Analytical Solution of the Broer-Kaup Equations by the Differential Transform Method

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ABSTRACT.

The Broer-Kaup system is one of the basic systems in studying and understanding mathematical and physical applications, as it contributes to understanding and studying difficult physical phenomena such as wave correlations, as this system consists of a set of non-linear partial differential equations that define variables across space and time.By using and integrating the differential transformation method (DTM) with the ADM analysis method, we provide a complete methodology for solving the Broer-Kaup equation in this study.The DTM differential transformation method has been widely used to solve many differential equations in many different mathematical and physical fields and applications. We have overcome the difficulties resulting from the nonlinear part of the Purer-Cup equation by integrating the DTM differential transformation method with the ADM analysis method, which improves efficiency and accuracy. Arithmetic. We have found the good performance and effective flexibility of our proposed method by implementing numerical and theoretical tests. The study and analysis of the Brewer Coup system is an interesting topic for researchers in different fields, especially in mathematical, engineering and physics applications. Absorbing and understanding this system makes it easy and possible to develop new applications and more complex understanding of difficult physical phenomena that are involved in different fields.

Keywords :Partial Differential equation, Broer-Kaup Equations, Differential transformation method, Adomain Decomposition Method

1. INTRODUCTION

Partial differential equations(PDE), are one of the important branches of mathematics that fall within the applied sciences and that play a pivotal role in the mechanism of perceiving and understanding natural phenomena and studying them in various applications and scientific and engineering fields. The partial differential equation can be considered as a relationship between the dependent variable (the function) And independent variables (partial derivatives), and they can also be used in physical and mathematical computational systems by describing the application at a specific time and place, such as heat transfer, wave motion, fluid flow, and other natural physical phenomena [1-3]

One of the important mathematical systems concerned with understanding and interpreting non-linear mathematical phenomena in applied mathematics and physics is known as the Broer-Kaup equation system This system includes a community of nonlinear partial differential equations and also contains a set of rather difficult and complex vocabulary. This system is also of great importance and effectiveness in understanding and interpreting wave interactions and other difficult and complex basic physical phenomena. The most important features that distinguish the Broer-Kaup system of equations are its comprehensive comprehension and ability to analyze and interpret solution solutions, which provides a good approach to comprehending and understanding complex physical phenomena. And its applications are diverse in a wide range of fields, for example, fluid motion, engineering, plasma physics, and its applications in multidisciplinary fields. It is clear that understanding and studying the Broer-Kaup system of equations is a thought-provoking challenge. Researchers because it requires a deep understanding of nonlinear interactions and their various applications.[4][7-5]

The differential transformation method is considered within of the approximate(numerical) methods that require numerical calculations, and it is an important tool to solve ordinary differential equations(ODE). The differential transformation method(DTM) uses polynomial mathematical functions to find the best solution near to the precise solution. The differential transformation method differs from the rest of the other methods that require complex operations in calculating the derivatives in each step, as in the Taylor expansion series. Also, the differential transformation method(DTM) does not require creating these calculations, which are considered somewhat complicated, as the differential transformation method(DTM) is an Recursiveness method, and thus we obtain The Tyler series has a high degree of structure, which is important in solving complex mathematical problems[8-14]

The Adomian analysis method is considered one of the methods that is characterized by a scientific-mathematical method that divides mathematical solutions into multiple and small partial sub-solutions, where it deals with each solution separately until the problem is solved, and by finding solutions from the sub-solutions iteratively, which makes the mathematical solutions improve, and after hybridization With the differential transformation method, the Adomian analysis method becomes more useful in dealing with complex problems represented by the presence of the nonlinear part in the differential equations, as the differential transformation method facilitates finding solutions to the differential equations by converting them into mathematical equations, where the DTM method has been integrated. And ADM, which in turn has become a powerful and effective fundamental image for solving complex mathematical problems that exist in many different engineering and mathematical sciences. [15-19]

2. Differential Transformation Method (DTM)

One of the important mathematical procedures in solving differential equations, which is involved in many diverse scientific and engineering fields, is the differential transformation method (DTM). Its effective potential and ability to solve differential equations has its beginnings with applied mathematics, by avoiding some of the problems and difficulties experienced by classical methods, like the differential transformation method. (DT M) understands important basics for any approach and able to solve ordinary differential equations in this introductory period [20, 21]

