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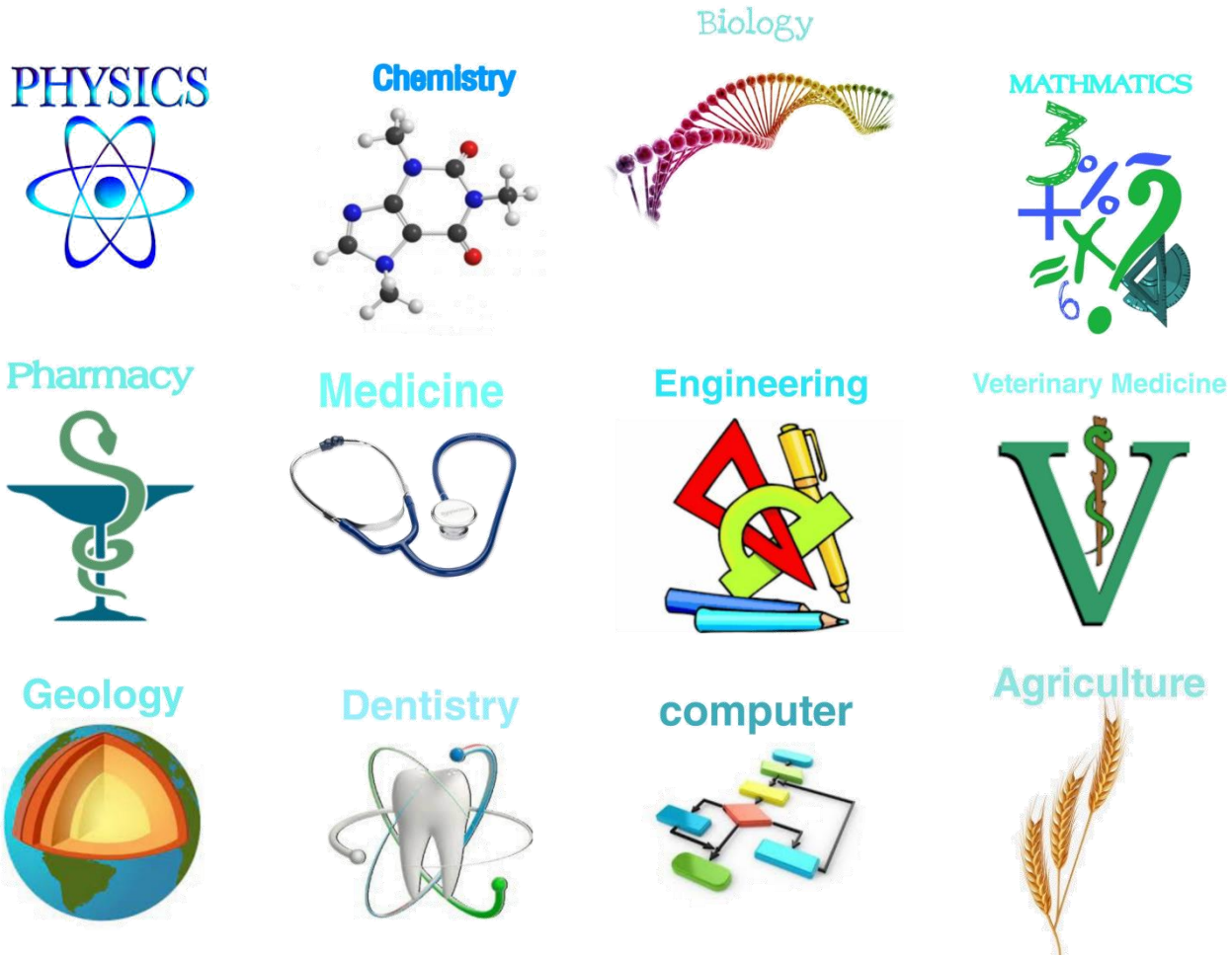

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Index

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Electron Microscope Study of Human Cerebral Cortex (Left Superior Temporal Gyrus), in different age groups and gender

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Abstract:

Human cerebral cortex is the outer folded neuronal layer and represents major part of the cerebrum with enormous functions. It is a laminar structure, easily visualized grossly. Previous studies showed that the Superior Temporal gyrus is one of the thickest cerebral cortex regions, reaching (about 4 mm). The Electron microscope study was made on 6 samples taken to measure the neuronal soma dimension of the large pyramidal cells present in the internal pyramidal cortical layer V in different age groups and gender. Aging process was obvious on the large pyramidal cells of the cerebral cortex, in which their neuronal soma dimensions showed shrinkage with age progression. But statistically there was no differences in the values between males and females.

Keywords: Cerebral Cortex; Electron Microscope; large Pyramidal cells; Neuronal Soma Dimension.

Background: The largest part of brain is the cerebrum and each cerebral hemisphere is covered by a highly folded layer of tissue called the Cerebral Cortex [(1)]. The Superior Temporal gyrus includes an area (within the Sylvian fissure) called the primary auditory cortex involved in hearing [(2)]. The functions of the left temporal lobe are not limited to low-level perception but extend to comprehension, naming, verbal memory and other language functions [(3)]. Histologically; there is a laminar pattern in the Cerebral cortex which is characterized to have six distinct layers numbered with Roman numerals from superficial to deep: Layer I is the molecular layer; layer II the external granular layer; layer III the external pyramidal layer; layer IV the internal granular layer; layer V the internal pyramidal layer; and layer VI the multiform, or fusiform layer. There are no actual borders between the layers, and the pyramidal cells are the main cell type and can be extremely large in layer V [(4, 5, 6)]. The study of the cerebral cortex under electron microscope is of great importance and provides enormous valuable histological and neurological informations at different serial powers that exceed the limit of light microscope, and most histological descriptions of the Cerebral cortex in the text books and literatures are referred to the whole brain [(7, 8)].

The aim of the study: is to measure the differences in neuronal soma dimension of the pyramidal cell in males and females of different age groups using Electron microscope in a specific Cerebral cortex region.

Methods: This study was carried out during the period from March 2011 to July 2012. The six human cadavers included in the present study were dissected at the Forensic medicine unit in Tikrit Teaching Hospital. The Cerebral cortex samples were taken from the area below the Sylvian fissure at the left Superior Temporal gyrus. They were two in each age group: young (< **35** years old), middle (**35-55** years old) and old (> **55** years old); one male and one female for each age group. Histological slides were prepared for examination under the Transmission Electron Microscope at the Medicine College of Al-Nahrain University in which, large pyramidal cell in the internal pyramidal layer (layer V) was chosen in each sample using different magnification powers. Neuronal soma dimensions (NSD) [(9)] were measured which is the height × width of the cell in each sample was included using computer based measurements.

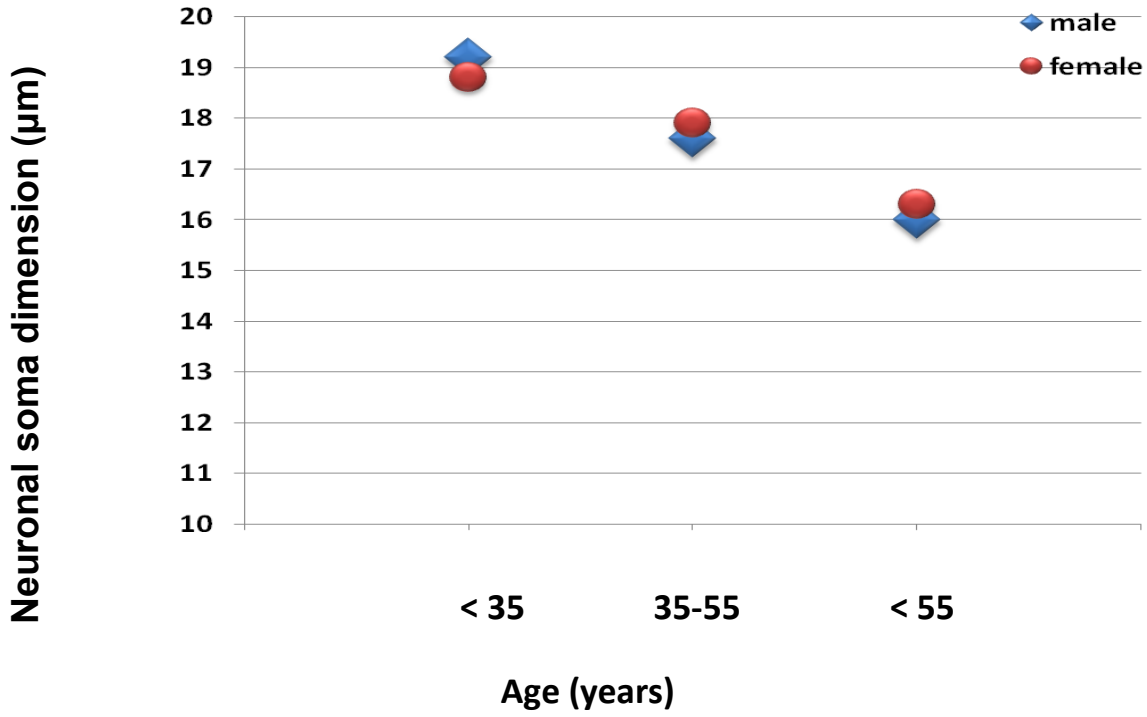
Results and Discussion: The large pyramidal cell with its nucleus in each sample was obviously seen and examined under several ascending powers of magnifications (Figure 1, 2, 3). It revealed that in the young age group (< **35** years old), the NSD of the males' sample **19.20** μm was more than the NSD of the females' **18.80** μm. While in group (**35-55** years old), the middle age group; NSD of the males' sample was **17.60** μm and was very close to the NSD of the females' sample **17.80** μm. In the old age group, group (> **55** years old), the males' sample had NSD of **16.00** μm; while the females' sample had NSD was **16.30** μm. In general, the NSD mean value of the males' pyramidal cells in the three age groups was **17.60** μm and was very close to the NSD mean value of the females' pyramidal cells in the three age groups **17.63** μm (Table 1).

Point value graphic presentation of the neuronal soma dimensions of the males' and the females' samples against the three age groups were plotted (Graph 1). It revealed that there is slight decrease in the NSD values with increasing age. In the young age group the males' NSD value was higher than the females'. While in the middle and old age groups the females' NSD values were higher than the males'. Although there was such difference, application of the Pearson's linear test revealed that there is no significant difference between the neuronal soma dimensions of the male and female samples in the three age groups, because the P value **0.979** was more than 0.05.

Table (1) Neuronal soma dimension values of pyramidal cells in layer V for each age group& gender.

Gender and Age groups		**No.***	**NSD** (μm)**	**Mean ± s.d (μm)**	
Male	**< 35**	1	19.20	**17.60**	1.60
	35-55	1	17.60		
	> 55	1	16.00		
Female	**< 35**	1	18.80	**17.63**	1.25
	35-55	1	17.80		
	> 55	1	16.30		
Total		6			

* No.: number of samples studied under electron microscope in each age group.
**NSD: Neuronal soma dimension.



Graph (1) Neuronal soma dimension values of cortical layer V pyramidal cells in different age groups and gender.

The dimensional values ranged from 16.00 µm in the old age group to 19.20 µm in the young age group agreed with the ranges obtained by Grazyana et al [10], who reported that the large pyramidal cell soma size is 16.2 µm to 20.8 µm, while the extralarge cell was 20.9 µm to 36.7 µm. Del & Marc [11] emphasized that, the largest pyramidal cells were found in cortical layers V, and their neuronal soma dimensions were about 15.7 µm ± 0.4. Also there was agreement with Togawa & Otsuka [12], who reported that, the pyramidal cell soma size in general was from 4 µm (0.004 mm) to 100 µm (0.1 mm). The mean values in the present study was 17.60 µm for the male samples and 17.63 µm for the females' for the three age groups, while the mean neuronal soma size was from 7.7 µm to 13.3 µm in layers I to VI done by

Smith et al [13].

Regarding the affect of age on the neuronal soma dimension of the large pyramidal cells, there was shrinkage in the somal size with age and this was totally agreed with the study of Rabinowicz et al [9] who said that, there was a decrease in neuronal soma size with the aging process. Henry [14] reported a little decrement change in neuronal size in twenty seven to thirty two years old age samples compared with young ones who were from six to nine years old and there was 50% loss in spines on the apical dendritic tufts of pyramidal cells. While Wang et al [15] mentioned that, although there is no overt loss of neurons during normal aging, but other, more subtle, changes occur in individual neurons. These include regression in soma size, loss or regression of dendrites and dendritic spines.

Regarding the gender there was no affect of this factor on the neuronal soma dimensions of the cortical layer V large pyramidal cells in this work and this was agreed with the study of Smiley et al [16] who achieved that males have similar neuronal soma dimension as females in the different layers of the cerebral cortex. Castejón et al [7] in a histo-morphometric study included select autopsy brains of six males and five females aged twelve to twenty four years old samples. In each brain, eighty six defined loci were analyzed for neuronal soma size the female group showed no difference of neuronal soma size than male. While Ehud & Jochen et al [17] disagreed with the present study results in that, laterality differences between the males and females were restricted to neuronal soma size of the cerebral cortex with significantly larger values in the female group in the left hemisphere. And he explained that, this gender difference may support female's right-handedness, language advantage, and tendency for bilateral activation patterns. However, the present study was also made in the left side of the brain and the female values of neuronal soma dimensions were also larger than the males' in the middle and old age groups, but in the young the males' value was higher. But statistically

there was no difference between the males' and females' neuronal soma dimensions of cortical layer V large pyramidal cells.

