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Some Properties on Infra Soft Nano $\alpha-$Open(Closed) Sets

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Abstract: In this paper, we explained a soft nano closed (open) set , referred to as an infra soft nano α closed (open) set and infra soft nano semi closed (open) and establish some fundamental features of this set. The connection between the infra soft nano-open(closed)set and other soft nano topological closed (open) sets is investigated. The findings mentioned in this study are preliminary and serve as and in the study are preliminary and serve as an introduction to more advanced research in theoretical and practical areas.

keywords: Nano soft open (closed)set, Nano soft topological space, Infra Nano soft topology, Infra Nana Soft α-open (closed) sets.

Introduction

In 1999, Molodtsov [13] developed custom soft sets, a novel mathematical approach to soft topology. He studied the union operator of open intersections and the difference and complement functions of two soft sets. However, Ali et al. found significant flaws in their definition. In 2003 [14] began to explore the basic principles and concepts of soft set theory . [7] developed new operators and methods to preserve many features and conclusions of soft set theory .In 2011 [8] and [18] defended the use of soft open sets in soft topological spaces (STS). There is a large body of research on topological terms in soft topology [1-3], [9] Many SOS(soft open set) and SCS (soft closed sets) are described in terms of parameter sets μ on some universe *U*. They are then extended to related soft topologies and their properties are examined. [4] introduced the concept of soft open sets in nano topological spaces, with are called nano-soft topological spaces. The separation axiom for NanoZ-topological space [11] defines soft NanoZ-open sets using NanoZ-topological space (SNZ) [10]. Soft topology has been extended to various structures with weaker or stronger topologies, one of which is the fundamental soft topological space (STS). Subsoft α-open sets on subsoft topological spaces have also been studied [4–7], [15–18]. Terms such as subsoft connectivity, subsoft local connectivity, and subsoft compactness are used in these studies.

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In this paper, we study infra soft nano $\alpha -$ interior $(\mathrm{ISN}_\alpha - \mathrm{int})$ infra soft nano α- closure $(\mathrm{ISN}_\alpha - \mathrm{cl})$ infra soft nano $\alpha - o$pen(ISN_αOS) infra soft nano α – closed (ISN_α-CS) sets, and establish some of their basic characteristics.

.This means that the infra nano soft topological spaces (INSTS)are flexible for discussing topological spaces and investigating the relationships between them. in INSTS many aspects of nano soft topological spaces are still valid valid, making it easier to establish specific links between certain topological notions. In this paper, we define Infra Soft Nano Topological Space (ISNTS) and Infra soft Nano $\alpha -$interior ($\mathrm{ISN}_\alpha -$int), infra Soft Nano α-closure (ISN_α -cl) with some examples and investigate many of their basic properties. we introduce the ideal of ISN_α-OS which form a class of ISOS. We give some characterizations of ISN_α-OS and ISN_α-CS and establish

some of their properties. Additionally, we demonstrate that this class of Soft Nano sets is closed under arbitrary unions and identity the conditions under which it is closed under finite intersection.

2.Preliminaries

Definition2.1 The group of soft parameters. In order to be able to define the soft group, the availability of primitive elements and an E group for the parameter are required. By assuming the existence of the set of all features of the primitive elements, and denoted it as P(U) , the existence of a nonempty subset of the parameter E. The ordered pair (F,A) means a soft set on the primitive elements, where F is a function defined by i.e.any soft set on p is a feature family of subsets. p, which shows that any soft group is not necessarily a grop.see[17]

Definition2.2 This definition dealt with a non-empty set of soft equivalence rows. Through it, the soft upper sets were defined, which include the union of all the soft equivalence rows that are part of the original set, so that if they intersect with the subset of the space, they are a non-empty set. As for the soft lower sets, they are the union of all soft parity rows that are part of the space group. As for the specified soft region, the resultant will be the difference between the upper soft sums and the soft lower sums, see[17]

Definition 2.3 In any soft nano topological space, the largest soft open set of the parameter can defined by finding the union of all the soft open sets of the parameter (A,E) or in short (A, E)the largest partial open soft nano group of the parameter(A,E), see[17]