The differential transformation method is based on a set of mathematical functions that are used in transformations, usually called differential mathematical transformations, which in turn convert differential equations into algebraic mathematical equations. The purpose of the transformation is to facilitate the differential equation, facilitate dealing with it, and find its solutions in an easier way, as these transformations accomplish finding mathematical limits between the function. The original and its derivatives, as the essential point in this topic is to employ an appropriate auxiliary parameter to represent the unknown function in the form of a series expansion, then use differential mathematical transformations to find multiple algebraic equations. It is also possible to employ a series of series expansion operations that are acquired by solving this system to reshape the solution to fundamental differential equation. [22-24]

DTM is characterized by its ability to interact with non-homogeneous differential-equations like non-linear equations. This type of equation is widespread in the real world, as this type is difficult to solve using analytical numerical methods. The DTM method transforms the differential equation into a system Mathematical algebraic equations. This method can be modeled on scientific fields such as engineering, mechanics, physics, and many other diverse fields. [25]

Comparing the characteristics with other solution methods and methods, the differential transformation method has many positives and advantages because it does not require specifying the field. DTM is a typical method for difficult and complex engineering applications or non-linear boundaries, unlike finite difference methods or finite elements as well, as the possibility of the DTM method To provide solutions close to the exact solution, which allows comparison between computational efficiency and the correctness of the solution. [26]

To sum up, the possibility of solving with the differential transformation method(DTM) is a successful flexible method for solve differential equations available in many scientific and engineering contexts, which allows of its applications through the possibility to converting differential equation into algebraic mathematical equations, which in turn allows for more effective and systematic methods and methods for solving them, especially solving systems. Controlled by dynamic differential equations, the DTM method is a very suitable tool that can deal with nonlinear terms and non-homogeneous differential equations.

Combining the Adomian analysis method (ADM) with the differential transformation method(DTM) is an effective way for find the solution to partial differential equations(PDES) with good efficiency, as the solution domain is divided into smaller subdomains and each domain is treated separately. The differential problem solution is obtained by iteratively optimizing solutions in different subdomains, each of which is solved analytically using the DTM method. DTM works to transform partial differential equations into algebraic mathematical equations, which allows accurate solutions within each subfield, as these two methods work on effective and successful processors to obtain faster convergence to the solution. [25]. We take into account that a mapping consists of two variables $w(x,t)$ assuming that it is possible to represent it.

We take into account that a function consists of two independent variables $w(x,t)$ and assume that it can be represented As the combine of two functions of one variable, i.e. $w(x,t) = f(x)g(t)$. Based on the properties of one-dimensional differential transform ,function $w(x,t)$ can be written as :

$$w(x,t) = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} W(i,j)\, x^i t^j \qquad (1)$$

Where $W(i,j)$ is named the spectrum of $w(x,y)$. Now we give the basic,The definitions and properties of 2-dimensional . If $w(x,t)$ and continuously differentiable With respect to time t in the domain of interest, then:

$$w(k,h) = \frac{1}{k!\,h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial t^h} w(x,t)\right]_{x_0=0,t_0=0} \qquad (2)$$

Where the spectrum function $W(k,h)$ is the transformation function, which is also called the T function. Let $w(x,y)$ be the original function while the capital letter $W(k,h)$ denotes the transformed function. Now we define the differential inverse transform of $W(k,h)$ as following:

$$w(x,t) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} W(k,h)(x-x_0)^k (t-t_0)^h \qquad (3)$$

Using equation (3) in equation (4), we have

$$w(k,h) = \frac{1}{k!h!}\left[\frac{\partial^{k+h}}{\partial x^k \partial t^h} w(x,t)\right]_{x_0=0,t_0=0} \quad x^k t^h$$

$$w(k,h) = \sum_{k=0}^{\infty} \sum_{h=0}^{\infty} W(k,h) x^k t^h \qquad (4)$$

Now from the definitions and equation (3) *and* (4) above, we can get some basic mathematical operations that are performed by the two-dimensional differential transformation in schedule 1. [23]

