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On Some Mappings in Intuitionistic Topological Spaces

[1]Saleh,B.H

[2]Yassen, S.R

[3]Amina, K. H

[1,2]Department of Mathematics, College of Education for Pure Sciences, Tikrit University, Tikrit, Iraq

[3]Materials Department, College of Engineering, Mustansiriyah University, Baghdad, Iraq.

[2]samer2017@tu.edu.iq

[3]Amina.kass@uomustansiriyah.edu.iq

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[2]samer2017@tu.edu.iq

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Abstract

This work presents new types of some generalized weak continuous, closed and open mappings like intuitionistic $Ig\alpha A*-$continuous$(resp.,Ig\alpha A*$ closed map , $Ig\alpha A*$ open map) , $Ig\beta A*-$continuous $(resp.,Ig\beta A*$ closed map , $Ig\beta A*$ open map), $IgPA*-$continuous $(resp.,IgPA*$ closed map , $IgPA*$ open map), $IgSA*-$continuous

$(resp.,IgSA*$ closed map , $IgSA*$ open map) and $LIg-$ continuous $(resp.,LIg-$ closed map , $LIg-$ open map) . Moreover, we introduced new types of some generalized weak irresolute mappings, Like intuitionistic

$Ig\alpha A*-$irresolute , $Ig\beta A*-$irresolute, $IgPA*-$irresolute, $IgSA*-$irresolute$-$, $LIg-$ irresolutewith studying some properties related them. Also, we studying some relationships among these concepts. Finally, relationships among generalized weak continuous mappings and generalized weak continuous mappings are studied.

Keywords: Weak shapes, irresolute function, strong function, perfectly function.

Introduction

The area for intuitionistic topological areas has progressed through a series of significant contributions by diverse academics. Zadeh's (1965) groundbreaking study on fuzzy sets laid the foundation for this subject. In 1983, Atanassov proposed the idea of fuzzy sets that were intuitionistic, which added another degree of complication to the idea. In 1996, Çoker presented intuitionistic sets as well as points, providing up new paths for investigation. In 1997, Çoker established intuitionistic fuzzy topological areas,

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and later, in 2000, he expanded on this structure to include intuitionistic topological spaces. In 2001, Bayhan and Çoker introduced the $T1$ and $T2$ separating criteria for intuitionistic topological spaces. Further expanding the theoretical terrain. Yaseen and Mohammad (2012) introduced Regular as well as Weak Regularly intuitionistic topological areas, whereas Jassim (2013) established entirely normal and weakly normal intuitionistic topological areas. Yaseen (2022) generalized some weak forms of irresolute mappings in intuitionistic topological spaces. Selvanayaki and Ilango (2015) established IGPR continuity and compactness in intuitionistic topological spaces, also H Jassim et al. (2015) introduced concept of generalized sets and mappings in intuitionistic topological spaces whereas Selvanayaki & Ilango (2016) investigated homeomorphism in these areas. Ilango & Albinaa (2016) and Kim et al. (2017) investigated intuitionistic closure in intuitionistic topological spaces, Prova & Hossain (2020) proposed intuitive fuzzy-based regularity, while Chae *et al.* (2020) used interval-valued intuitionistic sets in topological applications. Islam et al. (2021) contributed by developing notations for intuitionistic fuzzy r-regular areas and Isewid *et al.* (2021) introduced some properties of regular and normal space on topological graph space. Haque, Akhter, and Murshed (2022) presented intuitionistic subspace topology and explored its features. Previous research has not focused on the qualities of intuitionistic regular sub spaces. Also, Isewid and Yaseen (2022) were provided the notion of a function poorly open to intuition raster spaces and we get some new descriptions of the opening up of theweak functions among the intuitive point areas.

This paper fills a gap by using the structure of extended sets as well as maps in intuitionistic topological spaces. Jassim et al. (2015) provides a revolutionary discovery. This insight expands our comprehension of intuitionistic topological spaces and investigates some properties related them and adds a new level to the area's debate.

2. Basic Concepts

In this part we give some concepts which are needed in our work.

Definition 2.1 (Çoker, 1996): Assume the two sub sets B_T with B_F for a set that is not empty X where $B_T \cap B_F = \emptyset$. If $B = (B_T, B_F)$, then B is known as an intuitionistic set (IS) of X. Here, B_T represents a set of members, while B_F is considered the sets of nonmembers of B.

In actuality, B_T represents a subset from X accepting or allowing of a specific opinion, view, proposal, or strategy, while B_F is a subset for X rejecting or against a similar opinion, viewpoint, suggestion, or strategy, respectively. Assume $\emptyset_I = (\emptyset, X)$ as well as

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No: 7 Val:1 /11/ 2024

$X_I = (X, \emptyset)$, wherein $\emptyset I$ represents the intuitionistic empty subset and X_I represents the intuitionistic entire set of X. In overall, $B_T \cup B_F$ does not equal X and $IS(X)$ denotes the set of every one of the ISs of X.

Definition 2.2 (Çoker, 1996): Assume an $IS\ B$ associated with a non-empty set X, with $\xi \in X$.
(i) $\xi_I = (\{\xi\}, \{\xi\}^c)$ corresponds to an intuitionistic point (IP), while $\xi_{IV} = (\emptyset, \{\xi\}^c)$ is a dis appearing point of X.
(ii) For $\xi \in B_T$, so $\xi_I \in B$; while $\xi \notin B_F$, so $\xi_{IV} \in B$.IP (X) refers to are all intuitionistic points and vanishing points in X.

Definition 2.3. (Coker, 2000). If $B = (B_T, B_F)$ represents an intuitionistic collection in Y, subsequently the prior image of B according to h, represented by $h^{-1}(B)$, represents the intuitionistic subset in X that is determined by
$h^{-1}(B) = \langle h^{-1}(B_T), h^{-1}(B_F) \rangle$
$M = (h(\mathrm{M}_T), h(\mathrm{M}_F))$ represents an intuitionistic set corresponding to X. The image of M underneath h, indicated by $h(M)$, is the intuitionistic subset in Y that is determined by $h(M) = (h(\mathrm{M}_T), h - (\mathrm{M}_F))$ where $h - (\mathrm{M}_F) = Y - (h(X - \mathrm{M}_F))$.

Definition 2.4. (Bayhan and Çoker, 2001)
Suppose M and N represent two intuitionistic collections on X and Y, respectively. The products intuitionistic sets of M and N on $X \times Y$ is defined as $\times V = \langle \mathrm{M}_T \times \mathrm{N}_T, (\mathrm{M}_T{}^c \times \mathrm{N}_T{}^c) \rangle$, where $M = (M_T, M_F)$, and $N = (N_T, N_F)$. If (X, τ) and (Y, φ) are ITS, subsequently the product of the topologies $\tau \times \varphi$ on $X \times Y$ represents the intuitionistic topologies provided by the basis $B = \{M \times N : M \in \tau, N \in \varphi\}$.

Definition: 2.5 (Jassim *et al.*, 2015) Take (X, T) have an ITS, and allow $M = (M_T, M_F)$ have an IS in X, subsequently M will be considered to be:
Intuitionistic generalized pre-regular closed sets ($Igpr$-closed) are those where $pcl\ (A) \subseteq U$ whenever $M \subseteq U$ as well as U is intuitionistic open in X.
Intuitionistic semi weakly generalized pre-regular closed sets ($Isgr$-closed) are those where $clint(A) \subseteq U$ whenever $M \subseteq U$ as well as U is intuitionistic semi open in X.