Conclusion: In the Electron Microscope study there is a slight shrinkage in the size of the pyramidal cell, but there is no difference between the male' and female' samples in their neuronal soma dimensions.

Competing interests :

None

FUNDING:

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Figures:

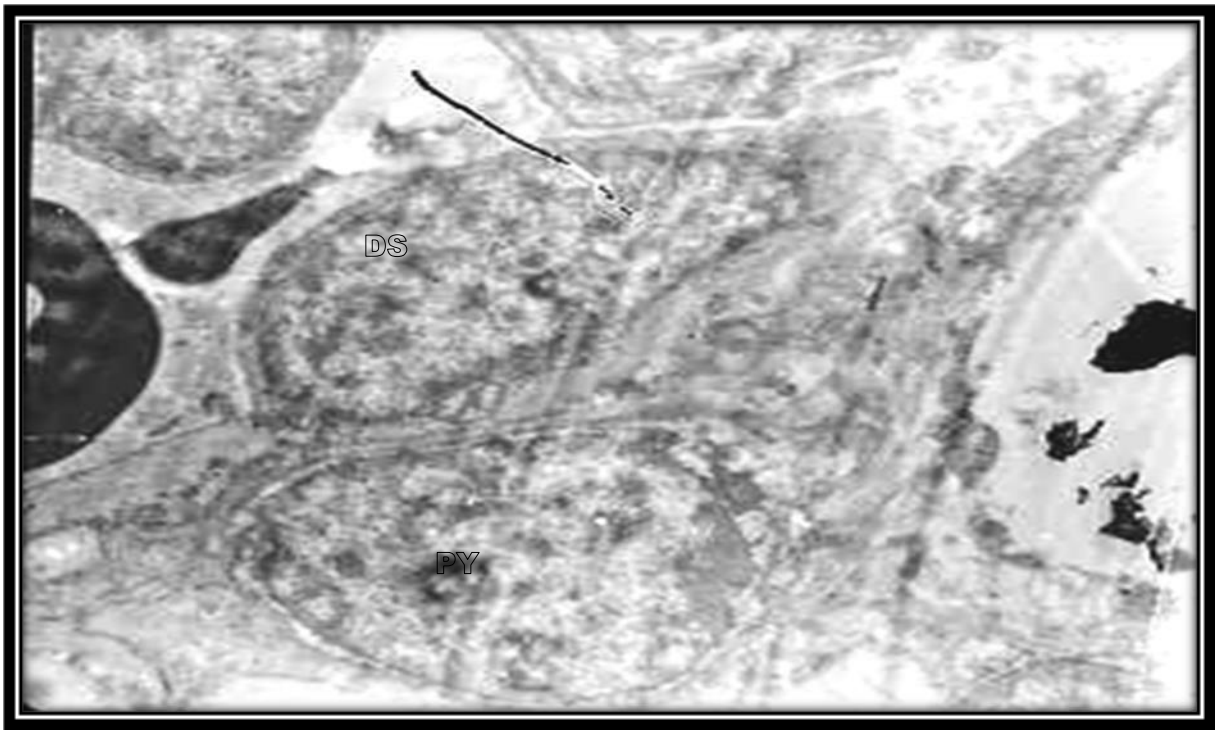

Figure (1) Electron microscope picture of cerebral cortex large pyramidal cell (PY) and a nearby dendritic section (DS). (X7900)

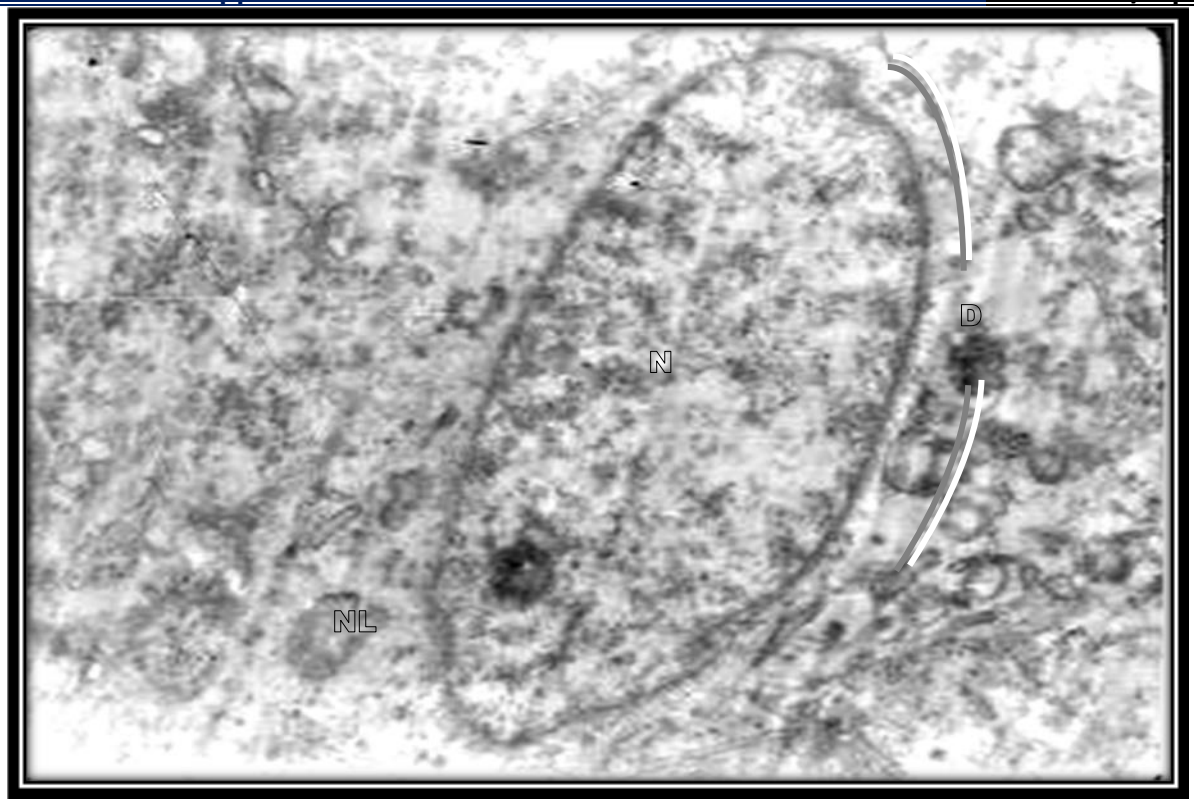

Figure (2) Electron microscope image of large pyramidal cell with nucleus(N), nucleolus(NL) and dendrite(D). (X10500

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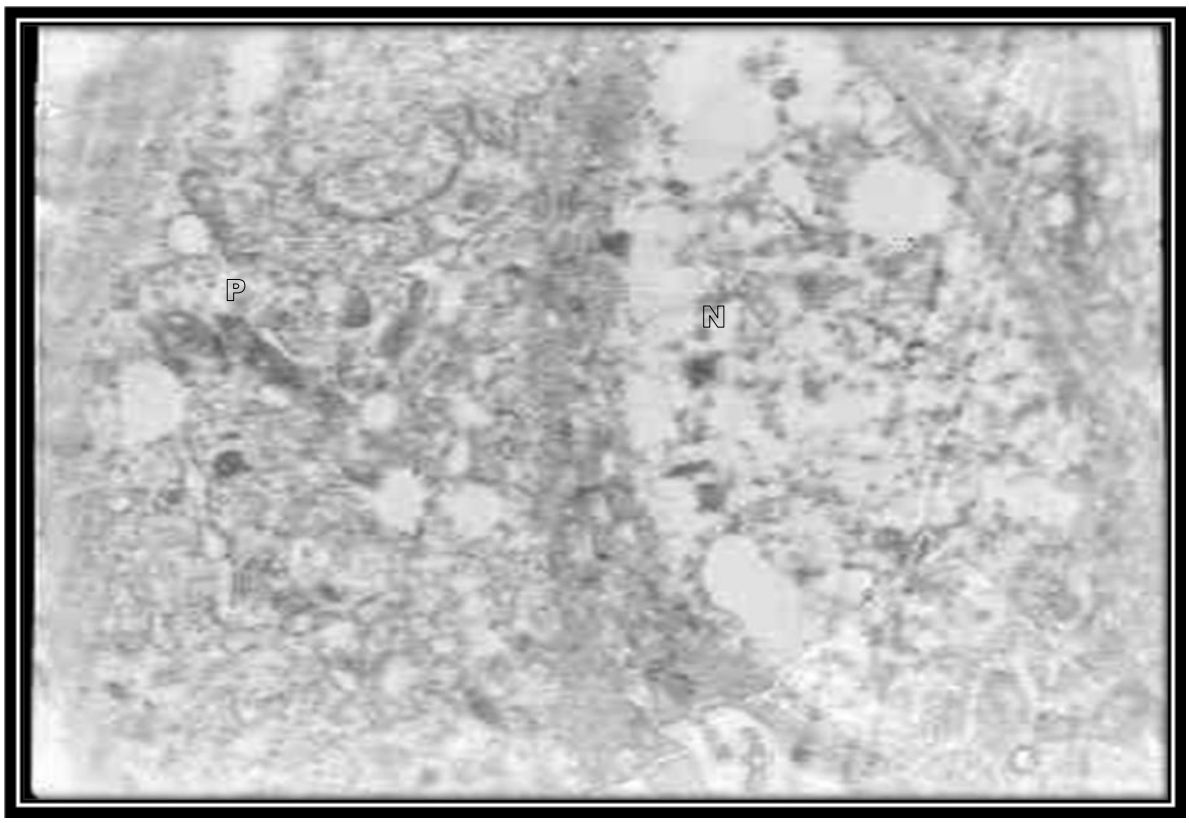

Figure (3) Electron microscope image of perikaryon (P) and nucleus (N) of a large pyramidal cell. (X10500)

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التقدير الطيفي لعقار هيدروكلوريد الكلوربرومازين بالأقتران التأكسدي مع كاشف البنزدين

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Spectrophotometric determination of chlorpromazine hydrochloride by oxidative coupling with benzdine reagent

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Abstract

A sensitive and accurate spectrophotometric method was developed for the determination of chlorpromazine hydrochloride (CPZH) by the oxidative coupling reaction. The method depends on the oxidation of the benzodine reagent using the oxidizing agent potassium periodate in an acidic medium and the addition of a chlorpromazine hydrochloride solution to it. 60 minutes, which is long enough to make many measurements. It gave the highest absorption at the wavelength of 426 nm and followed Beer's law in the range of concentrations 1.25-25 µg/ml. The molar absorptivity was 7177 L. $Mol^{-1}.cm^{-1}$ and sandal index 0.049 µg. Cm^{-2} The method was accurate and compatible, as the recovery 103.024%, the method was successfully applied to chlorpromazine hydrochloride in the pharmaceutical preparation.

الملخص

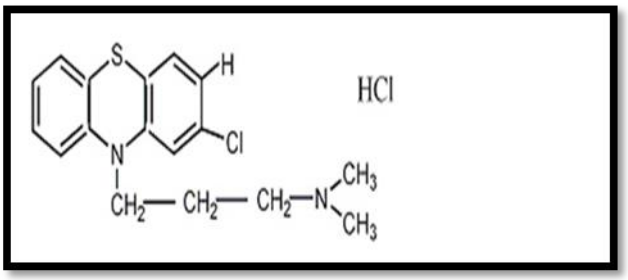
طورت طريقة طيفية حساسة ودقيقة لتقدير هيدروكلوريد الكلوربرومازين (CPZH) بتفاعل الاقتران التأكسدي، تعتمد الطريقة على أكسدة كاشف البنزدين باستخدام العامل المؤكسد بيريودات البوتاسيوم في وسط حامضي وإضافة محلول هيدروكلوريد الكلوربرومازين إليه، بعد إكمال الإضافات يتكون الناتج النهائي للتفاعل بني اللون مباشرة وأثبت استقراريتة أكثر من 60 دقيقة وهي مدة كافية لإجراء العديد من القياسات . وأعطى أعلى امتصاص عند الطول الموجي426 نانومتر ويتبع قانون بير في مدى التراكيز 1.25 –25

مكغم/ مل . وكانت قيمة الامتصاصية المولارية 7177 لتر. مول$^{-1}$.سم$^{-1}$ ودلالة ساندل 0.049 مكغم . سم$^{-2}$ وكانت الطريقة على درجة من الدقة والتوافقية فقد كانت قيمة الاسترجاعية 103.024% وقد طبقت الطريقة بنجاح على هيدروكلوريد الكلوربرومازين في المستحضر الصيدلاني .