Table 1.Show some transforms for multi types of functions

Original function	**Transformed function**
$w(x,t) = u(x,t) \pm v(x,t)$	$W(k,h) = U(k,h) \pm V(k,h)$
$w(x,t) = cu(x,t)$	$W(k,h) = cU(k,h)$
uu_x	$W(k,h) = \sum_{r=0}^{k} \sum_{s=0}^{h} U(r,h-s)(k-r+1)\, U(k-r+1,s)$
$w(x,t) = \frac{\partial}{\partial x} u(x,t)$	$W(k,h) = (k+1)U(k+1,h)$
$w(x,t) = \frac{\partial}{\partial x} v(x,t)$	$W(k,h) = (k+1)V(k+1,h)$
$w(x,t) = \frac{\partial}{\partial t} u(x,t)$	$W(k,h) = (h+1)U(k,h+1)$
$w(x,t) = \frac{\partial}{\partial t} v(x,t)$	$W(k,h) = (h+1)V(k,h+1)$
$(uv)_x$	$W(k,h) = \sum_{r=0}^{k} \sum_{s=0}^{h} (r+1)U(r+1,h-s)V(k-r,s) + \sum_{r=0}^{k} \sum_{s=0}^{h} (r+1)V(r+1,h-s)U(k$
u_{xxx}	$(k+1)(k+2)(k+3)U(k+3,h)$

3. Application method

(Broer-Kaup system) Consider the following, [27]

$$u_t + uu_x + v_x = 0 \qquad (5)$$

$$v_t + u_x + (uv)_x + u_{xxx} = 0 \qquad (6)$$

Having exact solution defined by :

$$u(x,t) = 1 - 2\tanh\,(t-x)$$

$$v(x,t) = 1 - 2\tanh^2(t-x)$$

with the initial conditions:

$$u(x,0) = 1 + 2\tanh(x) \quad (7)$$

$$v(x,0) = 1 - 2\tanh^2(x) \quad (8)$$

Taking the differential transform (DTM) of system (5-6) and using the relevant operations in table(1) we get:

$$(h+1)\mathrm{U}(k,h+1) + \sum_{r=0}^{k}\sum_{s=0}^{h} \mathrm{U}(r,h-s)(k-r+1)\mathrm{U}(k-r+1,s) + (k+1)V(k+1,h) = 0 \quad (9)$$

$$(h+1)V(k,h+1) + (k+1)\mathrm{U}(k+1,h) + \sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)\mathrm{U}(r+1,h-s)V(k-r,s) + \sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)V(r+1,h-s)\mathrm{U}(k-r,s) + (k+1)(k+2)(k+3)\mathrm{U}(k+3,h) = 0 \quad (10)$$

From the initial conditions, we use differential transformation (DTM) to convert the initial conditions for u and v as :

$$u(x,0) = 1 + 2\tanh(x) \text{ to } U(k,0) \quad , k = 0,1,2,3 \ldots \quad (11)$$

$$v(x,0) = 1 - 2\tanh^2(x) \text{ to } V(k,0) \quad , k = 0,1,2,3 \ldots \quad (12)$$

Such that

$$\mathrm{U}(k,0) = \frac{1}{k!}\left[\frac{d^k u(x,0)}{dx^k}\right]_{x=0} \quad k = 0,1,2,3 \ldots \ldots \quad (13)$$

$$V(k,0) = \frac{1}{k!}\left[\frac{d^k v(x,0)}{dx^k}\right]_{x=0} \quad k = 0,1,2,3 \ldots \ldots \quad (14)$$

From equation (13) we get $\mathrm{U}(k,0)$:

$\mathrm{U}(0,0) = 1$	$\mathrm{U}(1,0) = 2$	$\mathrm{U}(2,0) = 0$	$\mathrm{U}(3,0) = -0.6666666667$
$\mathrm{U}(4,0) = 0$	$\mathrm{U}(5,0) = 0.2666666667$	$\mathrm{U}(6,0) = 0$	

And From equation (14) we get $V(k,0)$:

$V(0,0) = 1$	$V(1,0) = 0$	$V(2,0) = -2$	$V(3,0) = 0$
$V(4,0) = 1.333333333$	$V(5,0) = 0$	$V(6,0) = -0.7555555556$	

Equation (9) becomes as follow:

$$U(k,h+1) = \frac{1}{(h+1)} * \left[-\sum_{r=0}^{k}\sum_{s=0}^{h} \mathrm{U}(r,h-s)(k-r+1)\mathrm{U}(k-r+1,s) - (k+1)\ V(k+1,h)\right]$$

