Estimation of the reliability using Lognormal Distribution

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Abstract

This study uses the Lognormal distribution to assess the reliability of deturbidity and the level of residual chlorine in the water of Al Rashidiya Water Center in Baghdad, Iraq. The maximum likelihood method was used for estimating distribution parameters in the MATLAB program, and the Kolmogorov-Smirnov test for Goodness of Fit was applied at a significant level of 0.05. It was concluded that the average turbidity was 9.389 and the average was 9, which is within the legal range, the average chlorine was 0.0296 and the average was 0.03 which is well below the permissible level. The study also discovered that the average effectiveness of deturbation was 71.764%, which is sufficient but should be improved. the reliability for deturbation was 100% and for the residual chlorine 0.96, suggesting that the Rashidiya Water Center was reliable during the research period. The researchers recommend conducting frequent inspections of water treatment facilities and plants in Iraq and comparing failure distributions to determine the optimal distribution of all water impurities.

Keywords: Reliability, Efficiency, Lognormal Distribution, Maximum likelihood Estimator, Water Treatment Center.

1. Introduction

All drinking water treatment stations in Iraq work by legal limitations. As a result, frequent evaluations must be done to research and analyze the quality of these stations ' outputs. Reliability can be broadly defined for a water treatment plant as the probability that a plant will meet the maximum allowable limits of the treatment standards within a specified time [1] One of the most widely used methods for assessing reliability is using the Lognormal distribution. The Lognormal distribution is a general probability distribution that can be used in assessing reliability and in assessing risk. for a Tigris River water treatment center. The site's water treatment center reliability in terms of depollution is affected by several factors; Among the most important of these factors are the wide variations in water quantity and quality, such as the amount of rain falling, and the torrents that end in the Tigris River. In the Rashidiya Water Center, one of the most important factors that hurt the operation of the center and thus reduce reliability is rain, torrential rains and illegal discharge of sewage water.[2]

Al Rashidiya Water Center is located in the north of Baghdad, about 200 meters away from the Tigris River, with a design capacity of 2000 m3/h [3], as the project’s work mechanism includes drawing raw water from the river and then entering it into Relatively large ponds to be mixed with alum, after that the water is distributed to the sedimentation ponds, and then the water is transferred to the filtration ponds, the depth of each basin ranges (12-14)meters, a length of 4 meters, and a width of 5 meters, each basin consists of several layers of gravel and sand of varying thickness. Then, the water is pushed into ground tanks, in which the final chlorine(〖CL〗 _2) sterilization process takes place.[4]

2. Method

2.1 Reliability

The study was conducted during starting from 16/6/2021 to 15/6/2022, on daily (365) days data measured according to modern devices turbidity and chlorine $CL_2)$, where turbidity is measured before and after the treatment process, and the study relied on turbidity data after treatment only. Chlorine (CL_2) is measured after the treatment process to know the remaining amount of it in the treated drinking water, as it is used in the sterilization process.

The acceptable limits for discharged water after treatment are as follows [5][6]:

Turbidity: 15 NTU

Residual chlorine$CL_2)$: 3 mg/L

The reliability of the Rashidiya water center was determined using the Lognormal distribution, the Lognormal distribution can be used when the failure rate (the Exceedance Rate, in the case of water treatment plant) follows a monotonic trend. probability density function of The Lognormal distribution is: A positive random variable X is lognormally distributed(i,e) $X \sim Lognormal\ (\mu_x, \sigma^2{}_x)$, if the

natural Lognormal of X is normally distributed

with two parameters mean $\mu\mu$ and variance σ^2 [7] :

$$\mu = \ln\left(\frac{\mu_x^2}{\sqrt{\mu_x^2 + \sigma_x^2}}\right) \qquad \ldots (1)$$

$$\sigma^2 = \ln\left(1 + \frac{\sigma_x^2}{\mu_x^2}\right) \qquad \ldots (2)$$

x: turbidity or (chlorine) data.

μ: location parameter.

σ: scale parameter.