الكلمات الدالة :هيدروكلوريد الكلوربرومازين , بنزدين , تقدير طيفي

المقدمة

هيدروكلوريد الكلوربرومازين (CPZH) يعد من أهم المشتقات الفينوثيازينة التي تمتلك جهود تأين منخفضة حيث تعمل كواهبات للإلكترونات[1], ويكون بصيغة جزيئية هي $C_{17}H_{19}ClN_2S.HCl$ والوزن الجزيئي 355.33 g/mol وصيغتهُ التركيبية هي كالأتي [2].



الصيغة التركيبية لهيدروكلوريد الكلوربرومازين (CPZH)

هو مسحوق أبيض اللون عديم الرائحة قابل للذوبان في الماء وكذلك سهل الذوبان في الكلوروفورم والكحول ولكنهُ يذوب في الأيثر جزيئياً وتكون درجة أنصهارهُ 196 درجة مئوية ويحفظ في أوعية `مغلقة وبعيدة عن الضوء لأن عند تعرضهُ للضوء والهواء يتفكك وقد أعطى اعلى امتصاص بين `الطول الموجي 239– 306 نانومتر[3] ومن الأسماء العلمية هي

2-Chloro-10 (dimethyl amino propyl) phenothiazine hydrochloride

أما الأسماء التجارية حسب الشركات المصنعة لها

Largactil, Taroctyl, Marazine, Chloractil, Hepanil and Thorazine .

حيث توجد المستحضرات الصيدلانية ل(CPZH) على هيئة حقن وأقراص [4].وقد تم تقدير هيدروكلوريد الكلوربرومازين بتقنيات تحليلية مختلفة منها الطرائق الطيفية [5 - 7] وطرائق البريق الكيميائي [8] وطرائق الحقن الجرياني [9] والطرائق RP– HPLC [10].

الجزء العملي

الأجهزة المستخدمة

1- جهاز المطياف مزدوج الحزمة نوع UV-160 UV-Visible Spectrophotometer T90+ خلايا كوارتز بعرض مقداره 1سم.

2- جهاز حمام مائي.

3- ميزان حساس نوع ل Kern 770GS/G

4 – مسخن حراري(Harry Gestigkeit GmbH)

محاليل المواد المستخدمة

1 – محلول هيدروكلوريد الكلوربرومازين1000 مكغم/مل

حُضِّرَ بإذابة 0.1000 غرام من مسحوق هيدروكلوريد الكلوربرومازين في كمية من الماء المقطر ثم أكمل الحجم إلى حدّ العلامة في قنينة حجمية سعة 100 مل .

2- محاليل الكواشف:.

أ- محلول البنزدين $10^{-3} \times 1$ مولاري حُضِّرَ بإذابة 0.0092غرام من مسحوق حامض البنزدين في الايثانول ثم أكمل الحجم إلى حَدّ العلامة بالماء المقطر في قنينة حجمية سعةً 50 مل ويحفظ في قنينة معتمة .

ب- محلول بيريودات البوتاسيوم $10^{-2} \times 1$ مولاري حُضِّرَ بإذابة 0.0575 غرام من مسحوق `wa4raبيريودات البوتاسيوم في كمية معينة من الماء المقطر ثمّ اكمل الحجم الى حدّ العلامة في قنينة حجمية سعتهُ 25 مل.

ج ـ محلول حامض الهيدروكلوريك بتركيز تقريبي 1 مولاري حُضِّرَ باضافة 8.4 مل من محلول حامض الهيدروكلوريك 11.8 مولاري إلى كمية من الماء المقطر في قنينة حجمية سعة 100 مل وأكمل الحجم إلى حَدّ العلامة بالماء المقطر.

3- محلول المستحضر الدوائي

حُضِّرَ محلول المستحضر الصيدلاني (largeactil) بتركيز 250 مكغم/ مل بطحن 25 حبة من المستحضر الصيدلاني للحصول على وزن 25 ملغم ثم اذابتها في كمية معينة من الماء المقطر وترشيح المحلول وبعد ذلك يوضع الراشح في قنينة حجمية سعة 100 مل وأكمل الحجم إلى حَدِّ العلامة بالماء المقطر.

النتائج والمناقشة :.

الدراسة التمهيدية:

لوحظ انه عند إضافة 1 مل من محلول الكاشف بنزدين الى 1مل من محلول العامل المؤكسد بيريودات البوتاسيوم بوجود 0.6 مل من محلول حامض الهيدروكلوريك ثم إضافه 1 مل من محلول هيدروكلوريد الكلوربرومازين ذي التركيز 250 مكغم / مل يتكون ناتج بني اللون . وتَم قياس طيف الامتصاص للناتج الملون بعد تخفيفه بالماء المقطر في قنينة حجمية سعة 20 مل إلى حَد العلامة مقابل المحلول الصور**ي** وجد أنه يعطي أعلى امتصاص عند الطول الموجي 426 نانومتر في حين أنَّ المحلول الصوري لم يعطِي أي امتصاص في هذه المنطقة .

دراسة الظروف المثلى

بعد الحصول على طيف الامتصاص لمحلول التفاعل تم دراسة ظروف التفاعل التي تعطي افضل امتصاصية عند الطول الموجي المقترح

– اختيار العامل مؤكسد

تمَ أُخِذَ 1 مل من الكاشف البنزدين ذي التركيز 1×10^{-3} مولاري الى 1مل من كل من العوامل المؤكسدة المستخدمة ذي التركيز 1×10^{-2} مولاري بأستخدام 0.6 مل من حامض الهيدروكلوريك ذي التركيز 1 مولاري بأضافة 1 مل من محلول CPZH ذي التركيز 250 مكغم / مل والنتائج مدونة في الجدول 1 أدناه.

جدول (1) اختيار العامل المؤكسد

Oxidizing agent 1×10^{-2}M	Absorbance

Potassium chromate	0.308
Potassium Iodate	0.229
N-bromo succinimide	0.210
Potassium periodate	0.521
Sodium oxalate	0.267

تأثير كمية العامل المؤكسد

تَمَّتْ دراسة كمية تأثير العامل المؤكسد بأضافة 0.6 مل من محلول الكاشف بنزدين ذي التركيز 1×10^{-3}مولاري الى محلول العامل المؤكسد بيريودات البوتاسيوم ذي التركيز 1×10^{-2} مولاري من خلال إضافة أحجام متزايدة من 0.2 – 1.4 مل بوجود 0.6 مل من حامض هيدروكلوريك تركيزه مولاري منه إلى حجم1 مل من محلول هيدروكلوريد الكلوربرومازين ذي التركيز 250 مكغم/ مل الى قناني حجمية سعة 20 مل وإكمال الحجم إلى حَدِّ العلامة بالماء المقطر والنتائج مدونة في الجدول 2 أدناه.

جدول (2) تأثير كمية العامل المؤكسد

ml of 1×10^{-2} M KIO_4	0.2	0.4	0.6	0.8	1	1.2	1.4
Absorbance	0.309	0.370	0.415	0.491	0.541	0.451	0.314

لُوحِظَ من خلال النتائج المبينة في الجدول أعلاه أنَّ أفضل حجم للعامل المؤكسد يعطي أعلى امتصاص للناتج الملون هو 1 مل وتَمَّ اعتمادهِ في التجارب اللاحقة .

أختيار أفضل حامض

تَمَّتْ هذه الدراسة باستخدام 0.6 مل من الحوامض القوية والضعيفة ذي التركيز 1 مولاري واختيار الحامض الأفضل الذي يعطي أعلى قيمة للامتصاص والنتائج مدونة في جدول أدناه رقم (3) يوضح ذلك

جدول رقم (3) تأثير الحامض

1ml of acid 1M	Absorbance
HNO3	0.361
H_2 SO_4	0.162
HCI	0.540
CH_3COOH	0.191

لوحظ من الجدول أعلاه ان الحامض HCI أعطى أعلى قيمة للامتصاص الناتج ا لملون لذا تمّ استخدامهُ في التجارب اللاحقة.

تأثير الحامض

تمت هذه الدراسة باستخدام أحجام متزايدة تتروح من 0.1 – 1.2 مل من حامض الهيدروكلوريك المخفف ذي التركيز 1مولاري واختيار أفضل حجم الذي يعطي أعلى قيمة للامتصاص والنتائج مدونة في الجدول أدناه.

جدول رقم (4) تأثير الحامض

Ml of Acid	0.1	0.2	0.4	0.6	0.8	1	1.2
Absorbance	0.437	0.441	0.476	0.525	0.542	0.386	0.349

يلاحظ من الجدول أعلاه ان أفضل حجم لحامض الهيدروكلوريك هو0.8 مل لذا يتم استخدامه في التجارب اللاحقة .

تأثير كمية كاشف الاقتران

تَمَّتْ دراسة تأثير كمية كاشف الاقتران بأخذ أحجام مختلفة من محلول الكاشف البنزدين ذي التركيز 1×10^{-3} مولاري تتراوح بين 0.2 – 1.4 مل مع 1مل من محلول هيدروكلوريد الكلوربرومازين ذي التركيز

250 مكغم/ مل بوجود 1 مل من محلول بيريودات البوتاسيوم ذي التركيز 1×10^{-2} مولاري و0.8 مل من محلول حامض الهيدروكلوريك في قنينة حجمية سعةٌ 20 مل، والنتائج مدونة في الجدول4 أدناه .

جدول(4) تأثير كمية كاشف الأقتران

Ml of Reagent 1×10^{-3}	0.2	0.4	0.6	0.8	1	1.2	1.4
Absorbance	0.321	0.452	0.541	0.493	0.531	0.337	0.212

لُوحِظَ من النتائج المبينة في الجدول أعلاه أنّ حجم 0.6 مل من محلول كاشف البنزدين ذي التركيز 1×10^{-3} مولاري أعطى أعلى قيمة لذا تَمَّ اعتماده في التجارب اللاحقة .

تأثير زمن الأكسدة

وصلت شدة اللون إلى الحد الأقصى بعد تفاعل الكاشف بنزدين مع الدواء بوجود العامل المؤكسد بيريودات البوتاسيوم لمدة 5 دقائق , لذلك كانت 5 دقائق كافية لاستكمال الأكسدة , لذلك تم اعتمادها في التجارب اللاحقة

جدول (5) تأثير زمن الأكسدة

Time minutes	5	10	15	20	25	30
Absorbance	0.535	0.521	0.513	0.488	0.473	0.461

تسلسل الإضافات

تَمَّتْ دراسة تأثير تغيير تسلسل إضافة مواد التفاعل على امتصاص الناتج الملون باجراء عدد من التجارب، وقد وُجِدَ أنَّ تسلسل الإضافة (**III**) يحقق أعلى امتصاص للناتج الملون لذا تَمَّ اعتماده في التجارب اللاحقة والنتائج مدونة في الجدول 5 أدناه.

جدول (5) تسلسل الإضافات

Order Number	**Order of addition**	**Absorbance**
I	D + O + A + R	0.350

II	O + D + A + R	0.460
III	R + O + A + D	0.544
IV	D + R + O + A	0.420

إذ إنّ (R) محلول الكاشف بنزدين و(O) العامل المؤكسد محلول بيريودات البوتاسيوم و(A) محلول حامض الهيدروكلوريك. (D) محلول هيدروكلوريد الكلوربرومازين.

تأثير درجة الحرارة

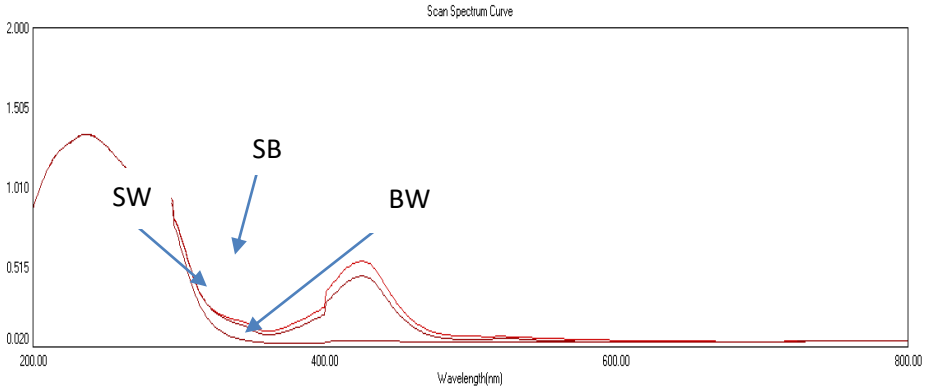
تَمَّتْ دراسة تأثير درجة الحرارة على امتصاص الناتج الملون المتكون واستقراريته باستخدام درجات حرارية 5 – 50 درجة مئوية وتم أعتماد 25 درجة مئوية والنتائج مدونة في الجدول 6 أدناه.

