الرادون في الماء. وتعتبر هذه التطورات حاسمة لتحسين تتبع ومراقبة أي مخاطر صحية مرتبطة بإشعاع اليورانيوم.

New Hybrid Explicit and Crank-Nicolson Method to Solve Wave Equation in Two Dimension

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Abstract

In this paper, we introduce the new hybrid numerical method to solve the wave equation in 2-Dim, in this technique we use two steps in the first one we apply the Explicit method to find all the node points and the second step we apply the C-N method to find the mid-point between any two point which are find in the step1. The results are more accuracy than other results.

الخلاصة

في هذا البحث نقدم الطريقة العددية الهجينة الجديدة لحل المعادلة الموجية في البعدين، في هذه التقنية نستخدم خطوتين، في الأولى نطبق الطريقة الصريحة لإيجاد جميع النقاط العقدية وفي الخطوة الثانية نطبق الطريقة الهجينة للعثور على نقطة المنتصف بين أي نقطتين تم العثور عليهما في الطريقة الصريحة.

Key-words; Explicit method, C-N method, wave equation, Hybrid method.

1- introduction:

There are many of numerical methods, where these methods are used to solve many of problems which can't solve by the analytic methods, so we are used the numerical methods to solve many problems in elasticity problems, diffusion problems, waves equations, … ADI numerical method is one of the numerical which is used to solve the wave equations (one -dimension and two-dimension). In (1993), S.J. Farlow, introduce the general form of wave equation [ ]. In (2014), Haneen F. Shareef, was used ADI method to solve Bi-Harmonic Equation. [1]. Hanan A. Alukaily used the ADI numerical method to find numerical solution of Fractional partial differential equation [2]. In (2020) Bushra and Awni are used ADI method for solving Heat diffusion problems [3]. And in (2023), Awni and Mohammed H. Rahim are solved the Wave equation in two-dimensional by Implicit numerical method.

2- Theoretical part

In this section we introduce the main steps to apply the hybrid method:

1. Make the mesh's for the region solution by vertically (Rows) and horizontally (columns).
2. Applying the Explicit method in the formula:

2-1. The common formula of the correct calcution method.

$$(1-k)w_{i,j+1} = (4-2k)w_{i,j} - (1-k)w_{i,j-1} - w_{i-1,j} - w_{i+1,j} \quad (1)$$

To find all node points in the grid (mesh)-

3. Applying the C-N

2-2 The common formula of C-N.

$$w_{i,j+1} = w_{i,j} + \frac{1}{4}k\left(w_{i+1,j} - w_{i-1,j}\right) + \frac{1}{4}k\left(w_{i+1,j+1} - w_{i-1,j+1}\right) \quad (2)$$

To find all mid-points between every two points which are find there in the 2nd step.

3- Particular part:

In this part we applying the Algorithm hybrid method to solve the wave equation as shown in the following example:

We solve the first example by the normal method and the second example is solve by hybrid method

Example:

The following the tow dimension wave equation:

$$\frac{1}{6}w_{tt} = w_{xx} + w_{yy} \quad (3)$$

When

$t \in [0,1]$, $x \in [0,2]$

With the boundary conditions;

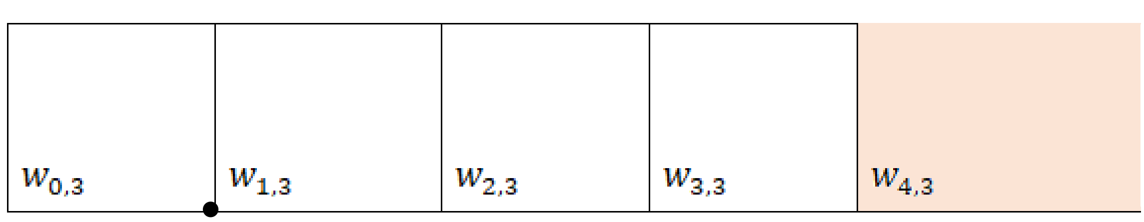
$$w(0,t) = w(1,t) = 1 \quad (4)$$

And initial conditions;

$$w(x,0) = F(x) = \cos x\ , \qquad w'(x,0) = g(x) \quad (5)$$

$s.t \quad 0 < x < 2 \quad , h = 0.5 \quad , \Delta t = 0.5\ sec$

Step1:



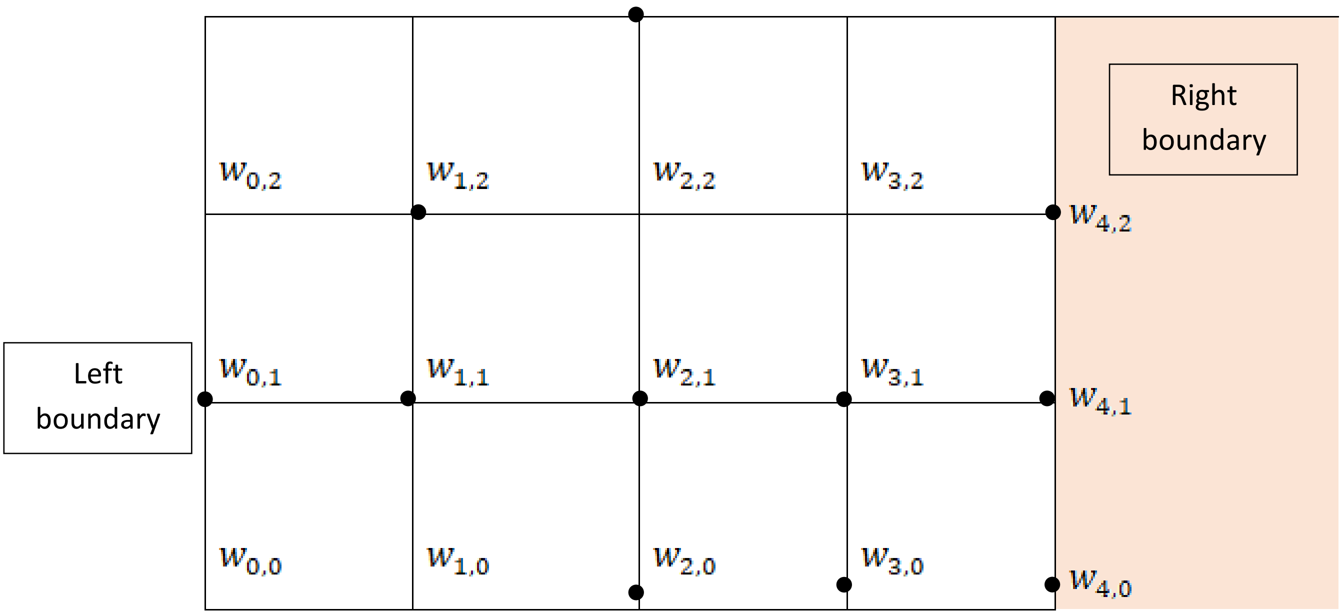


Fig (1-1): initial condition distribution of initial boundary condition mesh

Step2:

Applying the following formula;

$$(1-k)w_{i,j+1} = (4-2k)w_{i,j} - (1-k)w_{i,j-1} - w_{i-1,j} - w_{i+1,j} \quad (6)$$
$$\forall i = 1,2,3,\ldots\ldots..n\,, \quad \forall j = 0,1,2,3,\ldots\ldots\ldots.m$$

And we have

$$w_{i,-1} = w_{i,1} - 2\,\Delta t\, g\, i \;\;, \quad \forall i = 1,2,3,\ldots\ldots\ldots\ldots.n \qquad (7)$$

First find the value of (k)

$$k = \frac{h^2}{c^2(\Delta t)^2} = \frac{(0.5)^2}{16\,(0.5)^2} \Rightarrow k = 0.0625$$