Definition2.4 A family Ω of soft sets over A with bas a parameter set is said to be an infra soft topology on A if it is closed under finite intersection and Φ is a member of Ω. The triple ðA, Ω,BÞ is called an infra soft topological space (briefly, ISTS). We called a member of Ω an infra soft open set and called its complement an infra soft closed set. We called ðA, Ω, BÞ stable if all its infra soft open sets are stable and called finite (resp., countable) if A is finite (resp., countable). Proposition 15 (see [17]). Let ðA, Ω, BÞ be an ISTS. The.

Definition 2.5[10]

Let $(U, \tau_R^Z(X), \mathfrak{E})$ or $(U, \tau_R^Z(X_{\mathfrak{E}}))$ be a Soft Nano topological space then subset ϖ of U is said soft Nano-z open set if ϖ satisfices the following conditions:

$1 - \forall a \in \varpi, \mathfrak{E})\exists\ SNO\ G_{\mathfrak{E}}$ or $(G.\mathfrak{E})$ such that $a \in G_{\mathfrak{E}} \sqsubseteq SNCl(\varpi, \mathfrak{E})$.

$2 - G_{\mathfrak{E}}$ or $(G, \mathfrak{E})\ U_{\mathfrak{E}} \Leftrightarrow (\varpi, \mathfrak{E}) = (U, \mathfrak{E})$.

3. Infra Soft Nano α −open(closed) sets

Definition3.1 A Nano Soft $(N_{\mathrm{E}}, \mathrm{F})$ of universal set U with parameter F is said to be ISN$_\alpha$-OS (ISN$_\alpha$-CS)set if $(N_{\mathrm{E}}, \mathrm{F}) \subseteq$N int $(Ncl^*(N\ int(N_{\mathrm{E}}, \mathrm{F})$) ([$(Ncl\ (N\ int^*(Ncl(N_{\mathrm{E}}, \mathrm{F}))] \subseteq\ (N_{\mathrm{E}}, \mathrm{F})$
The class of all ISN$_\alpha$-OS (ISN$_\alpha$-CS)sets in U_F will be denoted as ISN$_\alpha$-OS (U_F) [ISN$_\alpha$-CS (U_F)]

Definition3.2 Let us consider two Nano soft sets (N_{E}, Z) *and* $(N_{\mathrm{E}}, \mathrm{F})$, we can then define nano soft closure and Nano soft interior over Universal set U with parameter K. In the following way

$*$ ISN$_\alpha cl(N_{\mathrm{E}}, \mathrm{F}) = \cap \{(N_{\mathrm{E}}, Z) : (N_{\mathrm{E}}, Z) \supseteq (N_{\mathrm{E}}, \mathrm{F}), (N_{\mathrm{E}}, Z)$ is an ISN$_\alpha$–CS of U }called an ISN$_\alpha$–cl.

$*$ ISN$_\alpha -\ int(N_{\mathrm{E}}, \mathrm{F}) = \cup \{\ \{(N_{\mathrm{E}}, Z) : (N_{\mathrm{E}}, Z)\ \subseteq (N_{\mathrm{E}}, \mathrm{F}), (N_{\mathrm{E}}, Z)$ is an ISN$_\alpha$ -OS in U}is called on (ISN$_\alpha$ −int)

$*$ ISN$_S\ cl(N_{\mathrm{E}}, \mathrm{F}) = \cap \{(N_{\mathrm{E}}, Z) : (N_{\mathrm{E}}, Z)\ \supseteq (N_{\mathrm{E}}, \mathrm{F}), (N_{\mathrm{E}}, Z)$

Is an ISN_S CS of U is called an Infra Nano soft semi closure.