3. Some Generalized continuous Mappings in ITS

In this part, we introduce a new generalized continuous mapping in ITS such as;
$Ig\beta A * C$- continuous, $IgPA * C$- continuous, $IgSA * C$ – continuous, $Ig\alpha A * C$ – continuous

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and $LIgC$ – continuous with studying relationship among them and some properties related them. we start with the following definition:

Definition 3.1. Assume that (X, I_τ) and (Y, I_σ) are two ITSs and h represents a mapping from (X, I_τ) into (Y, I_σ). Then a mapping $h: (X, I_\tau) \to (Y, I_\sigma)$ considers below:

$Ig\beta A*$ -continuous if $h^{-1}(M)$ is $Ig\beta A*C$ -set in X for every $Ig\beta A*C$ - set M in Y.
$IgPA*$ -continuous if $h^{-1}(M)$ is $Ig\beta A*C$ -set in X for every $IgPA*C$ - set M in Y.
$IgSA*$ -continuous if $h^{-1}(M)$ is $Ig\beta A*C$-set in X for every intuitionistic $IgSA*C$ - set M in Y.
$Ig\alpha A*$-continuous if $h^{-1}(M)$ is $Ig\alpha A*C$-set in X for every intuitionistic $Ig\alpha A*C$- set M in Y.
LIg -continuous if $h^{-1}(M)$ is $LIgC$-set in X for every $LIgC$ - set M in Y.

Theorem 3.2. Suppose that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping. Then the below are holds:

If h is LIg -continuous, then, h is $Ig\alpha A*$-continuous map.
If h is $Ig\alpha A*$ -continuous then, h is $IgSA*$-continuous map.
If h is $Ig\alpha A*$ -continuous then, h is $IgPA*$-continuous map.
If h is $IgSA*$ -continuous then, h is $Ig\beta A*$-continuous map.
If h is $IgPA*$ -continuous then, h is $Ig\beta A*$-continuous map.

Proof:

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $LIgC$-set in Y. Then, $int(\mathrm{M}) \subseteq \mathrm{U}$ whenever $\mathrm{M} \subseteq \mathrm{U}$, U is IOS and M is I$A*$ such that $\mathrm{M} = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IOS and $\aleph$ is ICS. Since, M is $LIgC$ – set, there exists an $Ig\alpha A*C$-set in Y, because every $LIgC$-set is $Ig\alpha A*C$-set. Now, since h is $LIgC$ -continuous as a result, $h^{-1}(M)$ is $LIgC$-set in $X$by assumption. Consequently, $h^{-1}(M)$ is an $Ig\alpha A*C$-set in X. Therefore, h is $Ig\alpha A*$-continuous map.

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $Ig\alpha A*C$-set in Y.Thus, $int_\alpha\big(cl_\alpha(int_\alpha(\mathrm{M}))\big) \subseteq \mathrm{U}$ whenever $\mathrm{M} \subseteq \mathrm{U}$, U is IαOS, and M is I$\alpha A*$ such that $\mathrm{M} = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IαOS and $\aleph$ is IαCS. Since, M is $Ig\alpha A*C$ set, there exists an $Ig\alpha A*C$-set in Y, because every $Ig\alpha A*C$ -set is $Ig\alpha A*C$-set. Since h is $Ig\alpha A*$-continuous as a result, $h^{-1}(M)$ is $Ig\alpha A*C$ -set in X by assumption. Consequently, $h^{-1}(M)$ is an $IgSA*C$-set in X. Therefore, h is $IgSA*$-continuous map.

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $Ig\alpha A*C$-set in Y.Hence, $int_\alpha\big(cl_\alpha(int_\alpha(\mathrm{M}))\big) \subseteq \mathrm{U}$ whenever $\mathrm{M} \subseteq \mathrm{U}$, U is IαOS, and M is I$\alpha A*$ such that $\mathrm{M} = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IαOS and $\aleph$ is IαCS. Since, M is $Ig\alpha A*C$ set, there exists an $Ig\alpha A*C$-set in Y, because every $Ig\alpha A*C$ -set is $IgPA*C$-set.

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Since h is $Ig\alpha A*$-continuous as a result, $h^{-1}(M)$ is $Ig\alpha A*C$ -set in X by assumption. Consequently, $h^{-1}(M)$ is an $IgPA*C$-set in X. Therefore, h is $IgPA*$-continuous map.
Considering that $h:(X,I_\tau)\to(Y,I_\sigma)$ be a mapping and M is IgSA $*$ C-set in Y.Thus, $cl_S(int_S(M))\subseteq U$ whenever $M\subseteq U$, U is ISOS and M is ISA $*$ such that $M=\mathcal{M}\cup\aleph$, where $\mathcal{M}$ is ISOS and $\aleph$ is ISCS. Since, M is $IgSA*C$ set, there exists an $Ig\beta A*C$-set in Y, because every $IgSA*C$ -set is $Ig\beta A*C$-set. Since h is $IgSA*$-continuous as a result, $h^{-1}(M)$ is $IgSA*C$-set in X by assumption. Consequently, $h^{-1}(M)$ is an $Ig\beta A*C$-set in X. Therefore, h is $Ig\beta A*$-continuous map.
Considering that $h:(X,I_\tau)\to(Y,I_\sigma)$ be a mapping and M is $IgPA*C$-set in Y.Thus $int_P(cl_P(M))\subseteq U$ whenever $M\subseteq U$, U is IPOS and M is IP$A*$ such that $M=\mathcal{M}\cup\aleph$, where $\mathcal{M}$ is IPOS and $\aleph$ is IPCS. Since, M is $IgPA*C$ set, there exists an $Ig\beta A*C$-set in Y, because every $IgPA*C$ -set is $Ig\beta A*C$-set. Since h is $IgPA*$-continuous as a result, $h^{-1}(M)$ is $IgPA*C$-set in X by assumption. Consequently, $h^{-1}(M)$ is an $Ig\beta A*C$-set in X. Therefore, h is $Ig\beta A*$-continuous map.

Remark 3.3. The converse of Theorem 3.2. is not true, As seen in the following examples.

Example 3.4. Assume $X=\{\xi,\lambda,\kappa\}$ with topology $I_\tau=\{\widetilde{X},\widetilde{\emptyset},M,N\}$, where $M=\langle\{\xi\},\{\lambda\}\rangle$,$N=\langle\{\xi,\kappa\},\{\lambda\}\rangle$, and $Y=\{15,17,19\}$ with topology $I_\sigma=\{\widetilde{X},\widetilde{\emptyset},P,W\}$ where $P=\langle\{19\},\emptyset\rangle, W=\langle\emptyset,\emptyset\rangle$ and $h:(X,I_\tau)\to(Y,I_\sigma)$ *be* a mapping defined by $h(\xi)=15, h(\lambda)=19, h(\kappa)=17$. Then, the following are fulfilled:
h is $IgPA*$-continuous but, h is not $Ig\alpha A*$-continuous, because for $D=\langle\{15,17\},\emptyset\rangle$ is $IgPA*C$-set in Y, $h^{-1}(D)=\langle\{\xi,\kappa\},\emptyset\rangle\notin Ig\alpha A*C$.
h is $IgSA*$-continuous but, h is not $Ig\alpha A*$-continuous, because for $L=\langle\{15\},\emptyset\rangle$ is $IgSA*C$-set in Y, $h^{-1}(L)=\langle\{\xi\},\emptyset\rangle\notin Ig\alpha A*C$.