$$w(x,0) = F(x) = \cos x$$

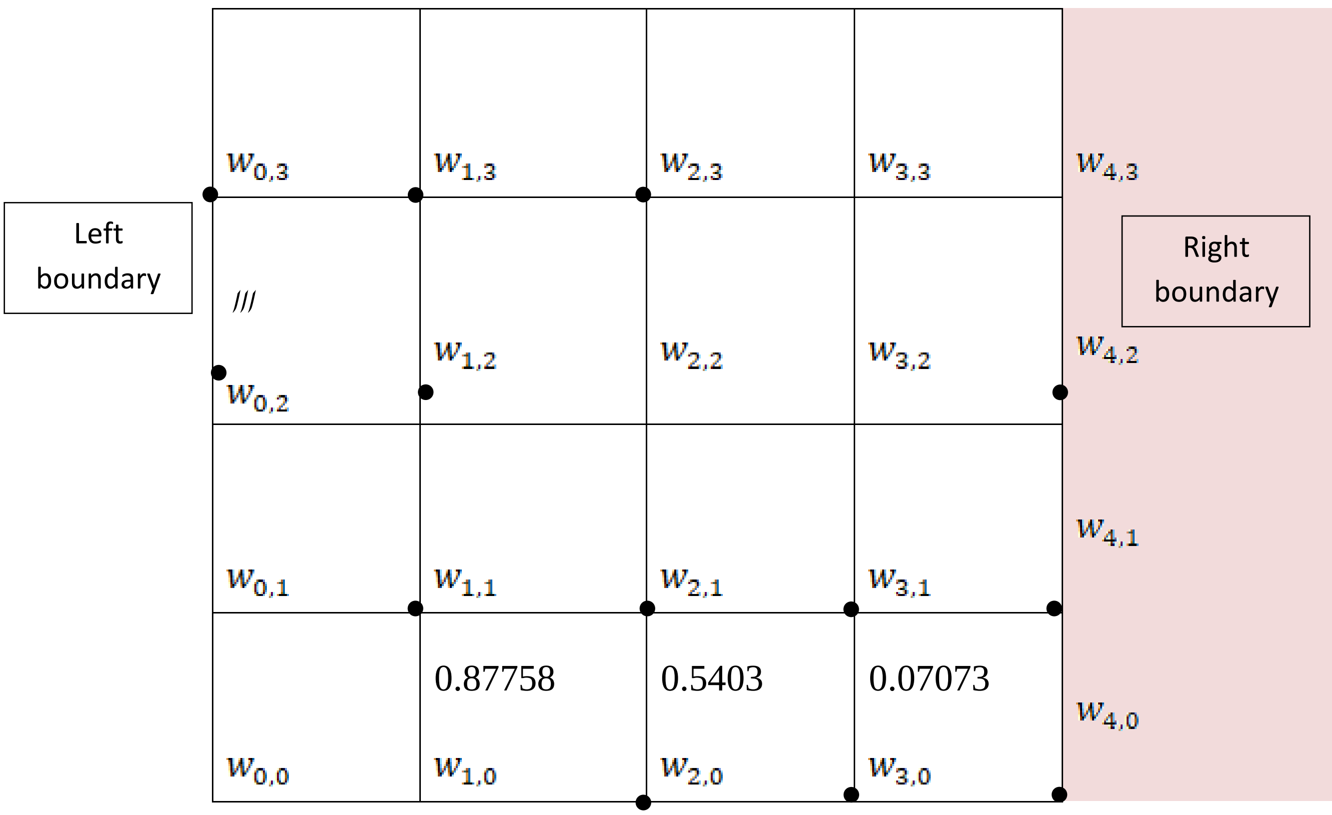
$$w_{(0,0)} = w_{0,0} = \cos 0 = 1$$

$$w_{(0.5,0)} = w_{1,0} = \cos(0.5) = 0.87758$$

$$w_{(1,0)} = w_{2,0} = \cos(1) = 0.5403$$

$$w_{(1.5,0)} = w_{3,0} = \cos(1.5) = 0.07073$$

$w = w_{4,0} = 1$



Fig(1-2): find the node points in level-1

Let j = 0 ، i = 1, 2, 3

$i = 1 \quad ,j = 0$

$$(1-K)w_{1,1} = (4-2K)\, w_{1,0} - (1-K)w_{1,-1} - w_{0,0} - w_{2,0} \qquad (8)$$