$*$ ISN$_S -\ int(N_{\mathrm{E}}, \mathrm{F}) = \cup \{(N_{\mathrm{E}}, Z) : (N_{\mathrm{E}}, Z)\ \subseteq (N_{\mathrm{E}}, Z), (N_{\mathrm{E}}, \mathrm{F})$ is an ISN_S OS in U is called an Infra Nano soft semi interior

Theorem3.3 A Nano soft(N_Υ, r) set *is* ISN$_\alpha$–OS (U_F) if and only if there exise a NSOS(N_Υ, F) such that

$$(N_\Upsilon, F) \subseteq (N_\Upsilon, r)\ \subseteq int(cl^*(N_\Upsilon, F))$$

proof: If $(N_\Upsilon, r) \in$ ISN$_\alpha$– OS (U_F)then $(N_\Upsilon, r) \subseteq Nint\left(Ncl^*\left(Nint(N_\Upsilon, r)\right)\right)$,

put $(N_\Upsilon, F) = Nint(N_\Upsilon, r)$ then

(N_Υ, r) is NSOS and $(N_\Upsilon, F) \subseteq (N_\Upsilon, r) \subseteq Nint(Ncl^*(N_\Upsilon, F))$. In other way , let (N_Υ, F)be a NSOS such that $(N_\Upsilon, F) \subseteq (N_\Upsilon, r) \subseteq Nint(Ncl^*(N_\Upsilon, F))$ therefore

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$Nint\big(Ncl^*(N_\Upsilon,F)\big) \subseteq Nint(Ncl^*(Nint(N_\Upsilon,r)))$. then $(N_\Upsilon,r) \subseteq Nint(Ncl^*(Nint(N_\Upsilon,r)))$

Theorem 3.4 A Nano soft set (N_Υ,F) is ISN_α – OS (U_F) if and only if a NSCS (N_Υ,r) such that $Ncl\big(N\,int^*(N_\Upsilon,r)\big) \subseteq (N_\Upsilon,F) \subseteq Ncl(N_\Upsilon,r)$.

Proof: If (N_Υ,F) is $\text{ISN}_\alpha\text{CS}\ (U_F)$ ther $Ncl(N\,int^*(Ncl(N_\Upsilon,F))) \subseteq (N_\Upsilon,\text{F})$. Put $(N_\Upsilon,B) = Ncl(N_\Upsilon,F)$ then $(N_\Upsilon,r)$is NSCS and $Ncl(N\,int^*(N_\Upsilon,r)) \subseteq (N_\Upsilon,\text{F})$. In other way . let (N_Υ,r) be a NSCS such that $Ncl(N\,int^*(N_\Upsilon,r)) \subseteq (N_\Upsilon,\text{F}) \subseteq (N_\Upsilon,r)$. This indicates that $Ncl(N\,int^*(Ncl(N_\Upsilon,\text{F})) \subseteq Ncl(N\,int^*(N_\Upsilon,r))$ then $Ncl\big(N\,int^*(N_\Upsilon,\text{F})\big) \subseteq Ncl(N_\Upsilon,\text{F})$.

Theorem3.5 Let (N_Υ,μ) be a Nano soft set over universe U and parameter set μ . then the following properties are true

$*$ $\text{INS}_s\, int(N_\Upsilon,\mu) = (N_\Upsilon,\mu) \cap N\,cl(Nint(N_\Upsilon,\mu)$.

$**$ $\text{INS}_s\, cl(N_\Upsilon,\mu) = (N_\Upsilon,\mu) \cup N\,int^*(Ncl(N_\Upsilon,\mu)$.

proof : * By the definition of $\text{INS}_s\ int$ is infra nano soft-semi open(INSSOS)) then $\text{INS}_s - int(N_\Upsilon,\mu) \subseteq Ncl^*(Nint\big(\text{INS}_s - int(N_\Upsilon,\mu)\big) \subseteq Ncl^*\big(Nint(N_\Upsilon,\mu)\big)$, also $\text{INS}_s - int(N_\Upsilon,\mu) \subseteq (N_\Upsilon,\mu) \subseteq Ncl^*(Nint\big((N_\Upsilon,\mu)\big) \Rightarrow (1)$