Example 3. 5. Assume $X=Y=\{\xi,\lambda,\kappa\}$ with topology $I_\tau=\{\widetilde{X},\widetilde{\emptyset},Q,R,K,W\}$, where $Q=\langle\emptyset,\{\xi\}\rangle, R=\langle\{\xi\},\emptyset\rangle, W=\langle\{\kappa\},\emptyset\rangle, K=\langle\{\xi,\kappa\},\emptyset\rangle$ and $Y=\{15,17,19\}$ with topology $I_\sigma=\{\widetilde{X},\widetilde{\emptyset},P,L,J,V\}$ where $P=\langle\emptyset,\{15\}\rangle, L=\langle\emptyset,\emptyset\rangle, J=\langle\{19\},\emptyset\rangle, V=\langle\{15,19\},\emptyset\rangle$ and $h:(X,I_\tau)\to(Y,I_\sigma)$

be a mapping defined by $h(\xi)=15, h(\lambda)=17, h(\kappa)=19$. Then, the following are fulfilled:
h is $Ig\beta A*$-continuous but, h is not $IgPA*$-continuous, because for $E=\langle\{15,19\},\emptyset\rangle$ is $Ig\beta A*C$-set in Y, $h^{-1}(E)=\langle\{\xi,\kappa\},\emptyset\rangle\notin IgPA*C$.
h is $Ig\beta A*$-continuous but, h is not $IgSA*$-continuous, because for $S=\langle\{17\},\emptyset\rangle$ is $Ig\beta A*C$-set in Y, $h^{-1}(S)=\langle\{\lambda\},\emptyset\rangle\notin IgSA*C$.

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No: 7 Val:1 /11/ 2024

Example 3. 6. Assume $X = Y = \{\xi, \lambda\}$ with topology $I_\tau = \{\widetilde{X}, \widetilde{\emptyset}, Q, R, W\}$, where $Q = \langle \emptyset, \{\xi\} \rangle$, $R = \langle \{\xi\}, \emptyset \rangle, W = \langle \{\lambda\}, \emptyset \rangle$ and $Y = \{15,17,19\}$ with topology I_σ = $\{\widetilde{X}, \widetilde{\emptyset}, P, J, K\}$ where $P = \langle \emptyset, \{15\} \rangle, J = \langle \emptyset, \emptyset \rangle, K = \langle \{19\}, \emptyset \rangle$ and $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping defined by $h(\xi) = 15, h(\lambda) = 17,$ $h(\kappa) = 19$. Then h is $Ig\alpha A*$-continuous but, h is not LIg-continuous, because for $T = \langle \{17\}, \emptyset \rangle$ is $Ig\alpha A* C$-set in Y, $h^{-1}(T) = \langle \{\lambda\}, \emptyset \rangle \notin LIgC$.

Proposition 3.7. Suppose a mapping $h: (X, I_\tau) \to (Y, I_\sigma)$ and for each subset $M \subseteq X$.Then, the below holds:

Whenever h is $Ig\alpha A* C$, thus $Ig\alpha A* \left(\text{int}_\alpha\left(\text{cl}_\alpha(\text{int}_\alpha(h(M)))\right)\right) \subset h(Ig\alpha A* \text{int}_\alpha\left(\text{cl}_\alpha(\text{int}_\alpha(M))\right))$.

Whenever h is $IgSA* C$, thus $IgSA* \text{int}_S(\text{cl}_S(h(M))) \subset h(IgSA* \text{int}_S(\text{cl}_S(M)))$.

Whenever h is $IgPA* C$, thus $IgPA* \text{int}_P(\text{cl}_P(h(M))) \subset h(IgPA* \text{int}_P(\text{cl}_P(M)))$.

Whenever h is $Ig\alpha A* C$, thus $Ig\alpha A* \text{cl}_\beta\left(\text{int}_\beta\left(\text{cl}_\beta(h(M))\right)\right) \subset h(Ig\alpha A* \text{cl}_\beta\left(\text{int}_\beta\left(\text{cl}_\beta(h(M))\right)\right))$.

Proof.

Assume that h is $Ig\alpha A* C$ and $M \subset X$. $\text{cl}_\alpha(\text{int}_\alpha(h(M)))$ is $Ig\alpha A* C$ in X and so $h(\text{cl}_\alpha(\text{int}_\alpha(M)))$ is $Ig\alpha A* C$ in (Y, I_σ). So, $h(M) \subset h(\text{cl}_\alpha(\text{int}_\alpha(M)))$,and hence $Ig\alpha A* (\text{int}_\alpha\left(\text{cl}_\alpha(\text{int}_\alpha(h(M)))\right) \subset Ig\alpha A* (\text{int}_\alpha(\text{cl}_\alpha(\text{int}_\alpha(h(\text{cl}_\alpha(\text{int}_\alpha(M))))))) \to$ (i)

Since $h(\text{cl}_\alpha(\text{int}_\alpha(M)))$ is $Ig\alpha A* C$ in (Y, I_σ).Hence,

$Ig\alpha A* (\text{int}_\alpha(\text{cl}_\alpha(\text{int}_\alpha(h(\text{cl}_\alpha(\text{int}_\alpha(M))))))) = h(\text{cl}_\alpha(\text{int}_\alpha(M))) \to$ (ii). Via (i) with (ii), we obtain for each subset $M \subseteq X$ $Ig\alpha A* \left(\text{int}_\alpha\left(\text{cl}_\alpha(\text{int}_\alpha(h(M)))\right)\right) \subset h(Ig\alpha A* \text{int}_\alpha\left(\text{cl}_\alpha(\text{int}_\alpha(M))\right))$.

Assume that h is $IgSA* C$ and $M \subset X$. $\text{int}_S(h(M))$ is $IgSA* C$ in X and so $h(\text{int}_S(M))$ is $IgSA* C$ in (Y, I_σ). So, $h(M) \subset h(\text{int}_S(M))$,

and hence $IgSA* \left(\text{cl}_S(\text{int}_S(h(M)))\right) \subset Ig SA* (\text{cl}_S(\text{int}_S(h(\text{cl}_S(\text{int}_S(M)))))) \to$ (i)

Since $h(\text{cl}_S(\text{int}_S(M)))$ is $Ig SA* C$ in (Y, I_σ).Hence,

$Ig SA* (\text{cl}_S(\text{int}_S(h(\text{cl}_S(\text{int}_S(M)))))) = h(\text{cl}_S(\text{int}_S(M))) \to$ (ii). Via (i) with (ii), we obtain for each subset $M \subseteq X$ $IgSA* IgSA* \text{int}_S(\text{cl}_S(h(M))) \subset h(IgSA* \text{int}_S(\text{cl}_S(M)))$.

Similarly, we can prove (3) and (4) as same way.