And find the value of $W_{1,-1}$ by eq (7)

$$w_{1,-1} = w_{1,1} - 2\, \Delta t\; g\; i$$

$$w_{1,-1} = w_{1,1} - 2\,(0.5)(2)$$

$$w_{1,-1} = w_{1,1} - 2 \qquad (9)$$

$$(1-K)\, w_{1,1} = (4-2K)\, w_{1,0} - (1-K)\big(w_{1,1}-2\big) - w_{0,0} - w_{2,0} \quad (10)$$

$$2\,(1-K)w_{1,1} = (4-2K)w_{1,0} - w_{0,0} - w_{2,0} + 2(1-K) \quad (11)$$

Now, we substate the condition and the value of k in (11)

$$2\,(1-0.0625)w_{1,1} = \big(4-2(0.0625)\big)(\,0.87758) - 1 - 0.5403 + 2(\,1-0.0625)$$

$$1.875\, w_{1,1} = 3.73532$$

$$w_{1,1} = 1.99217$$

$i = 2$, $j = 0$

$$(1-K)w_{2,1} = (4-2K)w_{2,0} - (1-K)w_{2,-1} - w_{1,0} - w_{3,0} \quad (12)$$

And find the value of $w_{2,-1}$ by eq $w_{2,-1} = w_{2,1} - 2\,\Delta t\; g\; i$

$w_{2,-1} = w_{2,1} - 2\,\Delta t\; g_2$

$$w_{2,-1} = w_{2,1} - 2 \quad (13)$$

Substitute eq (12) in eq (11) to obtain;

$$(1-K)w_{2,1} = (4-2K)w_{2,0} - (1-K)\big(w_{2,1}-2\big) - w_{1,0} - w_{3,0} \quad (14)$$

Simplify and complete the process on eq (14)

$$2\,(1-K)w_{2,1} = (4-2K)w_{2,0} - w_{1,0} - w_{3,0} + 2(1-K) \quad (15)$$

$$2(1-0.0625)w_{2,1} = \big(4-2(0.0625)\big)(0.5403) - 0.87758 - 0.07073 + 2(1-0.0625)$$

$$1.875\, w_{2,1} = 3.02035$$

$$w_{2,1} = 1.61085$$

And soon we have:

$w_{3,1} = 0.3246$

$w_{1,2} = 4.57182$

$w_{2,2} = 3.64852$

$w_{3,2} = -1.51443$

$w_{1,3} = 11.94626$

$w_{2,3} = 10.20841$

$w_{3,3} = -11.542741$

Table (1-1): Exhibits the distribution of border and primary conditions and all nodal points on the rectangular network

Left boundary

$w_{0,3}$	11.94625 $w_{1,3}$	10.20841 $w_{2,3}$	-11.542741 $w_{3,3}$	$w_{4,3}$
$w_{0,2}$	4.57182 $w_{1,2}$	3.64852 $w_{2,2}$	-1.51443 $w_{3,2}$	$w_{4,2}$
$w_{0,1}$	1.99217 $w_{1,1}$	1.61085 $w_{2,1}$	0.3246 $w_{3,1}$	$w_{4,1}$
$w_{0,0}$	0.87758 $w_{1,0}$	0.5403 $w_{2,0}$	0.07073 $w_{3,0}$	$w_{4,0}$

Right boundary

Application of hybrid method in this section we solve the previous section by the hybrid method, and we compare between the results of these examples:

$$\Delta x = h = \frac{0.5}{2} = 0.25$$

$$K = \frac{C\,\Delta t}{\Delta x} = \frac{16(0.5)}{0.25} = 32$$

$\cos(0) = w_{0,0} = 1$
$\cos(0.25) = w_{1,0} = 0.96891$
$\cos(0.5) = w_{2,0} = 0.87708$
$\cos(0.75) = w_{3,0} = 0.731689$
$\cos(1) = w_{4,0} = 0.54032$
$\cos(1.25) = w_{5,0} = 0.66687$
$\cos(1.5) = w_{6,0} = 0.07074$
$\cos(1.75) = w_{7,0} = -0.17825$
$\cos(2) = w_{8,0} = -0.41615$

Applying the Crank-Nicholson generic formula of wave equation.

$$w_{i,j+1} = w_{i,j} + \frac{1}{4}K\left(w_{i+1,j} - w_{i-1,j}\right) + \frac{1}{4}K\left(w_{i+1,j+1} - w_{i-1,j+1}\right)$$