we have $Nint(N_\Upsilon,\mu) \subseteq (N_\Upsilon,\mu) \cap Ncl^*(Nint(N_\Upsilon,\mu)) \subseteq Ncl^*\big(Nint(N_\Upsilon,\mu)\big)$. by def. 3.2 $(N_\Upsilon,\mu) \cap Ncl^*(Nint(N_\Upsilon,\mu))$ is an INSSOS and $(N_\Upsilon,\mu) \cap Ncl^*(Nint(N_\Upsilon,\mu))$,then $(N_\Upsilon,\mu) \cap Ncl^*(Nint(N_\Upsilon,\mu)) \subseteq \text{INS}_s - int(N_\Upsilon,\mu) \Rightarrow (2)$from (1) &(2) we get (*).

** Similar to the above proof (*).

Theorem 3.7 Let us consider the Nano soft subset $(N_\Upsilon,\text{F})$of a nano Soft space $\text{NSS}(U_F)$, the following statements are hold:

$*$) If $(N_\Upsilon,\text{F}) \subseteq (N_\Upsilon,E) \subseteq Nint(Ncl^*((N_\Upsilon,F) \in \text{ISN}_\alpha\text{OS}(U_F)$ then $(N_\Upsilon,E) \in \text{ISN}_\alpha\text{OS}(U_F)$

$**$)if $N\,cl(Nint^*(N_\Upsilon,\text{F}) \subseteq (N_\Upsilon,E) \subseteq (N_\Upsilon,F)\ then\ (N_\Upsilon,F) \in \text{ISN}_\alpha\text{CS}(U_F)$ then $(N_\Upsilon,E) \in \text{ISN}_\alpha\text{CS}(U_F)$

proof: $*$)Let $(N_\Upsilon,\text{F}) \in \text{ISN}_\alpha\text{OS}(U_F)$then $\exists(N_\Upsilon,H)$ an SNOS such that $(N_\Upsilon,H) \subseteq (N_\Upsilon,\text{F}) \subseteq Nint(Ncl^*((N_\Upsilon,H)$ this implies that $(N_\Upsilon,H) \subseteq (N_\Upsilon,r)\ and\ (N_\Upsilon,F) \subseteq Nint(Ncl^*((N_\Upsilon,H)$.

$\Rightarrow$ $Nint(Ncl^*((N_\Upsilon,F) \subseteq Nint(Ncl^*((N_\Upsilon,H)$ and $(N_\Upsilon,H) \subseteq (N_\Upsilon,\gamma) \subseteq Nint(Ncl^*((N_\Upsilon,H)$ then $(N_\Upsilon,\gamma) \in \mathrm{ISN}_\alpha \mathrm{OS}(U_F)$

$**$)Same as the proof of $*$

Proposition3.8 Let (N_Υ,F) and (N_Υ,E)be two nana soft sets in (U_k) and $(N_\Upsilon,\mathrm{F}) = (N_\Upsilon,E)$then the following statements are true:

1. $\mathrm{ISN}_\alpha - int(N_\Upsilon,F)$ is the largest $\mathrm{ISN}_\alpha\mathrm{OS}$ Contained in (N_Υ,F)
2. $\mathrm{ISN}_\alpha - int(N_\Upsilon,F) \subseteq (N_\Upsilon,F)$
3. $\mathrm{ISN}_\alpha - int(N_\Upsilon,F) \subseteq \mathrm{ISN}_\alpha - int(N_\Upsilon,E)$
4. $\mathrm{ISN}_\alpha - int(\mathrm{ISN}_\alpha - int(N_\Upsilon,F)) = \mathrm{ISN}_\alpha - int(N_\Upsilon,F)$
5. $(N_\Upsilon,F) \in \mathrm{ISN}_\alpha\mathrm{OS}(U_F)$ iff $\mathrm{ISN}_\alpha - int(N_\Upsilon,F) = (N_\Upsilon,F)$

proposition3:9 Let $(N_\Upsilon,F)$of Jand (N_Υ,E)be two nano soft sets in $\mathrm{NSS}(U_F)$ and $(N_\Upsilon,F) \subseteq (N_\Upsilon,F)$then the following statements are true:

1. $\mathrm{ISN}_\alpha cl(N_\Upsilon,F)$is the smallest $\mathrm{ISN}_\alpha CS$ containing (N_Υ,F)
2. $(N_\Upsilon,F) \subseteq \mathrm{ISN}_\alpha cl(N_\Upsilon,F)$
3. $\mathrm{ISN}_\alpha cl(N_\Upsilon,F) \subseteq \mathrm{ISN}_\alpha cl(N_\Upsilon,E)$
4. $\mathrm{ISN}_\alpha cl\big(\mathrm{ISN}_\alpha cl(N_\Upsilon,F)\big) = \mathrm{ISN}_\alpha cl(N_\Upsilon,F)$
5. $(N_\Upsilon,F) \in \mathrm{ISN}_\alpha \mathrm{cl}(U_F)$ iff $\mathrm{ISN}_\alpha cl(N_\Upsilon,F) = (N_\Upsilon,F)$

Theorem3.10 Let (N_Υ,F)be a nano soft set of NSS (U_F) then the following

assertions are true:

1. $(\mathrm{ISN}_\alpha int(N_\Upsilon,F)^c = \mathrm{ISN}_\alpha cl(N_\Upsilon,F)$
2. $(\mathrm{ISN}_\alpha cl(N_\Upsilon,F)^c = \mathrm{ISN}_\alpha int(N_\Upsilon,F)$
3. $\mathrm{ISN}_\alpha int(N_\Upsilon,F) \subseteq (N_\Upsilon,F) \cap Nint(Ncl^*(Nint(N_\Upsilon,F)$
4. $\mathrm{ISN}_\alpha cl(N_\Upsilon,F) \supseteq (N_\Upsilon,F) \cup Ncl(Nint^*(Ncl(N_\Upsilon,F)$

Proof:

1. $(\mathrm{ISN}_\alpha int(N_\Upsilon,F)^c = \cup \{(N_\Upsilon,E): (N_\Upsilon,E) \subseteq (N_\Upsilon,F), (N_\Upsilon,E)$ is an $\mathrm{ISN}_\alpha\mathrm{OS}$ of $(U_F)^c$ $= \mathrm{ISN}_\alpha cl(N_\Upsilon,F)$
2. Similarly of 1
3. Since $(N_\Upsilon,F) \supseteq \mathrm{ISN}_\alpha int(N_\Upsilon,F)$ and $\mathrm{ISN}_\alpha int(N_\Upsilon,F)$ is an $\mathrm{ISN}_\alpha\mathrm{OS}$. Hence $Nint(Ncl^*(Nint(N_\Upsilon,F) \supseteq \mathrm{ISN}_\alpha int(N_\Upsilon,F)$ then $\mathrm{ISN}_\alpha int(N_\Upsilon,F) \subseteq (N_\Upsilon,F) \cap Nint(Ncl^*(Nint(N_\Upsilon,F))$
4. Similarly of 3

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Corollary3.11 Let (N_{Υ}, F)be a nano soft set of NSS (U_F). then the following assertions are true:

•)If $(N_{\Upsilon}, F)$is a NSOS, then $\text{ISN}_{\alpha}int(N_{\Upsilon}, F) \subseteq Nint(Ncl^*(Nint(N_{\Upsilon}, F)).$

••) If $(N_{\Upsilon}, F)$is a NSCS, then $\text{ISN}_{\alpha}cl(N_{\Upsilon}, F) \supseteq Ncl(Nint^*(Ncl(N_{\Upsilon}, F))$

Theorem 3.12 *)The arbitrary Union of an $\text{ISN}_{\alpha}OS$ is $\text{ISN}_{\alpha}OS$

**) The arbitrary intersection of an $\text{ISN}_{\alpha}CS$ is $\text{ISN}_{\alpha}CS$

Proof: Let $\{(N_{\Upsilon}, D)\}$ be a family of $\text{ISN}_{\alpha}OS$. then for every j ,
$(N_{\Upsilon}, F)_j \subseteq Nint(Ncl^*(Nint(N_{\Upsilon}, F)_j) \; and \cup (N_{\Upsilon}, F)_j \cup (Nint(Ncl^*(Nint(N_{\Upsilon}, F)_j))) \subseteq (Nint(Ncl^*(Nint(N_{\Upsilon}, F)_j)))$

Hence $(N_{\Upsilon}, F)_j$ is $\text{ISN}_{\alpha}OS$.