جدول (6) تأثير درجة الحرارة

Temp C°	5	10	15	20	25	30	35	40	45	50
Absorbanc	0.461	0.467	0.485	0.501	0.543	0.538	0.508	0.491	0.483	0.470

طيف الامتصاص النهائي

تَمَّ قياس طيف الامتصاص النهائي بعد الوصول إلى الظروف المثلى وهي استخدام 1 مل من محلول هيدروكلوريد الكلوربرومازين ذي تركيز 250 مكغم/ مل و0.6 مل من الكاشف بنزدين ذي التركيز $3^{-}10^{*}1$ مولاري و1 مل من العامل المؤكسد بيريودات البوتاسيوم ذي التركيز $2^{-}10^{*}1$ مولاري و0.8 مل من حامض الهيدروكلوريك ذي التركيز 1 مولاري ودرجة الحرارة 25 – 30 درجة مئوية وزمن 5 دقائق بعد إتمام الإضافات وفق الظروف المثلى للتفاعل وإكمال الحجم إلى حَدِّ العلامة في قنينة حجمية سعة 20 مل بالماء المقطر، تَمَّ قياس امتصاص الناتج البني اللون مقابل محلوله الصوري الذي يعطي أعلى امتصاص عند الطول الموجي 426 نانومتر – شكل رقم (1)



شكل (1) طيف الامتصاص النهائي لمحلول النموذج

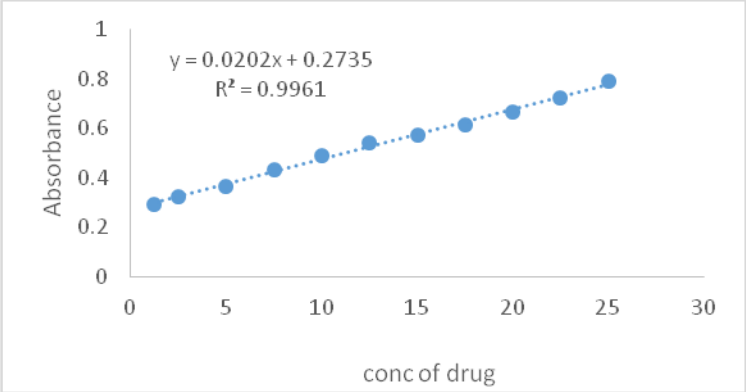
SW يُمَثلُ طيف امتصاص محلول هيدروكلوريد الكلوربرومازين مقابل الماء المقطر.

SB يمثل طيف امتصاص محلول هيدروكلوريد الكلوربرومازين مقابل المحلول الصوري

BW يمثل امتصاص المحلول الصوري مقابل الماء المقطر.

طريقة العمل المعتمدة ومنحني المعايرة

إلى سلسلة قناني حجمية سعة 20 مل تحتوي على تراكيز متزايدة من 1.25 – 25 مكغم/ مل من محلول هيدروكلوريد الكلوربرومازين باضافة 0.1 – 2 مل بتركيز 250 مكغم/ مل أُضيفَ 0.6 مل من محلول الكاشف بنزدين ذي التركيز 1×10^{-3}مولاري و1 مل من محلول المادة المؤكسدة بيريودات البوتاسيوم ذي التركيز 1×10^{-2} مولاري و 0.8 مل من حامض الهيدروكلوريك ذي التركيز 1 مولاري وتم التخفيف الى العلامة بالماء المقطر وتم الانتظار 5 دقائق عند درجة حرارة 25 درجة مئوية . تَمّ رسم الطيف وقياس امتصاص المحاليل مقابل المحلول الصوري عند الطول الموجي 426 نانومتر، والشكل رقم (2)، يمثل منحني المعايرة الذي يتبع قانون بير في حدود التراكيز 1.25 –25 مكغم/ مل من محلول هيدروكلوريد الكلوربرومازين ويحصل انحراف عن قانون بير عند تركيز اكثر من 25 مكغم / مل ، وأعطى معامل ارتباط مقداره 0.9961 وتَمَّ حساب قيمة الامتصاصية المولارية وبلغت قيمتها 717766 لتر / مول$^{-1}$.سم$^{-1}$ وقيمة دلالة ساندل 0.049 مكغم . سم^{2}



شكل (2) منحني المعايرة لتقدير هيدروكلوريد الكلوربرومازين بطريقة

الاقتران التأكسدي مع كاشف البنزدين بوجود العامل المؤكسد بيريودات البوتاسيوم

الدقة والتوافقية

تَمَّ استخدام الظروف المثلى في طريقة العمل لاختبار دقة منحني المعايرة وتوافقيته، فقد تَمَّ أخذ خمس قراءات لثلاث كميات مختلفة من محلول هيدروكلوريد الكلوربرومازين ضمن حدود قانون بير في منحني المعايرة ومن حساب معدل الاسترجاعية والانحراف القياسي النسبي (RSD)وجد أنَّ الطريقة ذات دقة عالية معدل الاسترجاعية 102.99% وذات توافقية عالية.

جدول (7) الدقة والتوافقية

Conc.of CPZH µg ml	Conc.of CPZH µg/ml (observed)	Recovery,*%	Average of Recovery%	RSD*,%
7.5	7.797	103.96	103.024	0.371
12.5	13.24	105.94		0.221
15	14.87	99.174		0.150

حساب حَدّي الكشف النوعي والكمي

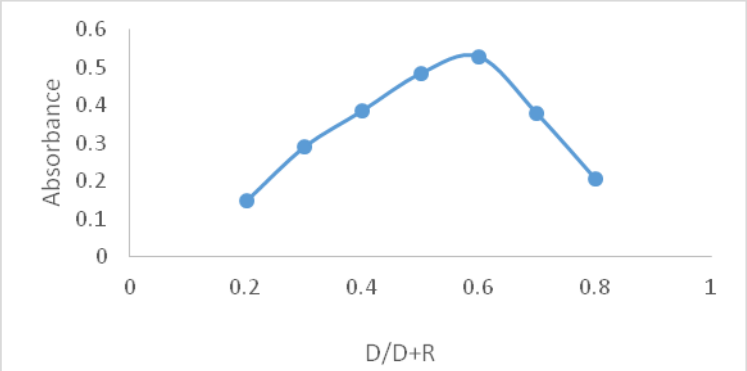
تَمَّ حساب حَدِّ الكشف لتقدير هيدروكلوريد الكلوربرومازين عند الطول الموجي 426 نانومتر من خلال قياس محلول بلانك مقابل الماء بمقدار 6 قراءات حيث كانت قيمتي حد الكشف النوعي والكمي كما يظهر في الجدول 8.

جدول (8) حد الكشف

الحد الكمي LOQ	الحد النوعي LOD	الانحراف القياسي SD	الميل Slope
0.6930	0.2287	0.0014	0.0202

طبيعة الناتج المتكون

لمعرفة طبيعة المحلول البني اللون المتكون ونسبة ارتباط العقار مع الكاشف طبقت طريقتان التغيرات المستمرة (طريقة جوب) وطريقة النسبة المولية . في كلا الطريقتين يكون تركيز كل من محلول هيدروكلوريد الكلوربرومازين ومحلول الكاشف بنزدين في نفس التركيز وهو 1×10^{-3} مولاري في طريقة جوب في سلسلة من قنان حجمية سعة 20 مل تم وضع أحجام مختلفة من محلول CPZH تتراوح بين 1 – 4 مل وأُضيفت مكملات هذه الأحجام إلى حجم 4 – 1 مل من محلول الكاشف ثم أُضِيفَ 1 مل من محلول بيريودات البوتاسيوم ذي التركيز 1×10^{-3} مولاري و0.8 مل من محلول حامض الهيدروكلوريك تركيزه 1 مولاري وتَمَّ التخفيف إلى حَدِّ العلامة بالماء المقطر وتَمَّ قياس امتصاص هذه المحاليل عند الطول الموجي 426 نانومتر والشكل رقم (3) يبين أنَّ النسبة هي1: 1

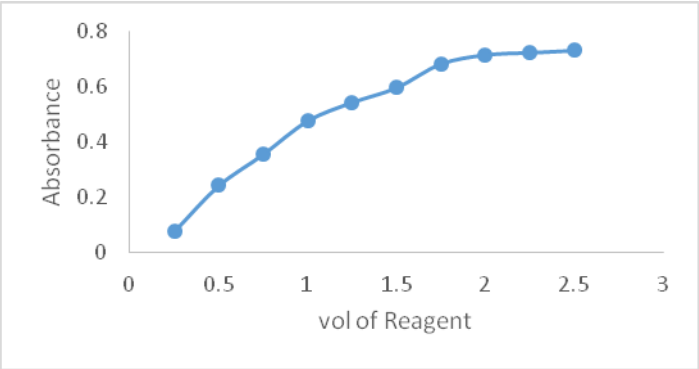


شكل (3) طريقة جوب لتقدير هيدروكلوريد الكلوربرومازين

مع الكاشف بنزدين بوجود العامل المؤكسد بيريودات البوتاسيوم

أمَّا في طريقة النسبة المولية تَمَّ وضع 2 مل من العقار في سلسلة قناني حجمية سعة 20 مل وأُضيفَ اليها الكاشف بأحجام تتراوح بين 5 – 0.5 مل ثم إضافة 1 مل من محلول بيريودات البوتاسيوم ذي التركيز 1×10^{-2} مولاري و 0.8 مل حامض الهيدروكلوريك ذي التركيز1 مولاري وأكمل الحجم إلى

حَدِّ العلامة بالماء المقطر ثم قيسَ الامتصاص لهذه المحاليل عند الطول الموجي 426 نانومتر مقابل المحلول الصوري لِكُلٍ منها ، وجدَ أنّ النسبة المولية تتفق مع طريقة التغيرات المستمرة وتحقق النسبة 1:1.



الشكل (4) طريقة النسبة المولية لتقدير هيدروكلوريد الكلوربرومازين مع الكاشف بنزدين بوجود بيريودات البوتاسيوم

التطبيقات

الطريقة المباشرة

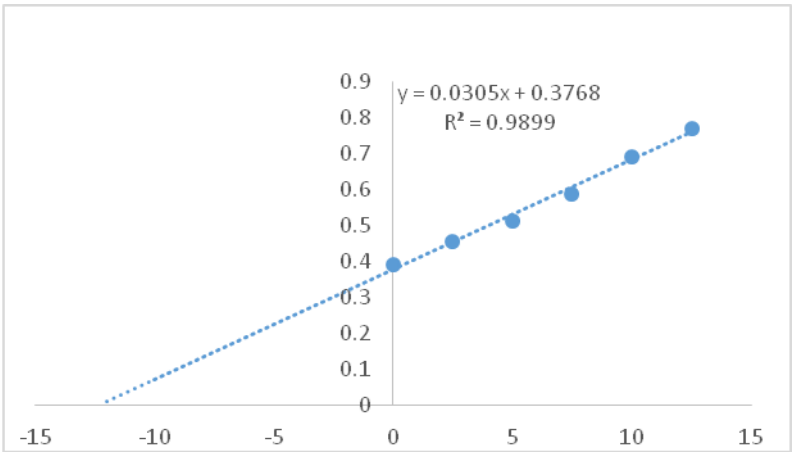
أخذت ثلاثة تراكيز مختلفة من محلول المستحضر largeactil 100mg مبين تحضيرها هي 12.5, 10 , 5 مكغم / مل وعوملت المحاليل بنفس الخطوات المتبعة عند تحضير منحني المعايرة وتَمّ قياس الامتصاص لها عند الطول الموجي 426 نانومتر مقابل المحلول الصوري وتَمَّ حساب معدل خمسة قياسات لِكُلِ تركيز وتَمّ حساب الاسترجاعية والنتائج مدونة في الجدول أدناه.

جدول (9) الطريقة المباشرة

Conc of CPZH , µg/ml	Recovery , %	Average recovery , %
5	95.54	100.44
10	102.22	
12.5	103.56	

طريقة الإضافات القياسية

أُجْرِيَتْ هذه الطريقة لغرض بيان كفاءة الطريقة المقترحة ودقتها واثبات ان الطريقة خالية من التداخلات طبقت طريقة الاضافة القياسية في تقدير هيدروكلوريد الكلوربرومازين في المستحضر الصيدلاني largeactil . تضمنت الطريقة اضافة حجم ثابت 1 مللتر من محلول المستحضر الصيدلاني المحضر بتركيز 250 مكغم/ مل في سلسلة من قناني حجمية سعة 20 مللتر ، ثم اضافة حجوم متزايدة 0 ،0.2 ، 0.4 ، 0.6 ،0.8 ، 1 مللتر من المحلول هيدروكلوريد الكلوربرومازين القياسي ذي تركيز 250 مكغم / مل، وتم معاملة المحاليل اعلاه بنفس طريقة العمل المستخدمة عند تحضير منحنى المعايرة .