And to compensate for $i = 1,3,5,7$ *and for* $j = 0,1$ وعن $K = 32$ *using the results of the method*

$$w_{1,1} = w_{1,0} + 8\left(w_{2,0} - w_{0,0}\right) + 8\left(w_{2,1} - w_{0,1}\right)$$

$$w_{1,1} = 0.96891 + 8(0.87708 - 1) + 8(1.61085 - 1)$$

$$w_{1,1} = 0.96891 + (-0.98336) + 4.8868$$

$$w_{1,1} = 4.87235$$

$$w_{3,1} = w_{3,0} + 8\left(w_{4,0} - w_{2,0}\right) + 8\left(w_{4,1} - w_{2,1}\right)$$

$$w_{3,1} = 0.73169 + 8(0.54030 - 0.87708) + 8\left((1.3707264) - (1.61085)\right)$$

$$w_{3,1} = 0.73169 + (-2.69424) + (-1.9209888)$$

$$w_{3,1} = -3.88354$$

And soon we have:

$$w_{5,1} = -6.970802$$

$$w_{7,1} = -9.277814$$

$$w_{1,2} = 30.94731$$

$$w_{3,2} = -10.0645568$$

$$w_{5,2} = -28.7347652$$

$$w_{7,2} = 3.7300286$$

$$w_{1,3} = 99.42726$$

$$w_{3,3} = -21.562998$$

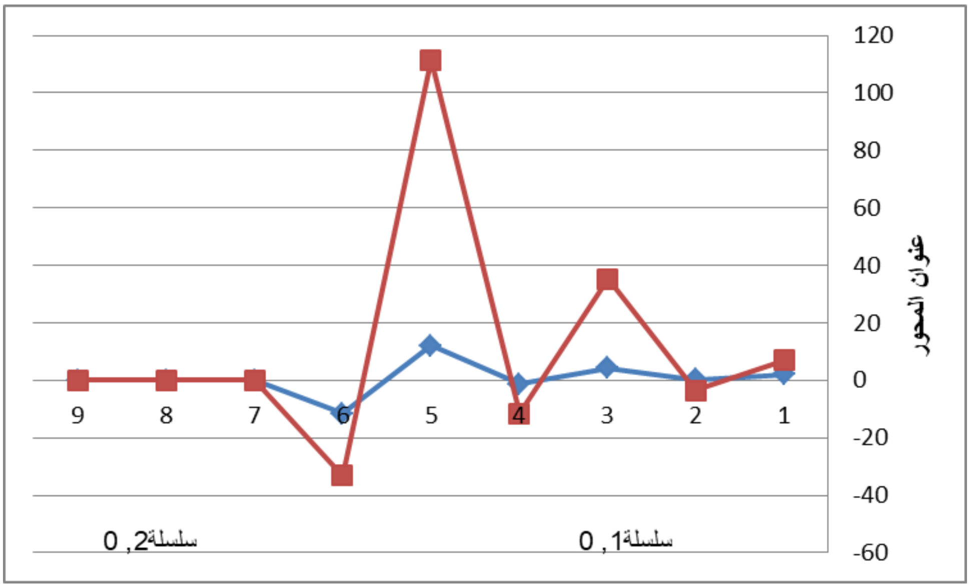
$$w_{5,3} = -202.2258704$$

$$w_{7,3} = 54.3064286$$

Table (1-2): below shows a comparison between the results of the original explicit method and the hybrid method

Point	Explicit Method Results	Hybrid Method Results	Difference
$w_{1,1}$	1.99217	4.87235	-2.8818
$w_{3,1}$	0.3246	-3.88354	4.20814
$w_{1,2}$	4.28608	30.94731	-26.66123
$w_{3,2}$	-1.41902	-10.0645568	-11.4835768
$w_{1,3}$	11.94626	99.42726	-87.481
$w_{3,3}$	-11.542741	-21.562998	-33.105739

Fig(1-3): comparison between the results of the original explicit method and the hybrid method.



Conclusions:

1. The results which obtaining by hybrid method is faster than the results for the original method.
2. The results of hybrid method are more accuracy than other results.
3. The hybrid method saves time and reduces effort.

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