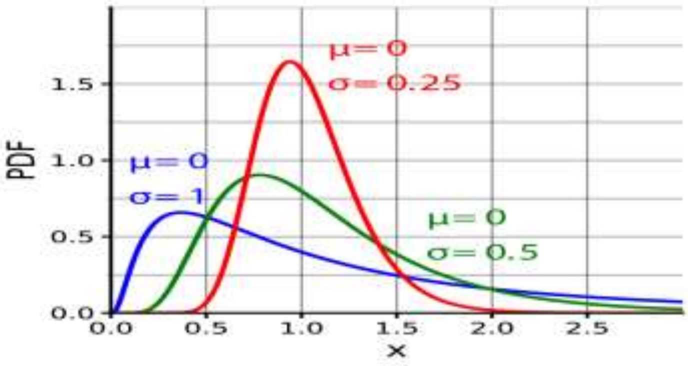

Figure 1: the probability density function of Lognormal distribution

The Cumulative distribution function is [8] ;

$$F_X(x) = \Phi\left(\frac{(lnx)-\mu}{\sigma}\right) \qquad ...(3)$$

Where Φ is the cumulative distribution function of N(0,1)

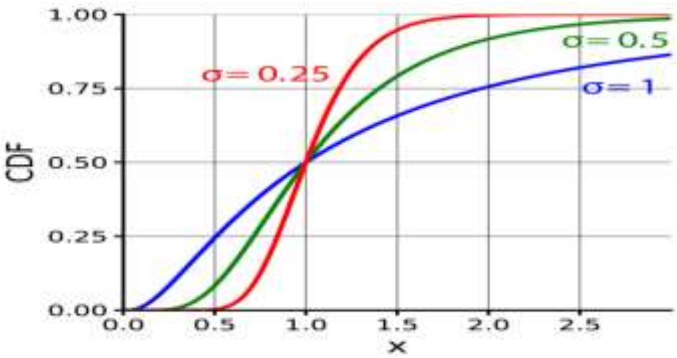

Figure 2: cumulative distribution function of the Lognormal distribution

The reliability function is given by [9]:

$$R(t) = 1 - \Phi\left(\frac{Logt - \mu}{\sigma}\right) \qquad ...(4)$$

We, however, want the Maximum Likelihood Estimator (MLE) μ and σ for the lognormal distribution, we can use the same procedure as for the normal distribution which is:

The log-likelihood function for a sample $\{x_1, \dots, x_n\}$ from a lognormal distribution is [7]:

$$LL = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(lnx_i - \mu)^2 - \sum_{i=1}^{n} lnx_i \qquad ...(5)$$

$$= -\frac{n}{2}\ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(lnx_i)^2 + \frac{\mu}{\sigma^2}\sum_{i=1}^{n} lnx_i - \frac{n\mu^2}{\sigma^2} - \sum_{i=1}^{n} lnx_i \qquad ...(6)$$

The log-likelihood function for a normal distribution is:

$$LL = -\frac{n}{2}\ln(2\pi) - \frac{n}{2}ln(\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(lnx_i - \mu)^2 \quad \dots (7)$$

Thus, the log-likelihood function for a sample $\{x_1, \dots, x_n\}$ from a lognormal distribution is equal to the log-likelihood function from $\{\ln x_1, \dots, \ln x_n\}$ minus the constant term $\sum \ln x_i$. Since the constant term doesn't affect which parameter values produce the maximum value of LL, we conclude that the maximum is achieved for the same values of μ and σ on the sample $\{\ln x_1, \dots, \ln x_n\}$ taken from a normal distribution, namely:

$$\mu = \frac{1}{n}\sum_{i=1}^{n} lnx_i \quad \dots (8)$$

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(lnx_i - \mu)^2 \quad \dots (9)$$

The MLE of reliability function is:

$$R(t)_{mle} = 1 - \Phi\left(\frac{Logt - \mu_{mle}}{\sigma_{mle}}\right) \quad \dots (10)$$

The mean, standard deviation and turbidity removal efficiency were calculated according to the following formula [1]:

$$eff = \left(1 - \frac{Turbidity_{out}}{Turbidity_{in}}\right) * 100 \quad \dots (11)$$

$Turbidity_{out}$: Turbidity after treatment.

$Turbidity_{in}$: Turbidity before treatment.

3. Results and discussion

Descriptive statistics were calculated, and it was found that the values of turbidity and chlorine were close to the acceptable standards according to the requirements [8] [8].

Table (1) Arithmetic Mean, St. deviation and CV for turbidity and residual chlorine.

Parameter	Unit	Mean	St. Deviation	CV
Turbidity	Unity	9.389	2.8520	0.3038
Chlorine	Mg/L	0.0296	0.0019	0.0625

To determine the reliability of turbidity and chlorine for Al-Rashidiya Water Center, using the log-normal distribution, and to ensure the goodness of fit, the Kolmogorov-Smirnov test was used at a significance level of α = 0.05.

Table (2) the results of estimating of the lognormal distribution,

Parameter	Unit	Parameter Distribution		Kolmogorov-Smirnov	
		μ	σ	Test Value	P-Value
Turbidity	Unity	-3.5219	0.0736	0.23152	0
Chlorine	Mg/L	2.2232	0.1491	0.53294	0

Table (2) shows the estimators μ and σ^2 of the lognormal distribution, which were estimated using the Maximum likelihood Estimator (MLE). and the results of the Kolmogorov-Smirnov test for the experimental data.