**) Similarly *)

Theorem 3.13 Let (Z_{Υ}, μ) be a nano soft set over the universes U parameter μ. then
$Nint^*(Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}int(Z_{\Upsilon}, \mu) \subseteq (Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}cl(Z_{\Upsilon}, \mu) \subseteq \text{N}cl^*(Z_{\Upsilon}, \mu)$

proof: Since $Nint^*(Z_{\Upsilon}, \mu) \subseteq (Z_{\Upsilon}, \mu)$ this continues that
$\text{ISN}_{\alpha}int(\text{ISN}_{\alpha}int^*(Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}int(Z_{\Upsilon}, \mu)$. then $\text{ISN}_{\alpha}int(\text{N}int^*(Z_{\Upsilon}, \mu) = \text{N}int^*(Z_{\Upsilon}, \mu)$ also $\text{N}int^*(Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}int(Z_{\Upsilon}, \mu) \ldots\ldots\ldots\ldots .*$

So using result $(Z_{\Upsilon}, \mu) \subseteq \text{N}cl^*(Z_{\Upsilon}, \mu)$, this $\text{ISN}_{\alpha}cl(Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}cl(\text{N}cl^*(Z_{\Upsilon}, \mu))$.then

$\text{ISN}_{\alpha}cl(\text{N}cl^*(Z_{\Upsilon}, \mu)) = \text{N}cl^*(Z_{\Upsilon}, \mu), so \; \text{ISN}_{\alpha}cl(Z_{\Upsilon}, \mu) \subseteq \text{N}cl^*(Z_{\Upsilon}, \mu)$ ………..**

from$*$ & $**$we will get

$Nint^*(Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}int(Z_{\Upsilon}, \mu) \subseteq (Z_{\Upsilon}, \mu) \subseteq \text{ISN}_{\alpha}cl(Z_{\Upsilon}, \mu) \subseteq \text{N}cl^*(Z_{\Upsilon}, \mu)$.

Theorem 3.14 Let (N_{Υ}, F) be a nano soft set of $(U, \lambda, \mathcal{F})$ then the following assertions hold:-

1- If (N_{Υ}, F) is an $\text{ISN}_{\alpha}OS(\text{ISN}_{\alpha}CS)$than (N_{Υ}, F) is a $\text{SN}_{\alpha}OS(\text{SN}_{\alpha}CS)$

2-If (N_{Υ}, F) is an $\text{ISN}_{\alpha}OS(\text{ISN}_{\alpha}CS)$,then (N_{Υ}, F) is a nano soft α^* open (Supra nano soft α-open) a nano soft α^* closed (Supra nano soft α- closed) set.

3- If (N_{Υ}, F) is an $\text{ISN}_{\alpha}OS(\text{ISN}_{\alpha}CS)$ then (N_{Υ}, F) is a nano soft Pre^*open (supra nano soft-preopen) (nano soft Pre^* closed (supra nano soft precloseal) set.

4-If (N_{Υ}, F) is an ISN$_{\alpha}OS$(ISN$_{\alpha}CS$) then (N_{Υ}, F) is a nano soft semi open (ISN_sOS) (nano soft semi* closed (ISN_sCS).

5- If (N_{Υ}, F) is a N SOS(NSCS) then (N_{Υ}, F) is ISN_sOS (ISN_sCS).

proof: By the definitions 2.1 , 2.2 and 3.1 and basic relationships with other nano soft sets, we can prove above results..