Theorem 3.8. Suppose that a two mapping $h: (X, I_\tau) \to (Y, I_\sigma)$ and $\varphi: (Y, I_\sigma) \to (Z, I_\rho)$ for each subset $M \subseteq X$.The below holds:

If both h and φ are LIg- continuous map then, $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is LIg- continuous map.

If both h and φ are $Ig\alpha A*$- continuous map then, $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $Ig\alpha A*$- continuous map.

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No: 7 Val:1 /11/ 2024

If both h and φ are $IgPA*$- continuous map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $IgPA*$-continuous map.
If both h and φ are $IgSA*$- continuous map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $IgSA*$-continuous map.
If both h and φ are $Ig\beta A*$- continuous map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $Ig\beta A*$-continuous map.

Proof.

Assume K be $LIgC$ set in (X, I_τ). Because, h is an LIg- continuous map, h(K) is $LIgC$ set in (Y, γ).Thus, φ(h(H)) is $LIgC$ –set in (Z, I_ρ) because, φ is LIg continuous map, so φ(h(H)) is $LIgC$ –set in (Z, I_ρ). In other words, φ ∘ h(H) = φ(h(H)) is $LIgC$ –set and therefore, φ ∘ h is $LIgC$- continuous map.

Assume K be $Ig\alpha A*C$ set in (X, I_τ). Because, h is an $Ig\alpha A*$- continuous map, h(K) is $Ig\alpha A*C$ set in (Y, γ).Thus, φ(h(H)) is $Ig\alpha A*C$ –set in (Z, I_ρ) because, φ is $Ig\alpha A*$ continuous map, so φ(h(H)) is $Ig\alpha A*C$ –set in (Z, I_ρ). In other words, φ ∘ h(H) = φ(h(H)) is $Ig\alpha A*C$ –set and therefore, φ ∘ h is $Ig\alpha A*$- continuous map.

Similarly, we can prove (3), (4) and (5) as same way.

Theorem 3.9. if a map $h: (X, I_\tau) \to (Y, I_\sigma)$ is a bijective map. The arguments that follow are equivalent:
(i) h is $Ig\alpha A*O$.
(ii) h is $Ig\alpha A*C$.
(iii) h^{-1}: $(Y, I_\sigma) \to (X, I_\tau)$ is $Ig\alpha A*$continuous.
Proof: $(i) \Rightarrow$ (ii)
If F is an $Ig\alpha A*C$-set in (X, I_τ) then $X - F$ is $Ig\alpha A*O$ in (X, I_τ) . But h is $Ig\alpha A*O$, thus $h(X - F)$ is $Ig\alpha A*O$ in (Y, I_σ). Hence, $h(< F_1, F_2 >) =< h(F_1), Y - h(X - F_2) >$ is $Ig\alpha A*O$ in (Y, I_σ) and so $< Y - h(X - F_2), h(F_1) >=< h(F_2), h(F_1) >$ is $Ig\alpha A*C$ in (Y, I_σ). Because $Y - h(X - F_2) = h(F_2), < Y - h(X - F_2), h(F_1) >=$
$< h(F_2), h(F_1) >$ is $Ig\alpha A*C$ in (Y, I_σ) . Therefore,h^{-1} is $Ig\alpha A*C$.
(ii) $\Rightarrow$ (iii)
Assume F be an $Ig\alpha A*C$ in (X, I_τ). Because h is bijective and $Ig\alpha A*C$, hence $h(F) = (h^{-1})^{-1}(F)$ is $Ig\alpha A*C$ in (Y, I_σ). Therefore h^{-1} is $Ig\alpha A*C$ -continuous.
(iii) $\Rightarrow (i)$
Assume F be an $Ig\alpha A*O$ in (X, I_τ). By assumption, $(h^{-1})^{-1}(F)$ is $Ig\alpha A*O$ in (Y, I_σ). Hence, $h(F)$ is $Ig\alpha A*O$ in (Y, I_σ). Thus, h is $Ig\alpha A*O$.

Theorem 3.10. if a map $h: (X, I_\tau) \to (Y, I_\sigma)$ is a bijective map. The arguments that follow are equivalent:
(i) h is $IgPA*O$.

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

(ii) h is $IgPA*C$.
(iii) $h^{-1}:(Y,I_\sigma) \rightarrow (X,I_\tau)$ is $IgPA*$continuous.
Proof: $(i) \Rightarrow$ (ii) If F is an $IgPA*C$-set in (X,I_τ) then $X-F$ is $IgPA*O$ in (X,I_τ) . But h is $IgPA*O$, thus $h(X-F)$ is $IgPA*O$ in (Y,I_σ). Hence,
$h(<F_1,F_2>) =< h(F_1),Y-h(X-F_2)>$ is $IgPA*O$ in (Y,I_σ) and so
$< Y-h(X-F_2),h(F_1)>=< h(F_2),h(F_1)>$ is $IgPA*C$ in (Y,I_σ). Because
$Y-h(X-F_2)=h(F_2),< Y-h(X-F_2),h(F_1)>=$
$< h(F_2),h(F_1)>$ is $IgPA*C$ in (Y,I_σ) . Therefore,h^{-1} is $IgPA*C$.
(ii) $\Rightarrow$ (iii) Assume F be an $IgPA*C$ in (X,I_τ). Because h is bijective and $IgPA*C$, hence $h(F)=(h^{-1})^{-1}(F)$ is $IgPA*C$ in (Y,I_σ). Therefore h^{-1} is $IgPA*C$ -continuous.
(iii) $\Rightarrow (i)$ Assume F be an $IgPA*O$ in (X,I_τ). By assumption, $(h^{-1})^{-1}(F)$ is $IgPA*O$ in (Y,I_σ). Hence, $h(F)$ is $IgPA*O$ in (Y,I_σ). Thus, h is $IgPA*O$.

Theorem 3.11. if a map $h:(X,I_\tau) \rightarrow (Y,I_\sigma)$ is a bijective map. The arguments that follow are equivalent:
(i) h is $IgSA*O$.
(ii) h is $IgSA*C$.
(iii) $h^{-1}:(Y,I_\sigma) \rightarrow (X,I_\tau)$ is $IgSA*$continuous.
Proof: obvious.

Theorem 3.12. if a map $h:(X,I_\tau) \rightarrow (Y,I_\sigma)$ is a bijective map. The arguments that follow are equivalent:
(i) h is $Ig\beta A*O$.
(ii) h is $Ig\beta A*C$.
(iii) $h^{-1}:(Y,I_\sigma) \rightarrow (X,I_\tau)$ is $Ig\beta A*$continuous.
Proof: obvious.

Theorem 3.13. if a map $h:(X,I_\tau) \rightarrow (Y,I_\sigma)$ is a bijective map. The arguments that follow are equivalent:
(i) h is $LIgO$.
(ii) h is $LIgC$.
(iii) $h^{-1}:(Y,I_\sigma) \rightarrow (X,I_\tau)$ is LIg continuous.
Proof: obvious.

Definition 3.14. A map $h:(X,I_\tau) \rightarrow (Y,I_\sigma)$ consider to be:
LIg –closed map if the image h(M) is LIg-closed in (Y,I_σ) for each $LIgC-$ set M in(X,I_τ).
$Ig\alpha A*$ –closed map if the image h(M) is $Ig\alpha A*$-closed in (Y,I_σ) for each $Ig\alpha A*C-$ set M in(X,I_τ).