شكل (5) منحني الإضافة القياسية لتقديرهيدروكلوريد الكلوربرومازين في القرص الدوائي (100 mg)

جدول (10) الإضافات القياسية

Type of Drug	Chlorpromazine present µg/ml	Chlorpromazine measured µg/ml	Recovery, (%)
Tablets Iargeactil Chlorpromazine 001 mg S.D.I Ira	12.5	12.8	102.60

الأستنتاج

تَمَّ تطوير طريقة طيفية حساسة ودقيقة لتقدير هيدروكلوريد الكلوربرومازين بتفاعل الاقتران التاكسدي، تعتمد الطريقة على أكسدة الكاشف باستخدام بيريودات البوتاسيوم في وسط حامضي وإضافة محلول عقار هيدروكلوريد الكلوربرومازين إليه، بعد إكمال الإضافات يتكون الناتج النهائي للتفاعل بني اللون مباشرة وأثبت استقراريتة أكثر من 60 دقيقة وهي مدة كافية لإجراء العديد من القياسات. وأعطى أعلى امتصاص عند الطول الموجي426 نانومتر ويتبع قانون بير في مدى التراكيز 1.25 –25 مكغم / مل. وكانت قيمة الامتصاصية المولارية 7177 لتر / مول$^{-1}$.سم$^{-1}$ ودلالة ساندل 0.049 مكغم . سم$^{-2}$ وكانت الطريقة على درجة من الدقة والتوافقية فقد كانت قيمة الاسترجاعية 103.024% وقيمة الانحراف القياسي النسبي ليس اكثر من 1.25 % وقد طبقت الطريقة بنجاح على هيدروكلوريد الكلوربرومازين في المستحضر الصيدلاني(largeactil)

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Best Simultaneous Approximation of bounded functions via linear Operators in $\mathcal{N}_2$

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Abstract

The purpose of suggested in this work presented concept proximinal best simultaneous approximation, proven the continuous of operator best simultaneous approximation in two-normed spaces and we find best simultaneous approximation of bounded function by using linear operators in two- normed space.

Keywords : Two-normed spaces , simultaneous proximinal

and continuous simultaneously operator .

1. INTRODUCTION

The notes of better approxima. has been principal presented via I. Singer(1974) [1], in Boszny discussed set of remark on simultaneous approximation (1978) [2] , Li and Watson proved a set of results about best simultaneous approximation (1996) & (1997) respectively [3,4] ,then Boyd and Vandenberghe explained characteristic of convex set (2004)[9] , and Mohebi achieve some properties of best simultaneous approximation (2005) [5] . Abu-sirhan presented a set of researches that include the best simultaneous approximation in $L^{\infty}(I,X)$, operator and functional spaces (2009) & (2012) respectively [6,7,8] , in the end ,I benefited greatly from the papers of the tow scholars Elumalai and makandeya , on best simultaneous approximation (2009)&(2013) [10,11] .

in this present working , we studied some marks of better simultane. approxima.of bounded mappings by using linear operators in two-normed space . This results which proven in 2-normed space. our main results are continuous and we find best simultaneous approximation set in two-normed space .

2. PRELIMINARIES

Definition 2.1 : $\mathcal{N}_2 : \mathcal{H} \times \mathcal{H} \to \mathcal{R}$ be real mapping which gratify the following properties

1) For $v_0, v_1 \in \mathcal{H}$, $\|v_0, v_1\|_{b,2} = 0$ if and only if v_0, v_1 are linearly dependent .

2) $\|v_0, v_1\|_{b,2} = \|v_1, v_0\|_{b,2}$,

3) $\|av_0, v_1\|_{b,2} = |a| \ \|v_0, v_1\|_{b,2}$,

4) $\|v_0, v_1 - v_2\|_{b,2} \leq \|v_0, v_1 - v_3\|_{b,2} + \|v_0, v_3 - v_2\|_{b,2}$, for every $v_0, v_1, v_2, v_3 \in \mathcal{H}$ and $a \in \mathbb{R}$ then $\mathcal{N}_2$ is named two – normed linear space .

Definition 2.2 : Let $\mathcal{N}_2$ be a two – normed linear space . for $\phi \neq \mathcal{B}, \mathcal{M} \subseteq \mathcal{N}_2$,

d$(\mathcal{B}, \mathcal{M})$ = $sup_{b \in \mathcal{B}}\{\|v_0, v_1 - m\|_{b,2}\}$ $m \in \mathcal{M}$, denotes the distance

from the set $\mathcal{B}$ to the set $\mathcal{M}$. If

$sup_{b \in \mathcal{B}}\{\|v_0, v_1 - m\|_{b,2}\}$ =$sup_{b \in \mathcal{B}}\{\|v_0, v_1 - m_0\|_{b,2}\}$.

Then , we say that a function $m_0 \in \mathcal{M}$ is named a better approximation from $\mathcal{B}$ to $\mathcal{M}$.

Definition 2.3 : Let $\mathcal{N}_2$ be a two – normed linear space on $\mathbb{F}$ ($real\ field$), $\mathcal{M}_q$ subspace of $\mathcal{N}_2$ and $(M \neq \emptyset)$ a subset of $\mathcal{M}_q$. for a bounded subset $\mathcal{Y}$ of $\mathcal{N}_2$. as define $\mathcal{N}_2$

$rad_M(\mathcal{Y}) = inf_{p \in M} sup_{a \in \mathcal{Y}} \|z, a - p\|_{b,2}$

for $\in {}^{\mathcal{N}_2}\!/_{\mathcal{M}_q}$, and $cent_M(\mathcal{Y}) = p_1 \in M$:

$sup_{a \in \mathcal{Y}} \|z, a - P_1\|_{b,2} = rad_M(\mathcal{Y})$ for every $z \in {}^{\mathcal{N}_2}\!/_{\mathcal{M}_q}$.

The number $rad_M(\mathcal{Y})$ is called the Chebyshev radius of $\mathcal{Y}$ with respect

to M and an element $p_1 \in cent_M(\mathcal{Y})$ is called a better simultaneous approximation.

Definition 2.4 : [9] : A subspace $\mathcal{M}$ of X (vector space) is called convex if $m_1, m_2 \in \mathcal{M}$ implies

D= $[w \in X , w = \jmath m_1 + (1 - \jmath)m_2 , 0 \leq \jmath \leq 1] \subset \mathcal{M}$.

Definition 2.5 : A 2-normed v is said to be continuous at (p, q) if

for a given $\in > 0$ there $\exists$ a $i > 0$ $\ni$

$|v(p, q) - v(w, d)| < \in$ whenever $\|p - w, q\|_{b,2} < i$ &

$\|w, q - d\|_{b,2} < i$ or $\|p - w, d\|_{b,2} < i$ and $\|p, q - d\|_{b,2}$.

then v is said to be continuous at each point the domain .

3. Auxiliary lemmas

Lemma 3.1 : Let $\mathcal{N}_2$ be two – normed linear space and $\mathcal{M}, \mathcal{B}$ be closed sub space of $\mathcal{N}_2$. Then

$$\|n, m \oplus b\|_{b,2} = \|n, m\|_{b,2} \oplus \|n, b\|_{b,2}$$

from $n \in \mathcal{N}_2$ and $m \in \mathcal{M}$, b $\in \mathcal{B}$.

Proof : For $v \in \|n, m \oplus b\|_{b,2}$, let $v_0 : \mathcal{N}_2 \longrightarrow \mathcal{M}$ and $v_1 : \mathcal{N}_2 \longrightarrow \mathcal{B}$ be such that $v(u) = \|v_0(u), v_1(u) \oplus m\|_{b,2}$ for all $u \in \mathcal{N}_2$, $m \in \mathcal{M}$.

It is clear that $v_0 \in \|n, m\|_{b,2}$ and $v_1 \in \|n, b\|_{b,2}$. Define

$\varphi : \|n, m \oplus b\|_{b,2} \longrightarrow \|n, m\|_{b,2} \oplus \|n, b\|_{b,2}$

from $n \in \mathcal{N}_2$ and $m \in \mathcal{M}$, b $\in \mathcal{B}$, by

$\varphi(v) = \|v_0, v_1 \oplus m\|_{b,2}$. It is clear that φ is onto isometry, noting that

$\|\varphi(v)\|_{b,2}$ = max $\{\|v_0, v_1 \oplus m\|_{b,2}\}$

= sup max $\{\|v_0(u), v_1(u) \oplus m\|_{b,2}\}$

= sup $\|v(u)\|_{b,2} = \|v\|_{b,2}$.

4. Main result

Theorem 4.1 : Let $\mathcal{M}$ be a closed sub space of two – normed linear space $\mathcal{N}_2$, for any $v_0, v_1 \in \mathcal{N}_2$, we have

$\|v_0(u), v_1(u) - w\|_{b,2} \leq \|v_0, v_1 - w\|_{b,2}$ $w \in \mathcal{M}$.

Proof : Since $\mathcal{M} \subseteq \mathcal{N}_2$, $w \in \mathcal{M}$, we need to proof

$\|v_0(u), v_1(u) - w\|_{b,2} \leq \|v_0, v_1 - w\|_{b,2}$ $w \in \mathcal{M}$.

$\|v_0(u), v_1(u) - w\|_{b,2} \leq$ max $\|v_0(u), v_1(u) - w\|_{b,2}$,

$\|v_1(u), v_0(u) - w\|_{b,2}$. We take super. two sides , we obtain

sup $\|v_0(u), v_1(u) - w\|_{b,2} \leq$ sup max $\{\|v_0, v_1 - w\|_{b,2}, \|v_1, v_0 - w\|_{b,2}\}$. Since $w \in \mathcal{M}$ was arbitrary , then

$sup\ \|v_0(u), v_1(u) - w\|_{b,2} \leq \|v_0, v_1 - w\|_{b,2}$.

Now, let $j > \sup \|v_0(u), v_1(u) - w\|_{b,2}$ for $u \in \mathcal{N}_2$, define

$\varphi(u)$
$= \{m \in \mathcal{M} : \max \{\|v_0(u), v_1(u) - m\|_{b,2}, \|v_1(u), v_0(u) - m\|_{b,2}\} \leq \sup \|v_0(u), v_1(u) - w\|_{b,2}\}$.

The $\emptyset \neq \varphi$ is $\underbrace{subset\ of\ \mathcal{M}}_{closed}$. Now , to show that

$\varphi(u)$ is convex for $\forall\ u \in \mathcal{N}_2$ and φ is lower semi continuous .

Let $\in \mathcal{N}_2$, $m_1, m_2 \in \varphi(u)$, and $0 \leq \delta \leq 1$.

max $\{\|v_0(u), v_1(u) - \delta m_1 - (1-\delta)m_2\|_{b,2}, \|v_1(u), v_0(u) - \delta m_1 - (1-\delta)m_2\|_{b,2}\}$

$\leq$ max
$\{\delta \|v_0(u), v_1(u) - m_1\|_{b,2} + (1-\delta)\|v_0(u), v_1(u) - m_2\|_{b,2}, \delta \|v_1(u), v_0(u) - m_1\|_{b,2} +$
$(1-\delta)\|v_1(u), v_0(u) - m_2\|_{b,2}\}$

$\leq \delta$ max $\{\|v_0(u), v_1(u) - m_1\|_{b,2}, \|v_1(u), v_0(u) - m_1\|_{b,2}\}$

$+ (1-\delta)\ max\{\|v_0(u), v_1(u) - m_2\|_{b,2}, \|v_1(u), v_0(u) - m_2\|_{b,2}\}$

$\leq \delta \sup \|v_0(u), v_1(u) - w\|_{b,2} + (1-\delta)\sup \|v_0(u), v_1(u) - w\|_{b,2} = \sup \|v_0(u), v_1(u) - w\|_{b,2}$.

To demonstration that φ is lower semi continuous , let p be an open set in $\mathcal{M}$.

to investeg ate $p^* = \{u \in \mathcal{N}_2 : \varphi(u) \cap p \neq \emptyset\}$.

It is to be exposed p^* is open . Let $e \in p^*$, then $\varphi(e) \cap p \neq \emptyset$. Hance , there exists an $m \in p$ such that

max$\{\|v_0(e), v_1(e) - m\|_{b,2}, \|v_1(e), v_0(e) - m\|_{b,2}\} \leq \sup \|v_0(u), v_1(u) - w\|_{b,2}$

By the definition of $\sup \|v_0(u), v_1(u) - w\|_{b,2}$, $\sup \|v_0(u), v_1(u) - w\|_{b,2} > inf_{q \in \mathcal{M}}$ max $\{\|v_0(e), v_1(e) - q\|_{b,2}, \|v_1(e), v_0(e) - q\|_{b,2}\}$, there exists $m' \in \mathcal{M}$ such that

max $\{\|v_0(e), v_1(e) - m'\|_{b,2}, \|v_1(e), v_0(e) - m'\|_{b,2}\}$

$< \sup \|v_0(u), v_1(u) - w\|_{b,2}$.

Now , $m \in p$, then there exists $\epsilon > 0$ such that

D$(m, \epsilon) = \{q \in \mathcal{M} : \|q - m\|_{b,2} < \epsilon\} \subseteq p$.

Let $i = \frac{\epsilon}{2\|m-m'\|_{b,2}}$ if $\|m-m'\|_{b,2} \geq 1\frac{\epsilon}{2}$ if $\|m-m'\|_{b,2} \leq 1$;

consider that $0 \leq i \leq 1$. Let $m'' = (1-i)m + i\,m'$, then

$\|m'' - m\|_{b,2} = i\,\|m-m'\|_{b,2} < \epsilon$, hance $m'' \in \mathcal{p}$.