Figure (3) shows that the reliability of meeting the specified turbidity threshold (15 units) was 0.96, with an exceedance probability of (0.04). This means that 362 times over the 365-day study period, a value equal to or less than 15 units was observed.

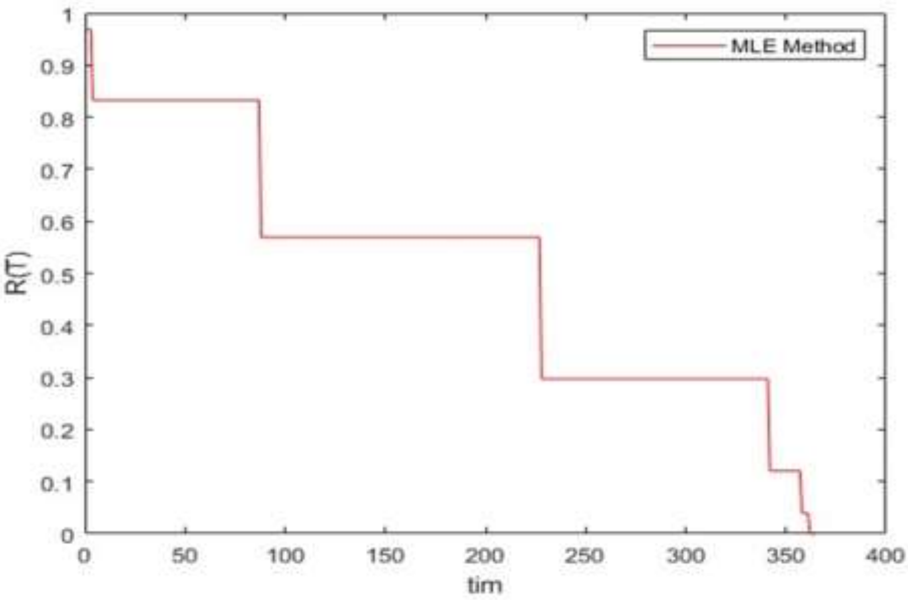

Figure 3: Turbidity reliability estimated using the Maximum likelihood Estimator (MLE) method.

Figure (3). depicts the dependability of estimations for turbidity levels using the Maximum Likelihood technique. The greatest reliability level was (0.96) at the smallest turbidity level, and the minimum reliability value was (0.04) at the maximum turbidity level, illustrating that reliability levels decrease as turbidity levels rise.

Graph of turbidity data in figure (4), we notice that the majority of the data falls within the permissible limit, and that some values fall outside the permissible limit, which is acceptable because the study period is relatively large, and the reason for this is due to rain and torrential rains that change the shape of the river water Tigris.

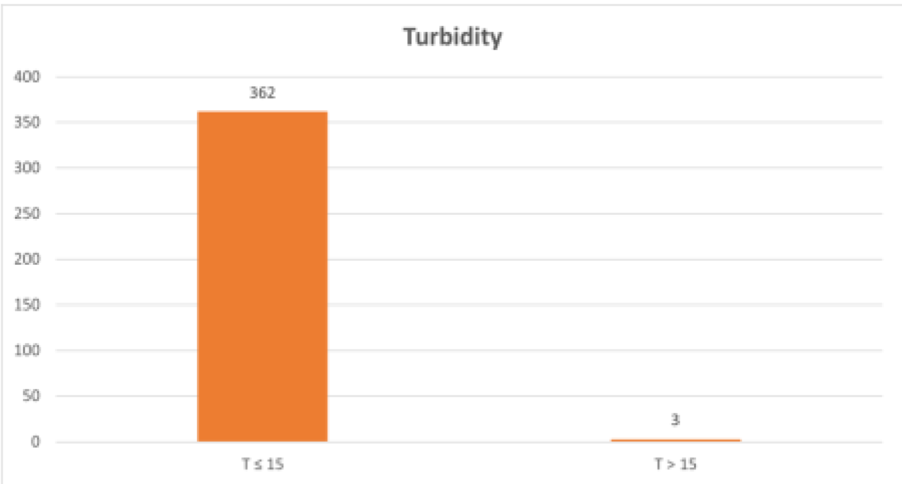

Figure (4) Graph of turbidity data.

Figure (4) shows a graph of the turbidity values and shows that (362) of the total values did not exceed the allowable treatment limit. While the permissible limit was exceeded (3) times only.