And equation (10) becomes as follow:

$$V(k,h+1) = \frac{1}{(h+1)}\left[-(k+1)\mathrm{U}(k+1,h) - \sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)\mathrm{U}(r+1,h-s)\ V(k-r,s) - \sum_{r=0}^{k}\sum_{s=0}^{h}(r+1)\ V(r+1,h-s)\mathrm{U}(k-r,s) - (k+1)(k+2)(k+3)\mathrm{U}(k+3,h)\right]$$

We obtain $\mathrm{U}(k,h)$ *for* $k = 0,1,2,3 \ldots \ldots \ldots$ *and* $h = 0,1,2,3 \ldots \ldots \ldots$

$\mathrm{U}(k,h)$	$h=0$	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
$k=0$	1	-2	0	0.6666666683	$-2.500000000 * 10^{-9}$	0.2666666510

$k=1$	2	0	-2	0	1.333333330	$2.360000000 * 10^{-7}$
$k=2$	0	2	0	-2.666666661	$-1.825000000 * 10^{-7}$	2.266667744
$k=3$	-0.6666666667	0	2.666666666	$1.333333333 * 10^{-8}$	-3.777777915	$6.600000000 * 10^{-7}$
$k=4$	0	-1.333333333	0	3.777777787	$-7.500000000 * 10^{-8}$	5.511111676
$k=5$	0.2666666667	0	-2.266666667	$1.333333333 * 10^{-8}$	5.511110932	$8.040000000 * 10^{-6}$

And we obtain $V(k,h)$ *for* $k = 0,1,2,3 \dots \dots \dots$ *and* $h = 0,1,2,3 \dots \dots \dots$

$V(k,h)$	$h=0$	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$
$k=0$	**1**	0	-2	0	1.333333309	$1.920000000 * 10^{-}$
$k=1$	0	4	$-5.000000000 * 10^{-9}$	-5.333333327	$-7.000000000 * 10^{-8}$	4.533334044
$k=2$	-2	0	8	0	-11.33333372	$3.780000000 * 10^{-}$
$k=3$	0	-5.333333333	$-5.000000000 * 10^{-9}$	15.11111133	$-1.275000000 * 10^{-6}$	-22.04439052
$k=4$	1.333333333	0	-11.33333334	$1.000000000 * 10^{-7}$	27.55555508	$6.600000000 * 10^{-}$
$k=5$	0	4.5333333333	$-5.000000000 * 10^{-8}$	-22.04444440	$7.750000000 * 10^{-7}$	49.13779022

$$u(x,t) = \sum_{k=0}^{5}\sum_{h=0}^{5} \mathrm{U}(k,h)x^k t^h = 1 + 8.040000000 * 10^{-6}\ x^5t^5 - 2.500000000 * 10^{-9}t^4 - 7.500000000 * 10^{-8}x^4t^4 + 1.333333333 * 10^{-8}x^5t^3 - \frac{4}{3}x^4t + 6.600000000 * 10^{-7}x^3t^5 + 1.333333333 * 10^{-8}x^3t^3 - 1.825000000 * 10^{-7}x^2t^4 + 2.360000000 * 10^{-7}xt^5 - 2t + 2x + 0.6666666683\,t^3 - 0.2666666510t^5 - 0.6666666667x^3 + 0.2666666667\,x^5 - 2xt^2 + 1.333333330x\,t^4 + 2x^2t - 2.666666661\,x^2t^3 + 2.266667744x^2t^5 + 2.666666666\,x^3t^2 - 3.77777791x^3t^4 + 3.777777787x^4t^3 - 5.511111676x^4t^5 - 2.266666667x^5t^2 + 5.511110932x^5t^4$$

$$v(x,t) = \sum_{k=0}^{5}\sum_{h=0}^{5} V(k,h)x^k t^h = 1 + 4*x*t - 5.333333327*x*t^3 + 4.533334044*x*t^5 + 8*x^2*t^2 - 11.33333372*x^2*t^4 + 15.11111133*x^3*t^3 - 22.04439052*x^3*t^5 - 11.33333334*x^4*t^2 + 27.55555508*x^4*t^4 - 22.04444440*x^5*t^3 + 49.13779022*x^5*t^5 + 1.333333333*x^4 - 2*t^2 + 1.333333309*t^4 - 2*x^2 + 1.920000000*10^{-7}*t^5 + 7.750000000*10^{-7}*x^5*t^4 + 68/15*x^5*t + 6.600000000*10^{-6}*x^4*t^5 - 5.000000000*10^{-8}*x^5*t^2 + 1.000000000*10^{-7}*x^4*t^3 - 1.275000000*10^{-6}*x^3*t^4 + 3.780000000*10^{-6}*x^2*t^5 - 16/3*x^3*t - 8.000000000*10^{-9}*x^3*t^2 - 7.000000000*10^{-8}*x*t^4 - 5.000000000*10^{-9}*x*t^2$$

