STUDY OF CHEMICAL AND HEALTH PROBLEMS BY KUFFI ISSA TRANSFORM

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Abstract

This paper introduces the Kuffi-Issa Transform (KIT), a new mathematical technique developed to deal with problems in chemistry, health, physics, and other sciences. To test the validity of this transform, three chemical and health examples are presented. The results show that this transform is a powerful tool for solving differential equations encountered in chemical and health problems.

1 Introduction

Integral transformations are used to solve differential equations, and are widely used in physics, engineering, and applied mathematics [1, 2, 3]. Through these transformations, a specific function in a domain such as time is transformed into another domain such as frequency. To solve these equations, several transformations have been introduced, including Laplace transform [4], Fourier transform [5], Elzaki transform [6], and Aboodh Transform [7]. Recently, new transformations have been introduced, including complex SEE transform [8], Emad-Sara transform [9], Mayan transform [10], and Jafari transform [11]. In this paper, we will introduce a new transformation (Kuffi Issa transform) to solve differential equations.

2 Preliminaries

2.1 Definitions and Theorems of Kuffi Issa Transform

Definition 2.1 *The Kuffi Issa transform of $f(t)$ denoted by $KI[.]$ is given by*

$$KI[f(t)] = (uv)^{i\beta} \int_{t=0}^{\infty} f(v^{i\gamma} t) e^{-u^{i\alpha} t} dt = F(u, v), \quad (1)$$

where u, v are parameters; $t \geq 0; \alpha, \beta, \gamma \in Z$.

Definition 2.2 *The Inverse Kuffi Issa transform of $F(u, v)$ denoted by $(KI)^{-1}[.]$ is given by*

$$(KI)^{-1}[KI[f(t)]] = \frac{1}{2\pi i} \lim_{\epsilon \to 0} \int_{\mu - i\epsilon}^{\mu + i\epsilon} (uv)^{i\beta} v^{i\gamma} e^{\frac{u^{i\alpha}}{v^{i\alpha}}} F(u, v) du = f(t). \quad (2)$$

Theorem 2.3 *(Linearity of KI Transform)*

If $KI[f_1(t)] = F_1(u, v)$ and $KI[f_2(t)] = F_2(u, v)$, then

$KI[c_1 f_1(t) \mp c_2 f_2(t)] = c_1 F_1(u, v) \mp c_2 F_2(u, v)$.

Proof. By using Eq (1) we obtain

$$KI[c_1 f_1(t) \mp c_2 f_2(t)] = (uv)^{i\beta} \int_0^{\infty} [c_1 f_1(v^{i\gamma} t) \mp c_2 f_2(v^{i\gamma} t)] e^{-u^{i\alpha} t} dt$$

$$= c_1 (uv)^{i\beta} \int_0^{\infty} f_1(v^{i\gamma} t) e^{-u^{i\alpha} t} dt \mp c_2 (uv)^{i\beta} \int_0^{\infty} f_2(v^{i\gamma} t) e^{-u^{i\alpha} t} dt$$

$$= c_1 F_1(u, v) \mp c_2 F_2(u, v).$$

Theorem 2.4 *Let C be a scalar and $f(t) = C$, then $KI[f(t)] = C u^{i(\beta - \alpha)} v^{i\beta}$*

Proof. By using Eq (1) we obtain

$$KI[C] = (uv)^{i\beta} \int_0^\infty Ce^{-u^{i\alpha}t}dt = C(uv)^{i\beta} \frac{e^{-u^{i\alpha}t}}{-u^{i\alpha}}|_0^\infty = Cu^{i(\beta-\alpha)}v^{i\beta}$$

Theorem 2.5 *If* $f(t) = t$, *then* $KI[f(t)] = u^{i(\beta-2\alpha)}v^{i(\beta+\gamma)}$

Proof. By using Eq (1) we obtain

$$KI[t] = (uv)^{i\beta} \int_0^\infty v^{i\gamma} te^{-u^{i\alpha}t}dt = u^{i\beta}v^{i(\beta+\gamma)} \int_0^\infty te^{-u^{i\alpha}t}dt$$

$$= u^{i(\beta-\alpha)}v^{i(\beta+\gamma)} \frac{e^{-u^{i\alpha}t}}{-u^{i\alpha}}|_0^\infty = u^{i(\beta-2\alpha)}v^{i(\beta+\gamma)}$$

Theorem 2.6 *If* $f(t) = e^{at}$, *then* $KI[f(t)] = \frac{(uv)^{i\beta}}{u^{i\alpha}-av^{i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[e^{at}] = (uv)^{i\beta} \int_0^\infty e^{a(v^{i\gamma}t)}e^{-u^{i\alpha}t}dt = (uv)^{i\beta} \int_0^\infty e^{-(u^{i\alpha}-av^{i\gamma})t}dt$$

$$= \frac{(uv)^{i\beta}}{-(u^{i\alpha}-av^{i\gamma})} e^{-(u^{i\alpha}-av^{i\gamma})t}|_0^\infty = \frac{(uv)^{i\beta}}{u^{i\alpha}-av^{i\gamma}}$$