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

$IgPA*$ –closed map if the image h(M) is $IgPA*$-closed in (Y, I_σ) for each $IgPA* C-$ set M in(X, I_τ).
$IgSA*$ –closed map if the image h(M) is $IgSA*$-closed in (Y, I_σ) for each $IgSA* C-$ set M in(X, I_τ).
$Ig\beta A*$ –closed map if the image h(M) is $Ig\beta A*$-closed in (Y, I_σ) for each $Ig\beta A* C-$ set M in(X, I_τ).

Theorem 3.15. Suppose that a two mapping $h: (X, I_\tau) \to (Y, I_\sigma)$ and φ: $(Y, I_\sigma) \to (Z, I_\rho)$ for each subset $M \subseteq X$.Then the below holds:
If both h and φ are LIg- closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is LIg - closed map.
If both h and φ are $Ig\alpha A*$- closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $Ig\alpha A*$- closed map.
If both h and φ are $IgPA*$- closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $IgPA*$ –closed map.
If both h and φ are $IgSA*$ - closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $IgSA*$ - closed map.
If both h and φ are $Ig\beta A*$- closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $Ig\beta A*$ - closed map.
Proof.
Assume K be an $LIgC-$ set in (X, I_τ) .Since h is LIg- closed map, h(K) is $LIgC-$ set in (Y, I_σ). Since φ is LIg- closed map, φ(h(K)) is $LIgC-$ set in (Z, I_ρ) .Thus, φ ∘ h is LIg - closed map.
Assume K be an $Ig\alpha A* C-$ set in (X, I_τ) .Since h is $Ig\alpha A*$- closed map, h(K) is $Ig\alpha A* C-$ set in (Y, I_σ). Since φ is $Ig\alpha A*$- closed map, φ(h(K)) is $Ig\alpha A* C-$ set in (Z, I_ρ) .Thus, φ ∘ h is $Ig\alpha A*$ - closed map.
Similarly, we can prove (3), (4) and (5) as same way.
Definition 3.16. A map $h: (X, I_\tau) \to (Y, I_\sigma)$ consider to be:
LIg –open map if the image h(M) is LIg-closed in (Y, I_σ) for each $LIgO-$ set M in(X, I_τ).
$Ig\alpha A* -$ open map if the image h(M) is $Ig\alpha A*$-closed in (Y, I_σ) for each $Ig\alpha A* O-$set M in(X, I_τ).
$IgPA* -$ open map if the image h(M) is $IgPA*$-closed in (Y, I_σ) for each $IgPA* O-$ set M in(X, I_τ).
$IgSA* -$ open map if the image h(M) is $IgSA*$-closed in (Y, I_σ) for each $IgSA* O-$ set M in(X, I_τ).
$Ig\beta A* -$ open map if the image h(M) is $Ig\beta A*$-closed in (Y, I_σ) for each $Ig\beta A* O-$ set M in(X, I_τ).

Theorem 3.17. Suppose that a two mapping $h: (X, I_\tau) \to (Y, I_\sigma)$ and φ: $(Y, I_\sigma) \to (Z, I_\rho)$ for each subset $M \subseteq X$.Then the below holds:
If both h and φ are LIg- closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is LIg - closed map.
If both h and φ are $Ig\alpha A*$- closed map then, φ ∘ h: $(X, I_\tau) \to (Z, I_\rho)$ is $Ig\alpha A*$- closed map.

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

If both h and φ are $IgPA*$- closed map then, $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $IgPA*$ –closed map.
If both h and φ are $IgSA*$ - closed map then, $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $IgSA*$ - closed map.
If both h and φ are $Ig\beta A*$- closed map then, $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $Ig\beta A*$ - closed map.

Proof.

Assume K be an $LIgO-$ set in (X, I_τ) .Since h is LIg- open map, h(K) is $LIgO-$ set in (Y, I_σ). Since φ is LIg- open map, φ(h(K)) is $LIgC-$ set in (Z, I_ρ) .Thus, $\varphi \circ h$ is LIg - open map.

Similarly, we can prove (2), (3), (4) and (5) as same way.

Remark 3.18. The following figure shows the relationships among the concepts discussed in this part that have been created.

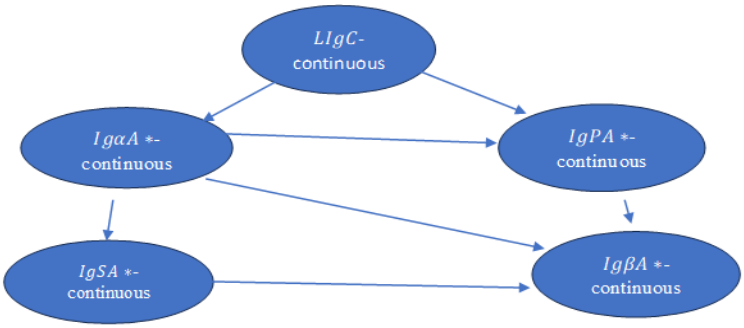

Fig 1. Showing the relationships among the continuous mappings that have been created

4. Some Generalized Irresolute Mappings in ITS

In this part, we began with the following definition:

Definition 4.1. Assume that (X, I_τ) and (Y, I_σ) are two ITSs and h represents a mapping from (X, I_τ) into (Y, I_σ). Then a mapping $h: (X, I_\tau) \to (Y, I_\sigma)$ considers below:

$Ig\beta A*$ -irresolute if $h^{-1}(M)$ is $Ig\beta A*O$ -set in X for every $Ig\beta A*O$ - set M in Y.

$IgPA*$ - irresolute if $h^{-1}(M)$ is $Ig\beta A*O$ -set in X for every $IgPA*O$ - set M in Y.

$IgSA*$ -irresolute if $h^{-1}(M)$ is $Ig\beta A*O$-set in X for every intuitionistic $IgSA*O$ - set M in Y.

$Ig\alpha A*$- irresolute if $h^{-1}(M)$ is $Ig\alpha A*O$-set in X for every intuitionistic $Ig\alpha A*O$- set M in Y.

LIg - irresolute if $h^{-1}(M)$ is $LIgO$-set in X for every $LIgO$ - set M in Y.

Theorem 4.2. *Suppose that* $h: (X, I_\tau) \to (Y, I_\sigma)$ *be a mapping.*

Then the below are holds:

If h *is* LIg *- irresolute, then,* h *is* $Ig\alpha A*$*- irresolute map.*

If h *is* $Ig\alpha A*$*- irresolute then,* h *is* $IgSA*$*- irresolute map.*

If h *is* $Ig\alpha A*$ *-continuous then,* h *is* $IgPA*$*- irresolute map.*

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

If h is $IgSA*$- irresolute then, h is $Ig\beta A*$- irresolute map.

If h is $IgPA*$- irresolute then, h is $Ig\beta A*$- irresolute map.

Proof:

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $LIgO$-set in Y. Then, $int(M) \subseteq U$ whenever $M \subseteq U$, U is IOS and M is $IA*$ such that $M = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IOS and $\aleph$ is ICS. Since, M is $LIgO - set$, so there exists an $Ig\alpha A*O$-set in Y, because every $LIgP$-set is $Ig\alpha A*O$-set. Now, since h is $LIgO$- irresolute as a result, $h^{-1}(M)$ is $LIgO$-set in $X$by assumption. Consequently, $h^{-1}(M)$ is an $Ig\alpha A*O$-set in X. Therefore, h is

$Ig\alpha A*$- irresolute map.