By the covexity of $\varphi(e)$, $m'' \in \varphi(e)$ and

$\max\left\{\|v_0(e), v_1(e) - m''\|_{b,2}, \|v_1(e), v_0(e) - m''\|_{b,2}\right\}$

$< \sup \|v_0(u), v_1(u) - w\|_{b,2}$

Now , let L be open ball of e such that

$\max \{\|v_0(e), v_1(e) - v_1(u)\|_{b,2}, \|v_1(e), v_0(e) - v_0(u)\|_{b,2}\} < \sup \|v_0(u), v_1(u) - w\|_{b,2}$
$-max\left\{\|v_0(e), v_1(e) - m''\|_{b,2}, \|v_1(e), v_0(e) - m''\|_{b,2}\right\}$

For any $u \in L$, we have

$\max\left\{\|v_0(u), v_1(u) - m''\|_{b,2}, \|v_1(u), v_0(u) - m''\|_{b,2}\right\}$

$\leq$max
$\{\|v_0(u), v_1(u) - v_1(e)\|_{b,2} + \|v_0(e), v_1(e) - m''\|_{b,2}, \|v_1(u), v_0(u) - v_0(e)\|_{b,2} +$
$\|v_1(e), v_0(e) - m''\|_{b,2}\}$

$\leq$ max $\{\|v_0(u), v_1(u) - v_1(e)\|_{b,2}, \|v_1(u), v_0(u) - v_0(e)\|_{b,2}\}$ + max
$\left\{\|v_0(e), v_1(e) - m''\|_{b,2}, \|v_1(e), v_0(e) - m''\|_{b,2}\right\}$

$\leq \sup \|v_0(u), v_1(u) - w\|_{b,2}$.

Hence , $m'' \in \varphi(u) \cap \mathcal{p}$, $u \in \mathcal{p}^*$, L $\in \mathcal{p}^*$ and $\mathcal{p}$ is open .

There exists $w \in \mathcal{M}$ such that $w(u) \in \varphi(u)$ for all $u \in \mathcal{N}_2$. Hance

$\max\left\{\|v_0(u), v_1(u) - w(u)\|_{b,2}, \|v_1(u), v_0(u) - w(u)\|_{b,2}\right\}$

$\leq \sup \|v_0(u), v_1(u) - w\|_{b,2}$ and max
$\left\{\|v_0, v_1 - w\|_{b,2}, \|v_1, v_0 - w\|_{b,2}\right\} \leq \sup \|v_0(u), v_1(u) - w\|_{b,2}$.

Thus, $\|v_0, v_1 - w\|_{b,2} \leq sup\|v_0(u), v_1(u) - w\|_{b,2}$, $w \in \mathcal{M}$.

Theorem 4.2 : Let $\mathcal{N}_2$ be two – normed linear space and $\mathcal{M}$ be a closed subspace of $\mathcal{N}_2$. Then

1. If $\|n,m\|_{b,2}$ from $n \in \mathcal{N}_2$ and $m \in \mathcal{M}$ is simultaneous. proximinal in $\mathcal{N}_2$, then $\mathcal{M}$ simultaneous. proximinal in $\mathcal{N}_2$.
2. If $\mathcal{M}$ has a continuous simultaneous. operator , then $\|n,m\|_{b,2}$ from $n \in \mathcal{N}_2$, $m \in \mathcal{M}$ *is* simultaneous. proximinal in $\mathcal{N}_2$ and has cont's simultaneous. proximity operator .

Proof : 1. Let $x_1, y_1 \in \mathcal{N}_2$. Define $v_{y_1} : \mathcal{N}_2 \longrightarrow \mathcal{N}_2$ and

$v_{x_1} : \mathcal{N}_2 \longrightarrow \mathcal{N}_2$ by $v_{y_1}(u) = y_1$, $v_{x_1}(u) = x_1$ for all $u \in \mathcal{N}_2$.

Since $\|n,m\|_{b,2}$ from $n \in \mathcal{N}_2$ and $m \in \mathcal{M}$ *is* simultaneous.

proximinal in $\mathcal{N}_2$, $\exists\, g \in \|n,m\|_{b,2}$ such that ,

max $\left\{\|v_{y_0}, v_{y_1} - g\|_{b,2}, \|v_{x_0}, v_{x_1} - g\|_{b,2}\right\}$ = sup $\|v_{y_1}, v_{x_1} - m\|_{b,2}$

= sup $\|v_{x_1}, v_{y_1} - m\|_{b,2} \leq \|x_1, y_1 - m\|_{b,2}$.

Then , for some $u_o \in \mathcal{N}_2$, we have

max $\left\{\|v_{y_0}(u_o), v_{y_1}(u_o) - g(u_o)\|_{b,2}, \|v_{x_0}(u_o), v_{x_1}(u_o) - g(u_o)\|_{b,2}\right\} \leq \|x_1, y_1 - m\|_{b,2}$. Hence $g(u_o)$ is a better simultaneo.

approx. for x_1, y_1 of $\mathcal{M}$.

2. Let $\mathcal{B} : \mathcal{N}_2 \oplus \mathcal{N}_2 \longrightarrow \mathcal{M}$ be a cont's simultaneous.

proximity operator for $\mathcal{M}$. Define

$\mathcal{B}' : \|n, n \oplus n\|_{b,2} \longrightarrow \|n,m\|_{b,2}$ from $n \in \mathcal{N}_2$ and $m \in \mathcal{M}$,

by $\mathcal{B}'(v) = \mathcal{B} \circ v$. $\mathcal{B}'$ can be redefined as

$\|n,n\|_{b,2} \oplus \|n,n\|_{b,2} \longrightarrow \|n,m\|_{b,2}$, and

$\mathcal{B}\,\|v_0, v_1 - m\|_{b,2}$ for all $m \in \mathcal{M}$. It is clear that

$\mathcal{B}'\,\|v_0, v_1 - m\|_{b,2} \in \|n,m\|_{b,2}$. Let $g \in \|n,m\|_{b,2}$, then

max $\left\{\|v_0, v_1 - \mathcal{B}(v_1)\|_{b,2}, \|v_0, v_0 - \mathcal{B}(v_0)\|_{b,2}\right\}$

$\leq$ max $\left\{\|v_0, v_1 - g\|_{b,2}, \|v_0, v_o - g\|_{b,2}\right\}$.

Thus , $\mathcal{B}\,\|v_0, v_1 - g\|_{b,2}$ is a better simultaneo. for v_0, v_1

from $\|n,m\|_{b,2}$ and then $\|n,m\|_{b,2}$ is simultaneous. proximinal

in $\mathcal{N}_2$. It is clear that $\mathcal{B}' : \mathcal{N}_2 \oplus \mathcal{N}_2 \longrightarrow \mathcal{M}$ is a continuous

simultaneo. proximity operator .

Theorem 4.3 : Let $\mathcal{M}$ be a simultaneo. sub space of a two – normed linear space $\mathcal{N}_2$. If $\mathcal{M}$ has a linear proximity operator ,

then $\| z, m \|_{b,2}$ is simultaneous. proximinal in $\| z, n \|_{b,2}$

from $z \in \mathcal{N}_2$ and $m \in \mathcal{M}$, $n \in \mathcal{N}_2$ and has a linear simultaneo. proximity operator .

Proof : Let $\varphi : \mathcal{N}_2 \oplus \mathcal{N}_2 \longrightarrow \mathcal{M}$ be a linear simultaneous. proximity operator for $\mathcal{M}$. Define anther operator

$\mathcal{B} : \| z, n \oplus n \|_{b,2} \longrightarrow \| z, m \|_{b,2}$ from $z \in \mathcal{N}_2$ and $m \in \mathcal{M}$, $n \in \mathcal{N}_2$,

given $\mathcal{B}(v) = \varphi \circ v$. we may write $\mathcal{B} : \| z, n \|_{b,2} \oplus \| z, n \|_{b,2} \longrightarrow \| z, m \|_{b,2}$, define by $\| v_0, v_1 - m \|_{b,2} = \varphi \circ \| v_0, v_1 - m \|_{b,2}$.

Since $\mathcal{B}$ is linear operator, we have

$\mathcal{B} \{ \delta \| v_0, v_1 - m \|_{b,2} + \lambda \| g_0, g_1 - m \|_{b,2} \}$

$= \delta \mathcal{B} \| v_0, v_1 - m \|_{b,2} + \lambda \mathcal{B} \| g_0, g_1 - m \|_{b,2}$, for all $m \in \mathcal{M}$

and $\| v_0, v_1 - m \|_{b,2}$ is a linear simultaneous. proximity operator

for $\| z, m \|_{b,2}$.

Theorem 4.4 : Let $\mathcal{N}_2$ be two – normed linear space , $\mathcal{M}$ closed

sub space of $\mathcal{N}_2$ and $\emptyset \neq Y \subseteq \mathcal{N}_2$. Then $\mathcal{M}$ is simultaneo. proximinal in $\mathcal{N}_2$ if and only if $\| y, m \|_{b,2}$ is simultaneo. proximinal

in $\| y, n \|_{b,2}$ for y $\in Y$ and $m \in \mathcal{M}$, $n \in \mathcal{N}_2$.

Proof : Assume that $\mathcal{M}$ is simultaneo. proximinal in $\mathcal{N}_2$,

we need to prove $\| y, m \|_{b,2}$ is simultaneo. proximinal

in $\| y, n \|_{b,2}$ for y $\in Y$ and $m \in \mathcal{M}$, $n \in \mathcal{N}_2$.

Let $v, g \in \| y, n \|_{b,2}$ for all y $\in Y$ and $n \in \mathcal{N}_2$. Since $\mathcal{M}$ is

simultaneo. proximinal in $\mathcal{N}_2$, the for any $u \in Y$ there exists

T$(u) \in \mathcal{M}$ such that

$\max\left\{\|v_0(u),v_1(u)-T(u)\|_{b,2},\|g_0(u),g_1(u)-T(u)\|_{b,2}\right\}$

$\leq \max\left\{\|v_0(u),v_1(u)-w\|_{b,2},\|g_0(u),g_1(u)-w\|_{b,2}\right\}$,

for all $w \in G$. In particular it holds for any $(u) \in G$,

$w \in \| y,m \|_{b,2}$. By Axiom of choice, the exists $T \in \| y,m \|_{b,2}$.

Hence $\max\left\{\|v_0,v_1-T\|_{b,2},\|g_0,g_1-T\|_{b,2}\right\}$

$\leq \max\left\{\|v_0,v_1-w\|_{b,2},\|g_0,g_1-w\|_{b,2}\right\}$, for all $w \in \| y,m \|_{b,2}$.

Then $\max\left\{\|v_0,v_1-T\|_{b,2},\|g_0,g_1-T\|_{b,2}\right\} = \|v,g-m\|_{b,2}$,

which implies $\| y,m \|_{b,2}$ is simultaneous. proximinal

in $\| y,n \|_{b,2}$ for all $y \in Y$ and $m \in \mathcal{M}$, $n \in \mathcal{N}_2$.

Converse : Assume $\| y,m \|_{b,2}$ is simultaneo. proximinal

in $\| y,n \|_{b,2}$ for $y \in Y$ and $m \in \mathcal{M}$, $n \in \mathcal{N}_2$, we need to prove

$\mathcal{M}$ is simultaneo. proximinal in $\mathcal{N}_2$.

Let $x_1,y_1 \in \mathcal{N}_2$. Set $v_{x_1}: Y \longrightarrow \mathcal{N}_2$, $v_{y_1}: Y \longrightarrow \mathcal{N}_2$,

define by $v_{x_1}(u) = x_1$, $v_{y_1}(u) = y_1$ for all $u \in Y$.

$\|v_{y_1},v_{x_1}-m\|_{b,2} = \sup \|v_{y_1}(u),v_{x_1}(u)-m\|_{b,2} = \|x_1,y_1-m\|_{b,2}$

for all $m \in \mathcal{M}$. Since $\| y,m \|_{b,2}$ for all $y \in Y$ and $m \in \mathcal{M}$

is simultaneous. proximinal in $\| y,n \|_{b,2}$ for all $y \in Y$ and $n \in \mathcal{N}_2$,

then there exists $g \in \| y,m \|_{b,2}$ such that

$\max\left\{\|v_{x_0},v_{x_1}-g\|_{b,2},\|v_{y_0},v_{y_1}-g\|_{b,2}\right\} = \|x_1,y_1-m\|_{b,2}$

$\forall\, m \in \mathcal{M}$. Choose $u_0 \in Y$ such that

$\max\left\{\|x_0,x_1-g(u_0)\|_{b,2},\|y_0,y_1-g(u_0)\|_{b,2}\right\}$

$\leq \|x_1,y_1-w\|_{b,2}$ for all $w \in G$. Then $g(u_0)$ is a better

simultaneo. approx. for x_1 and y_1 form $\mathcal{M}$.

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Best simultaneous approximation with some properties of Chebyshev centers in k-normed space

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Abstract

In this notes , we presented tow important properties to the best simultaneous approximation of bounded functions in k-normed space ,which are continuity and convex set and proved them .We have extensively discussed the Chebyshev centers and proved its monotheism in the same space .

Key words: best simultaneous approximation, k-normed space, Chebyshev

centers , convex set and bounded functions .

1. Introduction

The notion of best simultaneous approximation of functions in normed space and metric space, see for example [1, 2, 3].In (1989) Tanmoto [4] studied the problem of simultaneous approxima. of functions via continuous functions in finite dimensional normed space, in (1983) Prolla[5] discuss the concept of Non-expansive mappings and Chebyshev centers . T.Markandeya and D. Bharathi Khandagji [6] discuss the concept of simultan. approxima. in normed space, in (2014) . Narange [7,8,9,10,11,12] investigate the concept of best simultaneous approximation and Chebyshev centers.