Theorem 2.7 *If* $f(t) = e^{-at}$, *then* $KI[f(t)] = \frac{(uv)^{i\beta}}{u^{i\alpha}+av^{i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[e^{-at}] = (uv)^{i\beta} \int_0^\infty e^{-a(v^{i\gamma}t)}e^{-u^{i\alpha}t}dt = (uv)^{i\beta} \int_0^\infty e^{-(u^{i\alpha}+av^{i\gamma})t}dt$$

$$= \frac{(uv)^{i\beta}}{-(u^{i\alpha}+av^{i\gamma})} e^{-(u^{i\alpha}+av^{i\gamma})t}|_0^\infty = \frac{(uv)^{i\beta}}{u^{i\alpha}+av^{i\gamma}}$$

Theorem 2.8 *If* $f(t) = e^{ait}$, *then* $KI[f(t)] = \frac{(uv)^{i\beta}}{u^{i\alpha}-aiv^{i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[e^{ait}] = (uv)^{i\beta} \int_0^\infty e^{ai(v^{i\gamma}t)}e^{-u^{i\alpha}t}dt = (uv)^{i\beta} \int_0^\infty e^{-(u^{i\alpha}-aiv^{i\gamma})t}dt$$

$$= \frac{(uv)^{i\beta}}{-(u^{i\alpha}-aiv^{i\gamma})} e^{-(u^{i\alpha}-aiv^{i\gamma})t}|_0^\infty = \frac{(uv)^{i\beta}}{u^{i\alpha}-aiv^{i\gamma}}$$

Theorem 2.9 *If* $f(t)=e^{-ait}$, *then* $KI[f(t)]=\frac{(uv)^{i\beta}}{u^{i\alpha}+aiv^{i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[e^{-ait}]=(uv)^{i\beta}\int_0^\infty e^{-ai(v^{i\gamma}t)}e^{-u^{i\alpha}t}dt=(uv)^{i\beta}\int_0^\infty e^{-(u^{i\alpha}+aiv^{i\gamma})t}dt$$

$$=\frac{(uv)^{i\beta}}{-(u^{i\alpha}+aiv^{i\gamma})}e^{-(u^{i\alpha}+aiv^{i\gamma})t}|_0^\infty=\frac{(uv)^{i\beta}}{u^{i\alpha}+aiv^{i\gamma}}$$

Theorem 2.10 *If* $f(t)=cosat$, *then* $KI[f(t)]=\frac{u^{(\alpha+\beta)i}v^{i\beta}}{u^{2i\alpha}+a^2v^{2i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[\cos at]=KI\left[\frac{e^{ait}+e^{-ait}}{2}\right]=\frac{1}{2}[KI[e^{ait}]+KI[e^{-ait}]]=\frac{1}{2}[\frac{(uv)^{i\beta}}{u^{i\alpha}-aiv^{i\gamma}}+\frac{(uv)^{i\beta}}{u^{i\alpha}+aiv^{i\gamma}}]$$

$$=\frac{u^{(\alpha+\beta)i}v^{i\beta}}{u^{2i\alpha}+a^2v^{2i\gamma}}$$

Theorem 2.11 *If* $f(t)=sinat$, *then* $KI[f(t)]=\frac{au^{i\beta}v^{i(\beta+\gamma)}}{u^{2i\alpha}+a^2v^{2i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[\sin at]=KI\left[\frac{e^{ait}-e^{-ait}}{2i}\right]=\frac{1}{2i}[KI[e^{ait}]-KI[e^{-ait}]]=\frac{1}{2i}[\frac{(uv)^{i\beta}}{u^{i\alpha}-aiv^{i\gamma}}-\frac{(uv)^{i\beta}}{u^{i\alpha}+aiv^{i\gamma}}]$$

$$=\frac{au^{i\beta}v^{i(\beta+\gamma)}}{u^{2i\alpha}+a^2v^{2i\gamma}}$$

Theorem 2.12 *If* $f(t)=coshat$, *then* $KI[f(t)]=\frac{u^{(\alpha+\beta)i}v^{i\beta}}{u^{2i\alpha}-a^2v^{2i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[\cosh at]=KI\left[\frac{e^{at}+e^{-at}}{2}\right]=\frac{1}{2}[KI[e^{at}]+KI[e^{-at}]]=\frac{1}{2}[\frac{(uv)^{i\beta}}{u^{i\alpha}-av^{i\gamma}}+\frac{(uv)^{i\beta}}{u^{i\alpha}+av^{i\gamma}}]$$

$$=\frac{u^{(\alpha+\beta)i}v^{i\beta}}{u^{2i\alpha}-a^2v^{2i\gamma}}$$

Theorem 2.13 *If* $f(t)=sinhat$, *then* $KI[f(t)]=\frac{au^{i\beta}v^{i(\beta+\gamma)}}{u^{2i\alpha}-a^2v^{2i\gamma}}$

