	N_i	$Det\ R_k$ Graph	$Exp\text{-}R_k$ Graph	E	E^2
Dill	399	4.98	4.98	0	0
Chamomile	525	7.86	6.55	1.31	1.71
Poot nut	969	18.02	12.13	5.89	34.69
Slanderous	1125	21.59	14.051	7.539	56.83
Ginger	1153	22.23	14.37	7.86	61.78
Carway	1320	26.05	16.45	9.6	92.16
Gray roses	1349	26.71	16.8	9.91	98.20
Rose mary	1361	62.98	16.96	10.02	100.4
Tablet	1954	40.55	24.27	16.28	265.03
Snack	3256	70.34	40.555	29.785	887.14
Σ					1597.94

A visual representation, known as Figure 2.3, illustrates the relationship between the observed and predicted values in determining uranium and radon gas levels in water using the graph method. This graphical depiction offers a straightforward means to compare the actual measured values with the values estimated by the mathematical model represented by the graph. By visually examining the graph, one can assess the degree of accuracy and agreement between the two datasets, thereby evaluating the reliability and consistency of the graph model in estimating uranium and radon gas levels in water.

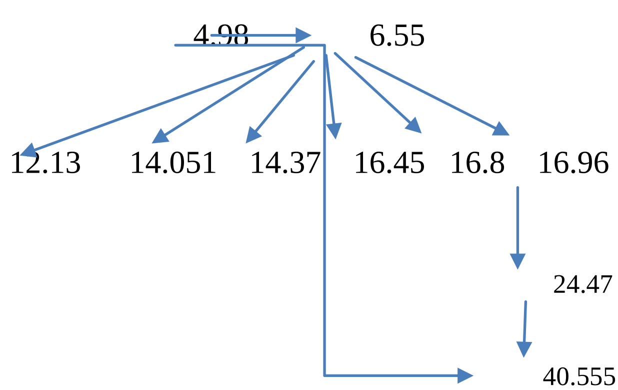

Fig. 2.3. shows the estimated and experimental values of uranium concentration effect on the water by using the graph method.

Conclusions

In this study, the linear regression method, the least squares method, and the histogram method were used to estimate the concentration levels of uranium and radon gases in water. The results obtained from these three methods were compared and analysed. The results indicated that the best method was the linear regression method, which showed the greatest effectiveness in estimating the concentrations of radon and uranium in water samples, as shown in Table 2.1, respectively. These results underscore the importance of choosing appropriate methods when developing models to estimate uranium and radon levels in water, as different methods may be more appropriate for different regions or sample types.

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بناء نماذج رياضياتية لتخمين تركيز اليورانيوم وغاز الرادون في الماء

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الخلاصه

الهدف من هذا البحث هو إنشاء نماذج رياضية دقيقة باستخدام الأساليب العددية لتقدير مستويات غازي اليورانيوم والرادون في الماء. لتحديد كميات اليورانيوم وغاز الرادون في الماء، تم استخدام عدة طرق رياضية: الانحدار الخطي، المربعات الصغرى الخطية، والرسم البياني. كشفت مراجعة النتائج عن وجود درجة عالية من التوافق للبيانات التجريبية. وتعد هذه الدراسة ضرورية لأنها تساعد في تطوير طرق دقيقة وفعالة لتقييم تركيزات اليورانيوم وغاز