In this paper, we presented an idea of the continuity property of the best simultaneous approximation of bounded function in k-normed space and its proof , as well as we know and prove the convexity property in the same space .In the last part of this article ,we touched on the Chebyshev centers and some important properties and theorems on it .

2. Preliminaries

Definition 2.1: Let $\check{E}$ be the normed space and M be a closed subspace of $\check{E}$. For $\check{t} \in \check{E}$, we have

$ḍ(t, M) = inf\ \{\|t - m\|_p : m \in M\}$.

then m is called a best approximant of $t \in Ě$.

Definition 2.2 Let F be the subset of k-$n$ormed space $À$ over real vector space and $t_1, t_2, \ldots, t_k \in À$. Then $ę \in F$ is named the better approxi. to $t_k \in À$ of F when

$$\|t_1, t_2, \ldots, t_{k-1} - ḟ\|_k = \underbrace{inf}_{ę \in F} \|t_1, t_2, \ldots, t_{k-1}, t_k - ę\|_k.$$

The family of all better approximat. into $À$ from F symbolized by $j_F(ę)$and we know it

$$j_F(ę) = \{ę \in F;\ \inf \|n_1, n_2, \ldots, n_k - ę\|_k\ \forall n_k \in À\}.$$

Definition 2.3: Let $Ě$ be a linear space over F, where F is the real or complex numbers field, dim $Ě > 1$ and let $\|•, \ldots, •\|_k : Ě \to \mathbb{R}^+ \cup \{0\}$ be a non-negative real-valued function on $Ě \times Ě$ with the following properties:

N1) $\| t_1, t_2, \ldots, t_k \|_k = 0$ if and only if $t_1, t_2, \ldots, t_k$ are linearly dependent vectors .

N2) $\| t_1, t_2, \ldots, t_k \|_k$ is invariant under any transformation .

N3) $\|t_1, t_2, \ldots, \beta t_k\|_k = |\beta| \| t_1, t_2, \ldots, t_k \|_k$ for all $\beta \in F$ and all $t_1, t_2, \ldots, t_k \in Ě$.

N4) $\left\|t_1, t_2, \ldots, t_{k-1}, t_k + ć\right\|_k \leq \|t_1, t_2, \ldots, t_{k-1}, t_k\|_k + \|t_1, t_2, \ldots, t_{k-1}, ć\|_k$.

The couple $(Ě, \|•, \ldots, •\|_k)$ is named an k-normed space .

Definition 2.4 Let $(Ě, \|•, \ldots, •\|_k)$ be a linear k-normed space and E be any bounded subset of $Ě$. An element $m^* \in M$ is said to be a best approximation to the set E, if

$$sup_{t \in E} \|t - m^*\| = inf_{m \in M}\{sup_{t \in E} \|t - m\|\}$$

Definition 2.5 Let $(Ě, \|.\|)$ be a L.N.S.. For a nonempty subset $Á$ of $Ě$ and $t \in Ě$,

$ḍ(t, Á) = inf_{à \in Á}\{\|ú - à\|_k\}$

denotes the distance from t to the set $Á$, if

$\|t - å\| = ḍ(t, Á)$,

then, we say that a function $å \in Á$ is named a better approxima. for $t \in Ě$. If each $t \in Ě$ has one or more better approxima. $å \in Á$, then $Á$ is named a proximinal subspace of $Ě$. If every $t \in Ě$ has only one better approximation $å \in Á$, then $Á$ is called a Chebyshev subspace of $Ě$.

$sup_{t\in E}\|t- m^*\| = inf_{m\in M}\{ sup_{t\in E} \|t-m\|\}$

Definition 2.6: A functional Ụ is a real valued mapping defined on $\grave{A}\times ß$, where $\grave{A}$ *and* ß are linear subspaces of a linear k-normed space $(\check{E}, \|\bullet,\dots,\bullet\|_k)$ is said to be continuous at $t_1,t_{2,\dots},t_k$ if for a given $\varepsilon > 0$ there exists a $\square > 0$ s.t.

$\|(t_1),(t_2),\dots,(t_{k-1}),Ụ(t_k) - Ụ(k)\|_k < \varepsilon$ whenever

$\|t_1,t_2,\dots,t_{k-1},t_k - k\|_k < \square$. Then Ụ is said to be continuous at each point .

Definition 2.7 : Let $\check{E}$ be a k-normed space and M a non-empty subset of $\check{g}_{(t)}$. For a bounded subset $\grave{A}$ of $\check{E}$ let us define

$rad_M(\grave{A}) = inf_{m\in M} sup_{\grave{a}\in\grave{A}}\|t_1,t_{2,\dots,}t_{k-1},\grave{a}-m\|_k$ for every $t_1,t_2,\dots,t_k \in \check{E}/\check{g}_{(t)}$ and

$cent_m(\grave{A}) = m_0 \in M: sup_{\grave{a}\in\grave{A}}\|t_1,t_{2,\dots,}t_{k-1},\grave{a}-m_0\|_k = rad_M(\grave{A})$ for every $t_1,t_2,\dots,t_k \in \check{E}/\check{g}_{(t)}$.

The $rad_M(\grave{A})$ is called the chebyshev radius of $\grave{A}$ with respect to M and an function $m_0 \in cent_m(\grave{A})$ is called chebyshev centre of $\grave{A}$ with respect to M .

2. Auxiliary lemmas

In this section, we introduced some lemmas which its need in our main theorems

Lemma *3.1* . Let $\check{G}$ be a convex subset of a convex k-normed space and $\mathcal{F}$ a bounded subset of $\check{E}$ and $\hat{g}_1^*, \hat{g}_2^* \in \check{G}$ are best simultaneous approximation to $\mathcal{F}$. Then $\hat{C}(\hat{g}_1^*, \hat{g}_2^*, \Upsilon)$ is a best simultaneous approximation in $\check{G}$ to $\mathcal{F}$ for every $\Upsilon \in \check{I}$ *where* $\check{I} = [0,1]$.

Proof . Since $\hat{g}_1^*, \hat{g}_2^* \in \check{G}$ are best simultaneous approximation to $\mathcal{F}$,

$sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{g}_1^*\|_k = \check{K}(\mathcal{F},\check{G}) = sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{g}_2^*\|_k$

Where $\check{K}(\mathcal{F},\check{G}) = inf_{\check{u}\in\check{K}}\|t_1,t_2,\dots,t_{k-1},\check{u} - t_k\|_k$.

For any $f\in\mathcal{F}, consider$ $\|t_1,t_2,\dots,t_{k-1},t_k - \hat{C}(\hat{g}_1^*,\hat{g}_2^*,\Upsilon)\|_k \le \Upsilon\|t_1,t_2,\dots,t_{k-1},t_k - \hat{g}_1^*\|_k$

$+ (1-\Upsilon)\|t_1,t_2,\dots,t_{k-1},t_k - \hat{g}_2^*\|_k$.

Implies

$sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{C}(\hat{g}_1^*,\hat{g}_2^*,\Upsilon)\|_k \le \Upsilon sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{g}_1^*\|_k + (1-\Upsilon) sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{g}_2^*\|_k$

$= \Upsilon\check{K}(\mathcal{F},\check{G}) + (1-\Upsilon)\check{K}(\mathcal{F},\check{G}) = \check{K}(\mathcal{F},\check{G}) \leq sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{C}(\hat{g}_1^*,\hat{g}_2^*,\Upsilon)\|_k$

As $\hat{C}(\hat{g}_1^*,\hat{g}_2^*,\Upsilon) \in \check{G}$ *by the convexity of* $\check{G}$.

Therefore , $sup_{f\in\mathcal{F}}\|t_1,t_2,\dots,t_{k-1},t_k - \hat{C}(\hat{g}_1^*,\hat{g}_2^*,\Upsilon)\|_k = \check{K}(\mathcal{F},\check{G})$, proving there by

$\hat{C}(\hat{g}_1^*,\hat{g}_2^*,\Upsilon)$ is a best simultaneous approximation in $\check{G}$ to $\mathcal{F}$ for every Υ.

Lemma 3.2 . If $\grave{A}$ is a bounded subset of a convex k-normed space $(\check{E},\|\bullet,\dots,\bullet\|_k)$,

then $Z(\grave{A})$ is a convex subset of $\check{E}$.

Where $Z(\grave{A}) = \{t_{\grave{A}} \in \check{E} : \sup_{\gamma\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\hat{u}_{\grave{A}} - \ddot{u}\|_k = \varkappa(\grave{A})\}$**,** is set of all centers of $\grave{A}$.

$\varkappa(\grave{A}) \equiv \inf_{t\in\check{E}}\sup_{\ddot{u}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\hat{u}-\ddot{u}\|_k$ is called the chebyshev radius of $\grave{A}$.

Proof . suppose $t_1,t_2,\dots,t_k \in \check{E}$, and $\hat{u},\ddot{u} \in Z(\grave{A})$,where $0 \leq \Upsilon \leq 1$.

Then $sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\hat{u}-\check{o}\|_k = \varkappa(\grave{A}) = sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\ddot{u}-\check{o}\|_k$.

Consider

$sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},u-\check{o}\|_k \leq sup_{\check{o}\in\grave{A}}[\Upsilon\|t_1,t_2,\dots,t_{k-1},\hat{u}-\check{o}\|_k + (1-\Upsilon)\|t_1,t_2,\dots,t_{k-1},\ddot{u}-\check{o}\|_k]$.

$\leq \Upsilon sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\hat{u}-\check{o}\|_k + (1-\Upsilon)sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\ddot{u}-\check{o}\|_k$,

as $0 \leq \Upsilon \leq 1$.

$\leq \Upsilon\varkappa(\grave{A}) + (1-\Upsilon)\varkappa(\grave{A}) = \varkappa(\grave{A}) \equiv \inf_{\acute{u}\in\check{E}}\sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\hat{u}-\check{o}\|_k$

$\leq sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},u-\check{o}\|_k$.

This gives $sup_{\check{o}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},u-\check{o}\|_k = \varkappa(\grave{A})$ and so $u \in Z(\grave{A})$.

Then $Z(\grave{A})$ is convex .

4.Main results

In this section, we proven some theorems which related better simultane. k-approxima. of bounded functions in k-normed space and Chebyshev centers proved its monotheism in the same space .

Theorem 4.1: Let $(\check{E},\|\bullet,\dots,\bullet\|_k)$ be a k-normed space, $M \subseteq \check{E}$ and $\grave{A}$ be abounded subset of $\check{E}$. Then the function $\dot{U}(a)$ define as

$sup_{\grave{a}\in\grave{A}}\|t_1,t_2,\dots,t_{k-1},\grave{a}-m\|_k$, $t_1,t_2,\dots,t_k \in \check{E}/\check{g}_{(t)}$, $m \in M$, $\grave{a} \in \grave{A}$ is a continuous .

Proof : Let $\grave{a} \in \grave{A}$ and $m, m_1 \in M$ we have

$$\|t_1, t_2, \dots, t_{k-1}, \grave{a} - m\|_k \leq \|t_1, t_2, \dots, t_{k-1}, \grave{a} - m_1\|_k + \|t_1, t_2, \dots, t_{k-1}, m_1 - m\|_k \quad , \; t_1, t_2, \dots, t_k \in \check{E} / \check{g}_{(t)} .$$

Then

$$sup_{\grave{a} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, \grave{a} - m\|_k \leq sup_{\grave{a} \in \grave{A}} \left\{ \|t_1, t_2, \dots, t_{k-1}, \grave{a} - m_1\|_k + \|t_1, t_2, \dots, t_{k-1}, m_1 - m\|_k \right\}.$$

If $\|t_1, t_2, \dots, t_{k-1}, m_1 - m\|_k \leq \epsilon$, then $\dot{U}(a,m) \leq \dot{U}(a, m_1) + \varepsilon$.

By exchanging m and m_1 we get

$$\dot{U}(a, m_1) \leq \dot{U}(a,m) + \varepsilon \Longrightarrow \|t_1, t_2, \dots, t_{k-1}, \dot{U}(a,m) - \dot{U}(a, m_1)\|_k$$

That is $\dot{U}(a,m)$ is continuous on $\check{E}$.

Theorem 4.2 . let $(\check{E}, \|\bullet, \dots, \bullet\|)$ be a linear k-normed space . Let $dim M = n$ subspace of $\check{E}$. Then there exists a best simultane. approxima.$m^* \in M$ to any given compact subset $\mathcal{F} \subset \check{E}$.

Proof. Since $\mathcal{F}$ be compact , there exists a finite constant $\check{G}$ such that $\|t_1, t_2, \dots, t_{k-1}, t_k - u\|_k \leq \check{G}$, for all $t_1, t_2, \dots, t_k \in \check{E}, u \in \mathcal{F}$

Now we define the subset $W \; of \; M \; as \; W \equiv W(0, 2\check{G})$. Then

$$inf_{m \in W} \|t_1, t_2, \dots, t_{k-1}, t_k - m\|_k = inf_{m \in M} \|t_1, t_2, \dots, t_{k-1}, t_k - m\|_k$$

Since W is compact , the continuous function $\tilde{\alpha}(m,u) = sup_{f \in \mathcal{F}} \|t_1, t_2, \dots, t_{k-1}, t_k - m\|_k$, attains its minimum over W for some $m^* \in M$, Which is the better simultane. Approxima. to $\mathcal{F}$.