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $Ig\alpha A*O$-set in Y.Thus, $int_\alpha(cl_\alpha(int_\alpha(M))) \subseteq U$ whenever $M \subseteq U$, U is IαOS, and M is $I\alpha A*$ such that $M = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IαOS and $\aleph$ is IαOS. Since, M is $Ig\alpha A*C$ set, so there exists an $IgSA*O$-set in Y, because every $Ig\alpha A*O$ -set is $IgSA*O$-set. Since h is $Ig\alpha A*$- irresolute as a result, $h^{-1}(M)$ is

$Ig\alpha A*O$ -set in X by assumption. Consequently, $h^{-1}(M)$ is an $IgSA*O$-set in X.

Therefore, h is $IgSA*$- irresolute map.

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $Ig\alpha A*O$-set in Y.Hence, $int_\alpha(cl_\alpha(int_\alpha(M))) \subseteq U$ whenever $M \subseteq U$, U is IαOS, and M is $I\alpha A*O$ such that $M = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IαOS and $\aleph$ is IαCS. Since, M is $Ig\alpha A*O$ set, so there exists an $Ig\alpha A*O$-set in Y, because every $Ig\alpha A*O$ -set is $IgPA*O$-set. Since h is $Ig\alpha A*$- irresolute as a result, $h^{-1}(M)$ is $Ig\alpha A*OC$ -set in X by assumption. Consequently, $h^{-1}(M)$ is an $IgPA*OC$-set in X. Therefore, h is $IgPA*$- irresolute map.

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $IgSA*O$-set in Y.Thus, $cl_S(int_S(M)) \subseteq U$ whenever $M \subseteq U$, U is ISOS and M is $ISA*$ such that $M = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is ISOS and $\aleph$ is ISCS. Since, M is $IgSA*O$ set, so there exists an $Ig\beta A*O$-set in Y, because every $IgSA*O$ -set is $Ig\beta A*O$-set. Since h is $IgSA*$- irresolute as a result, $h^{-1}(M)$ is $IgSA*O$-set in X by assumption. Consequently, $h^{-1}(M)$ is an $Ig\beta A*O$-set in X. Therefore, h is $Ig\beta A*$- irresolute map.

Considering that $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping and M is $IgPA*O$-set in Y.Thus $int_p(cl_p(M)) \subseteq U$ whenever $M \subseteq U$, U is IPOS and M is $IPA*$ such that $M = \mathcal{M} \cup \aleph$, where $\mathcal{M}$ is IPOS and $\aleph$ is IPCS. Since, M is $IgPA*O$ set, so there exists

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

an $IgA\beta * O$-set in Y, because every $IgPA * O$ -set is $IgA\beta * O$-set. Since h is $IgPA *$- irresolute as a result, $h^{-1}(M)$ is $IgPA * O$-set in X by assumption. Consequently, $h^{-1}(M)$ is an $IgA\beta * O$-set in X. Therefore, h is $IgA\beta *$- irresolute map.

Remark 4.3. The converse of Theorem 4.2. is not true, As seen in the following examples.

Example 4.4. Assume $X = \{\xi, \lambda, \kappa\}$ with topology $I_\tau = \{\tilde{X}, \tilde{\emptyset}, M, N, W\}$, where $M = \langle\{\xi\}, \{\lambda\}\rangle, N = \langle\{\kappa\}, \{\lambda\}\rangle, N = \langle\{\xi, \kappa\}, \{\lambda\}\rangle$, and $Y = \{15,17\}$ with topology $I_\sigma = \{\tilde{X}, \tilde{\emptyset}, P, W, L\}$ where $P = \langle\{15\}, \emptyset\rangle, W = \langle\emptyset, \emptyset\rangle, L = P = \langle\emptyset, \{17\}\rangle$, and $h: (X, I_\tau) \to (Y, I_\sigma)$ *be* a mapping

defined by $h(\xi) = 15, h(\lambda) = h(\kappa) = 17$. Then, the following are fulfilled:

h is $IgPA *$- irresolute but, h is not $Ig\alpha A *$- irresolute, because for $D = \langle\{17\}, \emptyset\rangle$ is $IgPA * O$-set in Y, $h^{-1}(D) = \langle\{\lambda, \kappa\}, \emptyset\rangle \notin Ig\alpha A * O$.

h is $IgSA *$- irresolute but, h is not $Ig\alpha A *$- irresolute, because for $E = \langle\{17\}, \emptyset\rangle$ is $IgSA * O$-set in Y, $h^{-1}(L) = \langle\{\lambda, \kappa\}, \emptyset\rangle \notin Ig\alpha A * O$.

Example 4.5. Assume $X = Y = \{\xi, \lambda, \kappa\}$ with topology $I_\tau = \{\tilde{X}, \tilde{\emptyset}, Q, R, K, W\}$, where $Q = \langle\emptyset, \emptyset\rangle, R = \langle\{\xi\}, \emptyset\rangle, W = \langle\{\kappa\}, \emptyset\rangle, K = \langle\{\lambda, \kappa\}, \emptyset\rangle$ and $Y = \{15,17,19\}$ with topology $I_\sigma = \{\tilde{X}, \tilde{\emptyset}, P, L, J, V\}$ where $P = \langle\emptyset, \{15\}\rangle, L = \langle\emptyset, \emptyset\rangle, J = \langle\{19\}, \emptyset\rangle, V = \langle\{15, 19\}, \emptyset\rangle$ and $h: (X, I_\tau) \to (Y, I_\sigma)$

be a mapping defined by $h(\xi) = 15 = h(\lambda), h(\kappa) = 19$. Then, the following are fulfilled:

h is $IgA\beta *$- irresolute but, h is not $IgPA *$- irresolute, because for $E = \langle\{15\}, \emptyset\rangle$ is $IgA\beta * O$-set in Y, $h^{-1}(E) = \langle\{\xi, \lambda\}, \emptyset\rangle \notin IgPA * O$.

h is $IgA\beta *$- irresolute but, h is not $IgSA *$- irresolute, because for $E = \langle\{15\}, \emptyset\rangle$ is $IgA\beta * O$-set in Y, $h^{-1}(S) = \langle\{\xi, \lambda\}, \emptyset\rangle \notin IgSA * O$.

Recall Example 3.6. We see that h is $Ig\alpha A *$- irresolute but, h is not LIg- irresolute, because for $T = \langle\{15\}, \emptyset\rangle$ is $Ig\alpha A * O$-set in Y, $h^{-1}(T) = \langle\{\xi\}, \emptyset\rangle \notin LIgO$.

Theorem 4.6. Assume $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping. Then the following fulfil:

If h is $Ig\alpha A *$- continuous and $Ig\alpha A * O$- then $h(M)$ is $Ig\alpha A * O(Y, I_\sigma)$, for every $M \in Ig\alpha A * (X, I_\tau)$.

If h is $IgPA *$- continuous and $IgPA * O$- then $h(M)$ is $IgPA * O(Y, I_\sigma)$, for every $M \in IgpA * (X, I_\tau)$.