Remark3.15 Any NSOS ⇒is supra nano soft α openset, ISN_{α}OS⇒ SN_{α}OS,

, SN_{α}OS⇒ SSN_{α}OS , SN_{α}OS⇒ SSN_{α}OS & S pre-OS, ISN_{α}OS⇒ $ISSN_{□}$OS and SSN_{pre}OS, SN_{pre}OS ⇒ SSN_{pre}OS , $ISSN_o$S ⇒ SSN_oS . there is no connections between $ISSN_o S$ *and* $SSN_{\text{pre}}S$

The following examples will show that the converses of remark need not to be true.

Example 316 Let $U = \{z, h, l, f\}$,$\mu = \{e_1, e_2, e_3\}$,
$(\kappa, \mu) = \{ (e_1, \{z\}), (\{e_2, l\}), (\{e_3, \{h, l\}\})\}$ be a soft set over U and $X = \{z, h\} \subseteq U$then $U/R = \{\{z\}, \{l\}, \{h, l\}\}$ and Nano soft sets
=
$\{ \sigma_{\mu}, U_{\mu}, (\{e_1, \kappa_1\}), (\{e_2, \kappa_1\}), (\{e_3, \kappa_1\}), (\{e_1, \kappa_{11}\}), (\{e_2, \kappa_{11}\}), (\{e_3, \kappa_{11}\}), (\{e_1, \kappa_8\}), (\{e_2, \kappa_8\}$
with $(\kappa_1, \mu) = \{ (\{e_1, \{z\}), (\{e_2, \{\kappa_1\}\}), (\{e_3, \{\kappa_1\}\})\}$,
$(\kappa_2, \mu) = \{ (\{\mu, \{h\}\})\}, (\kappa_3, \mu) = \{ (\mu, \{l\})\}$, $(\kappa_4, \mu) = \{\mu, \{f\}\}, (\kappa_5, \mu) = \{\mu, \{z, h\}\}, (\kappa_6, \mu) = \{\mu, \{z, l\}\}, (\kappa_7, \mu) = \{\mu, \{z, f\}\}, (\kappa_8, \mu) = \{\mu, \{h, l\}\}$
, $(\kappa_9, \mu) = \{\mu, \{h, f\}\}, (\kappa_{10}, \mu) = \{\mu, \{l, f\}\}$
, $(\kappa_{11}, \mu) = \{\mu, (\{z, h, l\})\}$, , $(\kappa_{12}, \mu) = \{\mu, (\{z, h, f\})\}$, $(\kappa_{13}, \mu) = \{\mu, (\{h, l, f\})\}$
, $(\kappa_{14}, \mu) = \{\mu, (\{z, l, f\})\}$

(κ_7, μ) is ISS NOS but it is not $SSN_{\text{pre}} - \text{OS}$.

(κ_9, μ) is ISSNOS but it is not a SNOS

(κ_{11}, μ) is a $SSN_{\text{pre}} - \text{OS}$ set but it is not ISSNOS.

(κ_{12}, μ) is $SSN_{\text{pre}} - \text{OS}$ but it is not (SNOS.

(κ_{12}, μ) is $SSN_{\text{pre}} - \text{OS}$ but it is not ISN_{α}OS

(κ_9, μ) is ISSNOS but it is not ISN_{α}OS

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Example 3.17 Let $U = \{z, l, r\}, \mu = \{e_1, e_2\}, \{\kappa, \mu\} = (e_1, \{l\}), (e_2, \{l, r\})$ be a soft set of U and $X = \{h, r\} \subseteq U$ with $U/R = \{\{l\}, \{z, r\}\}$ nano soft sets $(\tau_R(x)\mu) = \{\sigma_\mu, U_\mu, (e_1, \{h\}), (e_2, \{h\})\}$ with $(\kappa_1, \mu) = \{(\mu, \{h\})\}$, $(\kappa_2, \mu) = \{(\mu, \{l, r\})\}$

(κ_2, μ) is SSN_αOS but it is not ISN_αOS.

(κ_2, μ) is SSN_αOS but it is not SN_OS.

(κ_2, μ) is SN_αOS but it is not ISN_αOS.

(κ_2, μ) is SSN_OS but it is not ISN_αOS.

(κ_2, μ) is SN_{pre}OS but it is not ISN_αOS.

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