The reliability of lowering chlorine to the required standards (3 mg/L) was (100%), as indicated in Figure (5) with a chance of exceeding (0). This shows that no value of (3 mg/L) or above was reported over the 365-day research period.

Figure (5) depicts the Maximum Likelihood method reliability estimates; it's apparent from the figure that reliability amounts decrease as chlorine values increase, as the greatest reliability value was (1) at the smallest chlorine value and the smallest reliability value was (zero) at the greatest turbidity value.

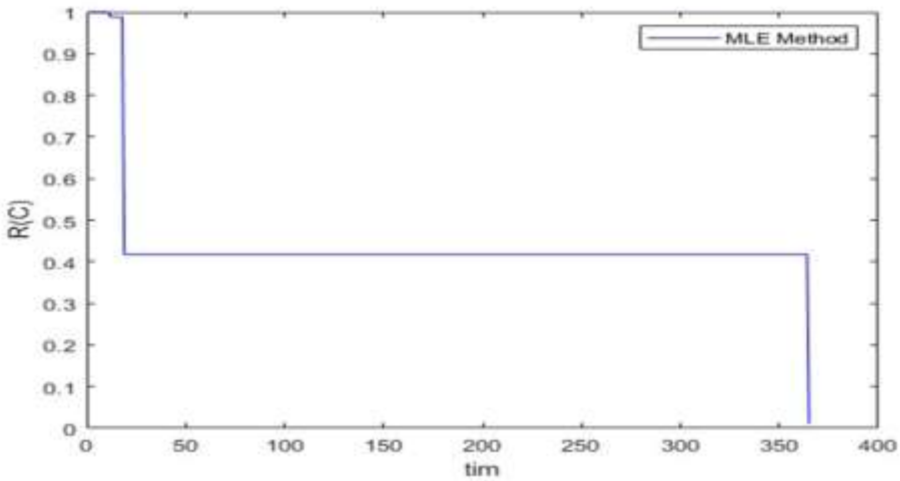

Figure 5: Chlorine reliability estimated using the Maximum likelihood Estimator (MLE) method.

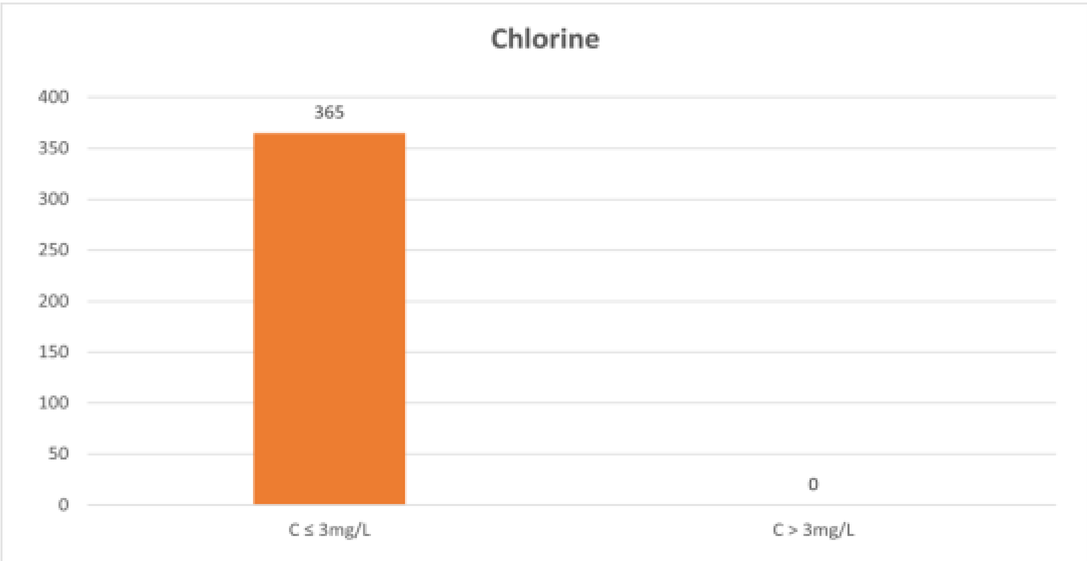

Figure (6) Chlorine data graph.

Because there were no cases of excess during the 365-day research interval, the chlorine measurements graph in Figure (6) shows that each value was less than the treatment-allowed limit.

4. Discussion

The study also discovered that the average turbidity removal effectiveness was 71.764%, which is adequate but has to be improved. The analysis revealed that the reliability value for turbidity removal was 100% and for remaining chlorine 0.96, suggesting that the Rashidiya water center was dependable during the research period. The report advises conducting frequent inspections of water treatment facilities and plants in Iraq and comparing failure distributions to determine the optimal distribution of all water impurities.

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