If h is $IgSA *$- continuous and $IgSA * O$- then $h(M)$ is $IgSA * O(Y, I_\sigma)$, for every $M \in IgSA * (X, I_\tau)$.

If h is $IgA\beta *$- continuous and $IgA\beta * O$- then $h(M)$ is $IgA\beta * O(Y, I_\sigma)$, for every $M \in IgA\beta * (X, I_\tau)$.

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

If h is LIg- continuous and $LIgO$- then $h(M)$ is $LIgO(Y, I_\sigma)$, for every $M \in \mathrm{L}IgO(X, I_\tau)$.

Proof:

Assume $M \in Ig\alpha A*(X, I_\tau)$.Thus $Q \in (X, I_\tau)$ such that $Q \subseteq M \subseteq Ig\alpha A*cl_\alpha(Q)$ and $h(Q) \subseteq h(M) \subseteq h(Ig\alpha A*cl_\alpha(Q))$. Because h is $Ig\alpha A*$- continuous and $h(\sigma) \in (Y, I_\sigma)$ and h is $Ig\alpha A*O$, hence, $h(Ig\alpha A*cl_\alpha(Q)) \subseteq Ig\alpha A*cl_\alpha(h(Q))$. Therefore, $h(M)$ is $Ig\alpha A*O(Y, I_\sigma)$.

Assume $M \in IgPA*(X, I_\tau)$.Thus $Q \in (X, I_\tau)$ such that $IgPA*int_P(Q) \subseteq Q \subseteq M$ and $h(IgPA*int_P(Q)) \subseteq h(Q) \subseteq h(M)$. Because h is $IgPA*$- continuous and $h(\sigma) \in (Y, I_\sigma)$ and h is $IgPA*O$, hence, $h(IgPA*int_P(Q)) \subseteq IgPA*int_P(h(Q))$. Therefore, $h(M)$ is $IgPA*O(Y, I_\sigma)$.

Similarly, we can prove (3), (4) and (5) as similar way.

Theorem 4.7. Assume $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping. Then the following fulfil:

If h is $Ig\alpha A*$- continuous and $Ig\alpha A*O$- then h is $Ig\alpha A*$ - irresolute.

If h is $IgPA*$- continuous and $IgPA*O$- then h is $Ig\alpha P*$ - irresolute.

If h is $IgSA*$- continuous and $IgSA*O$- then h is $IgSA*$ - irresolute.

If h is $Ig\beta A*$- continuous and $Ig\beta A*O$- then h is $Ig\beta A*$ - irresolute.

If h is LIg- continuous and $LIgO$- then h is LIg - irresolute.

Proof:

Assume $M \in Ig\alpha A*(Y, I_\sigma)$, so there exist is IOS $Q \subseteq Y$ such that $Q \subseteq M \subseteq Ig\alpha A*cl_\alpha(Q)$. Thus, $h^{-1}(Ig\alpha A*cl_\alpha(Q)) = Ig\alpha A*cl_\alpha(h^{-1}(Q))$ and so $h^{-1}(Q) \subseteq h^{-1}(M) \subseteq h^{-1}(Ig\alpha A*cl_\alpha(Q)) = Ig\alpha A*cl_\alpha(h^{-1}(Q))$. Now because h is $Ig\beta A*$-continuous, then $h^{-1}(Q)$ is an $Ig\alpha A*O$-set. Therefore, $h^{-1}(M)$ is h is $Ig\alpha A*$ - irresolute.

Assume $M \in IgPA*(Y, I_\sigma)$, so there exist is IOS $Q \subseteq Y$ such that $IgPA*int_P(Q) \subseteq Q \subseteq M$. Thus, $h^{-1}(IgPA*int_P(Q)) = IgPA*int_P(h^{-1}(Q))$ and so $h^{-1}(IgPA*int_P(Q)) \subseteq h^{-1}(Q) \subseteq h^{-1}(M) = IgPA*int_P(h^{-1}(Q))$. Now because h is $Ig\beta A*$- continuous, then $h^{-1}(Q)$ is an $IgPA*O$-set. Therefore, $h^{-1}(M)$ is h is $IgPA*$ - irresolute.

Similarly, we can prove (3), (4) and (5) as similar way.

Theorem 4.8. Assume $h: (X, I_\tau) \to (Y, I_\sigma)$ be a mapping. Then the following fulfil:

h is $Ig\alpha A*$- irresolute iff $h^{-1}(M)$ is $Ig\alpha A*$-closed map for every $Ig\alpha A*C$ –set M of Y.

h is $IgPA*$- irresolute iff $h^{-1}(M)$ is $IgPA*$-closed map for every $IgPA*C$ –set M of Y.

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

h is $IgSA*$- irresolute iff $h^{-1}(M)$ is $IgSA*$-closed map for every $IgSA*C$ –set M of Y.
h is $Ig\beta A*$- irresolute iff $h^{-1}(M)$ is $Ig\beta A*$-closed map for every $Ig\beta A*C$ –set M of Y.
h is LIg- irresolute iff $h^{-1}(M)$ is LIg-closed map for every $LIgC$ –set M of Y.
Proof:
Necessary condition: Assume $h:(X,I_\tau)\to(Y,I_\sigma)$ is $Ig\alpha A*$- irresolute, thus for each $Ig\alpha A*O$- set W of Y, $h^{-1}(W)$ i is $Ig\alpha A*O(X,I_\tau)$. Now, if M is any $Ig\alpha A*C$ set of (Y,I_σ), hence, $Y-M$ is $Ig\alpha A*O$. So, $h^{-1}(Y-M)$ is $Ig\alpha A*O$, but $h^{-1}(Y-M)=X-h^{-1}(M)$ and therefore, $h^{-1}(M)$ is $Ig\alpha A*C$.
Sufficient condition: Assume $M\in(Y,I_\sigma)$ be an $IgPA*C$-set. Then $h^{-1}(M)$ is $Ig\alpha A*C-$ set in X by assumption. Now, if W is any $Ig\alpha A*O(Y,I_\sigma)$, thus $Y-W$ is $Ig\alpha A*C$. So that, $h^{-1}(Y-N)=X-h^{-1}(N)$ is $Ig\alpha A*C$. Thus $h^{-1}(N)$ is $Ig\alpha A*O$. Therefore, h is $Ig\alpha A*$- irresolute.
Necessary condition: Assume $h:(X,I_\tau)\to(Y,I_\sigma)$ is $IgPA*$- irresolute, thus for each $IgPA*O$- set W of Y, $h^{-1}(W)$ i is $IgPA*O(X,I_\tau)$. Now, if M is any $IgPA*C$ set of (Y,I_σ), hence, $Y-M$ is $IgPA*O$. So, $h^{-1}(Y-M)$ is $IgPA*O$, but $h^{-1}(Y-M)=X-h^{-1}(M)$ and therefore, $h^{-1}(M)$ is $IgPA*C$.
Sufficient condition: Assume $M\in(Y,I_\sigma)$ be an $IgPA*C$-set. Then $h^{-1}(M)$ is $IgPA*C-$ set in X by assumption. Now, if W is any $IgPA*O(Y,I_\sigma)$, thus $Y-W$ is $IgPA*C$. So that, $h^{-1}(Y-N)=X-h^{-1}(N)$ is $IgPA*C$. Thus $h^{-1}(N)$ is $IgPA*O$. Therefore, h is $IgPA*$- irresolute.
Similarly, we can prove (3), (4) and (5) as similar way.