After expanding the series with additional terms and by substituting the values of x and t by fixing one of them and moving the other to some values chosen in the domain of the problem, we find the results that we will mention in the table:

Error results for the u series

x	DTM(x)	EXACT(x)	Error
-0.5	-0.2725260419	-0.270297905	2.23 e-3

-0.4	−0.1439555833	−0.143339932	6.16e-4
-0.3	−0.00118012504	−0.001040422	1.40e-4
-0.2	0.1561601252	0.1562019894	4.19e-5
-0.1	0.3272262085	0.3272489114	2.27e-5
0	0.5101562500	0.5101626752	6.4252e-6
0.1	0.7022412918	0.7022299328	1.14e-5
0.2	0.9001011253	0.9000832501	1.79e-5
0.3	1.099860126	1.099916750	5.66e-5
0.4	1.297323084	1.297770067	4.47e-4
0.5	1.488151042	1.489837325	1.69e-3
Mse			7.65e-7

x	DTM(x)	EXACT (x)	Error
-0.5	−0.5270833411	−0.523188312	3.90e-3
-0.4	−0.4350640024	−0.432595740	2.47e-3
-0.3	−0.3303280006	−0.328073541	2.25e-3
-0.2	−0.21087866672	−0.208735554	2.14e-3
-0.1	−0.07575933320	−0.074099134	1.66e-3
0	0.07500000054	0.0757656854	7.66e-4
0.1	0.2404360017	0.2401020754	3.34e-4
0.2	0.4187053366	0.4173747750	1.33e-3
0.3	0.6071380056	0.6052493596	1.89e-3
0.4	0.8022906767	0.8006640108	1.63e-3
0.5	1.000000019	1.0	1.90e-8
Mse			3.85e-6

Error results for the v series

x	DTM(x)	EXACT (x)	Error
-0.5	0.1930750809	0.1931716166	9.65e-5
-0.4	0.3464734527	0.3463868998	8.66e-5
-0.3	0.4990028510	0.4989590364	4.38e-5
-0.2	0.6440731721	0.6440024586	7.07e-5
-0.1	0.7738398951	0.7737029864	1.37e-4
0	0.880208333	0.8800296976	1.79e-4
0.1	0.9558378828	0.9556664935	1.71e-4
0.2	0.9951462726	0.9950083215	1.38e-4
0.3	0.9953138164	0.9950083215	3.05e-4
0.4	0.9572876593	0.9556664935	1.62e-3
0.5	0.8867860320	0.8800296976	6.76e-3
Mse			4.41e-6

x	DTM(x)	EXACT (x)	Error
-0.5	−0.1667362974	−0.160051317	6.68e-3
-0.4	−0.03116240238	−0.026165278	5.00e-3
-0.3	0.1159398975	0.1181103354	2.17e-3
-0.2	0.2717539474	0.2694791800	2.27e-3
-0.1	0.4302284243	0.4231555252	7.07e-3
0	0.583333338	0.5728954658	1.04e-2

0.1	0.7223160309	0.7112775720	1.10e-2
0.2	0.8389571808	0.8302739236	8.68e-3
0.3	0.9268267971	0.9220859660	4.74e-3
0.4	0.9825402288	0.9801325817	2.41e-3
0.5	1.007014154	1.0	7.01e-3
Mse			4.67e-5

4. CONCLUSION

From more complex systems than ever before and improving our understanding of fundamental physical processes as we continue to investigate and enhance this integrated approach, the (ADM) field analysis method and the (DTM) differential transformation method together provide a powerful method for solving the Broer-Kaup equations. Our research has proven the effectiveness of this integrated method in solving complex differential equations that control thermodynamics and fluid dynamics efficiently and accurately. The nonlinearities and bounds in the equations of the Broer-Kaup equations have proven to be formidable obstacles, but we have overcome them by combining the two approaches, and the results are robust and reliable. (DTM) is a powerful and flexible tool for academics in many different fields, including applied mathematics and engineering techniques.

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