Proof. By using Eq (1) we obtain

$$KI[\sinh at] = KI\left[\frac{e^{at}-e^{-at}}{2}\right] = \frac{1}{2}[KI[e^{at}] - KI[e^{-at}]] = \frac{1}{2}\left[\frac{(uv)^{i\beta}}{u^{i\alpha}-av^{i\gamma}} - \frac{(uv)^{i\beta}}{u^{i\alpha}+av^{i\gamma}}\right]$$

$$= \frac{au^{i\beta}\, v^{i(\beta+\gamma)}}{u^{2i\alpha}-a^2 v^{2i\gamma}}$$

2.2 Kuffi Issa Transform of derivatives of $f(t)$

$$KI[f'(t)] = (uv)^{i\beta}\int_0^\infty f'(v^{i\gamma}t)e^{-u^{i\alpha}t}dt,$$

$$= (uv)^{i\beta}\left[e^{-u^{i\alpha}t}\frac{f(v^{i\gamma}t)}{v^{i\gamma}}\Big|_0^\infty - \int_0^\infty f(v^{i\gamma}t)e^{-u^{i\alpha}t}\left(\frac{-u^{i\alpha}}{v^{i\gamma}}\right)dt\right]$$

$$= \frac{u^{i\alpha}}{v^{i\gamma}}F - f(0)u^{i\beta}v^{i(\beta-\gamma)}$$

3 Applications of Kuffi Issa Transform

Now we will provide a solution to some chemical and health problems using Kuffi Issa Transform

Application 3.1 *Consider the following Differential Equation*

$$\frac{dc}{dt} = -k_0,\ c(0) = c_0 \qquad (3)$$

By taking KI transform of Eq (3), we obtain

$$KI\left[\frac{dc}{dt}\right] = KI[-k_0]$$

$$\frac{u^{i\alpha}}{v^{i\gamma}}F - c_0u^{i\beta}v^{i(\beta-\gamma)} = -k_0u^{i(\beta-\alpha)}v^{i\beta},$$

$$F = (-k_0u^{i(\beta-\alpha)}v^{i\beta} + c_0u^{i\beta}v^{i(\beta-\gamma)}).\frac{v^{i\gamma}}{u^{i\alpha}},$$

$$F = -k_0u^{i(\beta-2\alpha)}v^{i(\beta+\gamma)} + c_0u^{i(\beta-\alpha)}v^{i\beta},$$

The analytical solution of (3) is

$$c(t) = F^{-1} = -k_0 t + c_0$$

Application 3.2 *Consider the following Differential Equation*

$$\frac{dc}{dt} = -k_1 c, c(0) = c_0, k_1 > 0 \quad (4)$$

By taking KI transform of Eq (4), we obtain

$$KI[\frac{dc}{dt}] = KI[-k_1 c]$$

$$\frac{u^{i\alpha}}{v^{i\gamma}} F - c_0 u^{i\beta} v^{i(\beta-\gamma)} = -k_1 F,$$

$$F = (c_0 u^{i\beta} v^{i(\beta-\gamma)}).\frac{v^{i\gamma}}{u^{i\alpha}+k_1 v^{i\gamma}},$$

$$F = c_0 \frac{(uv)^{i\beta}}{u^{i\alpha}+k_1 v^{i\gamma}},$$

The analytical solution of (4) is

$$c(t) = F^{-1} = c_0 e^{-k_1 t}$$

Application 3.3 *Consider the following Differential Equation*

$$\frac{dy}{dt}+\frac{y(t)}{200}=\frac{3}{4}, y(0)=20, \quad (5)$$

By taking KI transform of Eq (5), we obtain

$$KI[\frac{dy}{dt}+\frac{y(t)}{200}]=KI[\frac{3}{4}]$$

$$\frac{u^{i\alpha}}{v^{i\gamma}}F-y(0)u^{i\beta}v^{i(\beta-\gamma)}+\frac{1}{200}F=\frac{3}{4}u^{i(\beta-\alpha)}v^{i\beta},$$

$$F=\frac{4000(uv)^{i\beta}+150\,u^{i(\beta-\alpha)}v^{i(\beta+\gamma)}}{200\,u^{i\alpha}+v^{i\gamma}},$$

$$F=150u^{i(\beta-\alpha)}v^{i\beta}-\frac{130(uv)^{i\beta}}{u^{i\alpha}+\frac{1}{200}v^{i\gamma}},$$

The analytical solution of (5) is

$$y(t)=F^{-1}=150-130e^{\frac{-t}{200}}$$

4 Conclusion

In this paper, a new transformation (Kuffi-Issa Transform) is constructed and applied to chemical and health problems. The main advantage of KIT transformation is that it gives solutions with less effort without dealing with long and complicated calculations. These complex powers in this transformation are what gave ease in simplifying the solution, as well as generality to many integral transformations with two parameters. In addition, this transform has been proven to solve differential equations. In future studies, we can use this transform to deal with partial differential equations, and difference equations.

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