Theorem 4.9. Assume $h:(X,I_\tau)\to(Y,I_\sigma)$ be a mapping. Then the following fulfil:
If h is an $Ig\alpha A*$-irresolute then h is an $Ig\alpha A*$- continuous.
If h is an $IgPA*$-irresolute then h is an $IgPA*$- continuous.
If h is an $IgSA*$-irresolute then h is an $IgSA*$- continuous.
If h is an $Ig\beta A*$-irresolute then h is an $Ig\beta A*$- continuous.
If h is an LIg-irresolute then h is an LIg- continuous.
Proof.
Assume that M is an $Ig\alpha A*O(Y,I_\sigma)$. As every intuitionistic open set is $Ig\alpha A*O$,and so M is $Ig\alpha A*O(Y,I_\sigma)$. By hypothesis, h is $Ig\alpha A*$-irresolute. Thus, $h^{-1}(M)$ is $Ig\alpha A*O(X,I_\tau)$. Consequently, h is $Ig\alpha A*$- continuous.
Assume that M is an $IgPA*O(Y,I_\sigma)$. As every intuitionistic open set is $IgPA*O$, and so M is $IgPA*O(Y,I_\sigma)$. By hypothesis, h is $IgPA*$-irresolute. Thus, $h^{-1}(M)$ is $IgPA*O(X,I_\tau)$. Consequently, h is $IgPA*$- continuous.
Similarly, we can prove (3), (4) and (5) as similar way.

ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 7 Val:1 /11/ 2024

Remark 4.10. The converse of Theorem 4.9. is not true, As seen in the following examples.

Recall Examples 3.1.4. and 3.1.5. We see that the following:

h is $Ig\alpha A*$- continuous but not $Ig\alpha A*$- irresolute.

h is $Ig\alpha A*$- continuous but not $Ig\alpha A*$- irresolute.

h is $Ig\alpha A*$- continuous but not $Ig\alpha A*$- irresolute.

h is $Ig\alpha A*$- continuous but not $Ig\alpha A*$- irresolute.

Example 4.11. Let $X = \{\xi, \lambda\} = Y$, $I_\tau = \{\widetilde{X}, \widetilde{\emptyset}, \langle Q, \xi\rangle\}, I_\sigma = \{\widetilde{Y}, \widetilde{\emptyset}, \langle Q, \xi\rangle\}$ and define $h: (X, I_\tau) \to (Y, I_\sigma)$ by $h(\xi) = \xi, h(\lambda) = \lambda$ then h is an LIg- continuous but not an LIg- irresolute.

Theorem 4.12. Assume $h: (X, I_\tau) \to (Y, I_\sigma)$ and $\varphi: (Y, I_\sigma) \to (Z, I_\rho)$ are two mappings. Then the following holds:

If h and φ be both $Ig\alpha A*$- irresolute then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $Ig\alpha A*$- irresolute.

If h and φ be both $IgPA*$- irresolute then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $IgPA*$- irresolute.

If h and φ be both $IgSA*$- irresolute then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $IgSA*$- irresolute.

If h and φ be both $Ig\beta A*$- irresolute then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $Ig\beta A*$- irresolute.

If h and φ be both LIg- irresolute then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is LIg- irresolute.

Proof:

Let $F \subseteq Z$ is an $Ig\alpha A* O$ then $\varphi^{-1}(F)$ is $Ig\alpha A*$- irresolute so $(h^{-1}(\varphi^{-1}(F)))$ is $Ig\alpha A* O$ because, h, φ are both $Ig\alpha A*$- irresolute. Hence, $(\varphi \circ h)^{-1}(F) = (h^{-1}(\varphi^{-1}(F)))$ is an $Ig\alpha A* O$. Therefore, $\varphi \circ h$ is $Ig\alpha A*$- irresolute.

Let $F \subseteq Z$ is an $IgPA* O$ then $\varphi^{-1}(F)$ is $IgPA*$- irresolute so $(h^{-1}(\varphi^{-1}(F)))$ is $IgPA* O$ because, h, φ are both $IgPA*$- irresolute. Hence, $(\varphi \circ h)^{-1}(F) = (h^{-1}(\varphi^{-1}(F)))$ is an $IgPA* O$. Therefore, $\varphi \circ h$ is $IgPA*$- irresolute.

Similarly, we can prove (3), (4) and (5) as similar way.

Theorem 4.13. Assume $h: (X, I_\tau) \to (Y, I_\sigma)$ and $\varphi: (Y, I_\sigma) \to (Z, I_\rho)$ are two mappings. Then the following holds:

If h is an $Ig\alpha A*$- irresolute and φ is an $Ig\alpha A*$- continuous then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $Ig\alpha A*$- continuous.

If h is an $IgPA*$- irresolute and φ is an $IgPA*$- continuous then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $IgPA*$- continuous.

If h is an $IgSA*$- irresolute and φ is an $IgSA*$- continuous then $\varphi \circ h: (X, I_\tau) \to (Z, I_\rho)$ is $IgSA*$- continuous.

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No: 7 Val:1 /11/ 2024

If h is an $Ig\beta A*$- irresolute and φ is an $Ig\beta A*$- continuous then $\varphi \circ h : (X, I_\tau) \to (Z, I_\rho)$ is $Ig\beta A*$- continuous.

If h is an LIg- irresolute and φ is an LIg- continuous then $\varphi \circ h : (X, I_\tau) \to (Z, I_\rho)$ is LIg- continuous.

Proof.

Assume M is $Ig\alpha A * C(Z, I_\rho)$, so, $\varphi^{-1}(M)$ is $Ig\alpha A * C(Y, I_\sigma)$. Consequently, $h^{-1}(\varphi^{-1}(\bar{M})) = (\varphi \circ h)^{-1}(\bar{M})$ is $Ig\alpha A * O(X, I_\tau)$, because h is $Ig\alpha A*$-irresolute. Hence, $h^{-1}(\varphi^{-1}(M)) = (\varphi \circ h)^{-1}(M)$ is $Ig\alpha A * C(X, I_\tau)$ Therefore, $\varphi \circ h$ is also an $Ig\alpha A*$- continuous.

Assume M is $IgPA * C(Z, I_\rho)$, so, $\varphi^{-1}(M)$ is $IgPA * C(Y, I_\sigma)$. Consequently, $h^{-1}(\varphi^{-1}(\bar{M})) = (\varphi \circ h)^{-1}(\bar{M})$ is $IgPA * O(X, I_\tau)$, because h is $IgPA*$-irresolute. Hence, $h^{-1}(\varphi^{-1}(M)) = (\varphi \circ h)^{-1}(M)$ is $IgPA * C(X, I_\tau)$ Therefore, $\varphi \circ h$ is also an $IgPA*$- continuous.

Similarly, we can prove (3), (4) and (5) as similar way.

5. Conclusion

The article you cited presents the idea of a generalized weak continuous, closed and open mappings in intuitionistic topological environments and analyzes numerous results associated with this new sets. It implies that the concepts and results provided in the work can be applied to different kinds of topological spaces, like ideal topological spaces and others.

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