Theorem 4.3 . Let $\check{G}$ be a convex subset of a strictly convex k-normed space and $\mathcal{F}$ be a subset of $\check{E}$, there exists at most one best simultaneous approximation in $\check{G}$ to $\mathcal{F}$.

Proof . suppose $\hat{g}_1^*, \hat{g}_2^* \; \ni \; \hat{g}_1^* \neq \hat{g}_2^*$ are two best simultaneous approximation in $\check{G}$ to $\mathcal{F}$, i.e.

$$sup_{f \in \mathcal{F}} \|t_1, t_2, \dots, t_{k-1}, t_k - \hat{g}_1^*\|_k = \check{K}(\mathcal{F}, \check{G}) = sup_{f \in \mathcal{F}} \|t_1, t_2, \dots, t_{k-1}, t_k - \hat{g}_2^*\|_k$$

By Lemma 3.2 , $\hat{C}(\hat{g}_1^*, \hat{g}_2^*, \Upsilon) \in \check{G}$ is also a best simultaneous approximation to $\mathcal{F}$.

Then $\|t_1, t_2, \dots, t_{k-1}, t_k - \hat{C}(\hat{g}_1^*, \hat{g}_2^*, \Upsilon)\|_k = \check{K}(\mathcal{F}, \check{G})$ for every $\Upsilon \in \check{I}$.

Let $\Upsilon \in \check{I}$, since, there exists an function $f^* \in \mathcal{F}$ such that

$$\|f_1^*, t_2^*, \dots, t_{k-1}^*, t_k^* - \hat{C}(\hat{g}_1^*, \hat{g}_2^*, \Upsilon)\|_k = \check{K}(\mathcal{F}, \check{G}) \quad \dots\dots\dots\dots(1)$$

Now, $\|f_1^*, t_2^*, \dots, t_{k-1}^*, t_k^* - \hat{g}_1^*\|_k \leq \check{K}(\mathcal{F}, \check{G})$ and $\|f_1^*, t_2^*, \dots, t_{k-1}^*, t_k^* - \hat{g}_2^*\|_k \leq \check{K}(\mathcal{F}, \check{G})$, Because k-normed space be strictly convex , We have

$\|f_1^*, t_2^*, \dots, t_{k-1}^*, t_k^* - \hat{C}(\hat{g}_1^*, \hat{g}_2^*, \Upsilon)\|_k < \check{K}(\mathcal{F}, \check{G})$, unless $\hat{g}_1^* = \hat{g}_2^*$.

This contradicts (1) and hence the uniqueness .

Theorem 4.4 . Let $\grave{A}$ be a subset of a strictly convex k-normed space $(\check{E}, \|\bullet, \dots, \bullet\|)$. Then $Z(\grave{A})$ is at most a singleton .

Proof : suppose $\hat{u}, \ddot{u} \in Z(\grave{A})$, $\hat{u} \neq \ddot{u}$. Then $u \in Z(\grave{A})$ by Lemma 3 i.e.

$sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, u - \check{o}\|_k = \varkappa(\grave{A})$. Since $\grave{A}$ is a subset of $\check{E}$, there exists some $\acute{u} \in \grave{A}$ such that

$\|t_1, t_2, \dots, t_{k-1}, u - \acute{u}\|_k = \varkappa(\grave{A})$.

Since $\hat{u} \in Z(\grave{A}) \Rightarrow sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, \hat{u} - \check{o}\|_k = \varkappa(\grave{A}) \equiv \inf_{\check{g} \in \check{E}} \sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, \check{g} - \check{o}\|_k$. This gives $\|t_1, t_2, \dots, t_{k-1}, \hat{u} - \acute{u}\|_k \leq \varkappa(\grave{A})$. Similarly , $\ddot{u} \in Z(\grave{A})$ will give $\|t_1, t_2, \dots, t_{k-1}, \ddot{u} - \acute{u}\|_k \leq \varkappa(\grave{A})$. Since $\check{E}$ is strictly convex ,

$\|t_1, t_2, \dots, t_{k-1}, u - \acute{u}\|_k < \varkappa(\grave{A})$ This contradicts unless $\hat{u} = \ddot{u}$.

Theorem 4.5 Let $\grave{A}$ be a bounded subset of a uniformly convex k-normed space $(\check{E}, \|\bullet, \dots, \bullet\|)$. Then $Z(\grave{A})$ is at most a singleton .

Proof . suppose $\hat{u}, \ddot{u} \in Z(\grave{A})$, $\hat{u} \neq \ddot{u}$ i.e.,

$sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, \hat{u} - \check{o}\|_k = sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, \ddot{u} - \check{o}\|_k = \varkappa(\grave{A})$.

Then by Lemma 3.3 . $u \in Z(\grave{A})$ i.e.,

$sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, u - \check{o}\|_k = \varkappa(\grave{A}) \equiv \inf_{\acute{u} \in \check{E}} \sup_{\check{o} \in \grave{A}} \|t_1, t_2, \dots, t_{k-1}, \acute{u} - \check{o}\|_k$.

So, there exists a sequence $\{f_n\}$ in $\grave{A}$ such that

$\|t_1, t_2, \dots, t_{k-1}, u - f_n\|_k \to \varkappa(\grave{A})$ .....*

Also, $\|t_1, t_2, \dots, t_{k-1}, \hat{u} - f_n\|_k \leq \varkappa(\grave{A})$ for all . Now (*) implies that for every $d > 0$ there exists a positive integer g such that

$\|t_1, t_2, \dots, t_{k-1}, u - f_n\|_k > \varkappa(\grave{A}) - d$ for all $n \geq g$.

Let $\|t_1, t_2, \dots, t_{k-1}, \hat{u} - \ddot{u}\|_k = \varepsilon' > 0$. Since $(\check{E}, \|\bullet, \dots, \bullet\|)$ is uniformly convex , there exists a $d > 0$ such that

$\|t_1,t_2,\dots,t_{k-1},\hat{u}-f_m\|_k \le \varkappa(\grave{A})$, $\|t_1,t_2,\dots,t_{k-1},\ddot{u}-f_m\|_k \le \varkappa(\grave{A})$ and

$\|t_1,t_2,\dots,t_{k-1},u-f_m\|_k > \varkappa(\grave{A})-d$ imply $\|t_1,t_2,\dots,t_{k-1},\hat{u}-\ddot{u}\|_k < \varepsilon^{\cdot}$,

A contradiction and hence $\hat{u}=\ddot{u}$.

Remark 1. From simple to show that $f \in \check{E}$ that function in center of f and it's a unique .

Remark 2. If $f \in G$ satisfies the following condition

$$\|t_1,t_2,\dots,t_{k-1},f-t_k\|_k = \|t_1,t_2,\dots,t_{k-1},t_k-f\|_k$$

For all $t_1,t_2,\dots,t_k \in \check{E}$ then center of the function f exist and that function t_0 of best simultaneous approximation of f in $\check{E}$ and it's a unique .

Then center of f equal $\|t_1,t_2,\dots,t_{k-1},c+t\|_k$

Theorem 4.6 . Let $(\check{E},\|\bullet,\dots,\bullet\|_k)$ be k-normed space and t_0 be a best approximation of some one function g belong to subspace G of $\check{E}$. Then the function t_0 is best simultaneous approximation of f by functions of G

if and only if

$$\|t_1,t_2,\dots,t_{k-1},t_0-f\|_k = \|t_1,t_2,\dots,t_{k-1},\psi\|_k = \|t_1,t_2,\dots,t_{k-1},\psi\bullet t^*\|_k$$

Where $\psi \in G$.

Proof . Let center of f belong . Then $\|t_1,t_2,\dots,t_{k-1},c+t\|_k$ also belong to G is strictly monotone function $\mathscr{f}$ of $\|t_1,t_2,\dots,t_{k-1},t_0-c(f)\|_k$,where $c(f)$ center of f .

for every

$\forall\ \alpha > 0$ and $g > 0$ .Then every $\psi \neq 0$, $g \in G$ such that

$$\|t_1,t_2,\dots,t_{k-1},\psi(c(f)+g_1\|_k = |\psi|\|t_1,t_2,\dots,t_{k-1},(c(f))+g_1\|_k$$

$$= \mathscr{f}\ |\psi|\|t_1,t_2,\dots,t_k\|_k$$

Therefore

$$\mathscr{f}(t_1,t_2,\dots,|\psi|,|m|) = \|t_1,t_2,\dots,|\psi|,|m|\|_k$$

Hence

$$\|t_1,t_2,\dots,t_{k-1},\psi,s^*\|_k = \|t_1,t_2,\dots,t_{k-1},\psi(c(f))-s^*-\psi(c(f))\|_k$$

$$\le \|t_1,t_2,\dots,t_{k-1},\psi(c(f))\|_k + \|t_1,t_2,\dots,t_{k-1},s^*+\psi(c(f))\|_k$$

Imply

⇒ That s^* is best simultaneous approximation in $\check{E}$.

Theorem 4.7 . Let $(\check{E}, \|\bullet,\dots,\bullet\|_k)$ be k-normed space , M compact subspace of $\check{E}$ .Then M is simultaneous a proximinal in $\check{E}$.

Proof . Let $t_1, t_2, \dots, t_k \in \check{E}$ and define the function $\psi : M \to \mathbb{R}$ by

$$\psi(m) = \|t_1, t_2, \dots, t_{k-1}, , t_k - m\|_k$$

It clear that function ψ is continuous on M

Implies , there is $m_1 \in M$ such that

$$\psi(m_1) = \inf_{m\in M} \psi(m) = \inf_{m\in M} \|t_1, t_2, \dots, t_{k-1}, , t_k - m\|_k$$

⇒ m_1 best simultaneous approximation from M to function in $\check{E}$

⇒ M is simultaneous a proximinal in $\check{E}$.

Theorem 4.8. Let $(\check{E}, \|\bullet,\dots,\bullet\|_k)$ be k-normed space , M compact subspace of $\check{E}$ and $\{m_n\}_{n=1}^{\infty}$ be an simultaneous approximation sequence in M . If $\{m_n\}_{n=1}^{\infty}$ convergent to $m_1 \in M$ then m_1 best simultaneous approximation from M to a finite function in $\check{E}$.

Proof . Let $t_1, t_2, \dots, t_k \in \check{E}$ and k-norm function is continuous , We have

$$\|t_1 - m_1, t_2 - m_1, \dots, t_{k-1} - m_1, , t_k - m_1\|_k \le \|t_1 - m_n, t_2 - m_n, \dots, t_{k-1} - m_n, , t_k - m_n\|_k$$

$$\Rightarrow \|t_1 - m_1, t_2 - m_1, \dots, t_{k-1} - m_1, , t_k - m_1\|_k \le \lim_{n\to\infty} \|t_1 - m_n, t_2 - m_n, \dots, t_{k-1} - m_n, , t_k - m_n\|_k$$

$$\Rightarrow \|t_1 - m_1, t_2 - m_1, \dots, t_{k-1} - m_1, , t_k - m_1\|_k \le \|\lim_{n\to\infty}(t_1 - m_n), \lim_{n\to\infty}(t_2 - m_n), \dots, \lim_{n\to\infty}(t_{k-1} - m_n), \lim_{n\to\infty}(t_k - m_n)\|_k$$

We have $\lim_{n\to\infty} m_n = m_1$

Implies

⇒ m_1 best simultaneous approximation to $t_1, t_2, \dots, t_k$ in $\check{E}$.

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The effect of wastewater effluent on the water quality of the Tigris River passing through the city of Tikrit

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Abstract : in this study, researchers studied the chemical and physical Properties of the water from the Tigris River in Tikrit, Iraq. Through the period of the research's observational time (November 2021–October 2022) , the water's average temperature ranged from 48.0 to 13.0 **C**°, the water's pH ranged from 8.8 to 7.1, and the rate of total dissolved solids ranged from275 to 202 mg/ l .. The values of the ranged from 4.2-1.2 mg Biological oxygen demand / and the copper rate was NIL for all months. Tikrit city, Iraq; the Tigris River; water quality

Introduction: The hydrogen bond and polarity of water give it unique physical and chemical qualities that make it a good fit for living systems, The entire nature of the existence of the organisms in water is changed by changes in these attributes, since One of the most important factors in determining how water is put to different uses is its physical and chemical properties, as these are directly related to the environment in which they are found.(Balami et al.,2019), Water has distinctive physical and chemical properties, which made it important in uses for various purposes, and the organisms in it show high sensitivity in the change in its physical and chemical characteristics such as pH or the vital requirement for oxygen or others, these factors affect the nature of life for the organisms that live in it (Khalil et al.,2022) The group's primary goal was to investigate the chemical and physical characteristics of the Tigris River at Tikrit. (Sohrabi et al.,2021)

Objective of the study:

1- Studying some physical and chemical properties of the water of the Tigris River in different locations of the city of Tikrit to know the impact of existing pollutants.

Study stations (sampling stations) Studying Stations :

The first station (St. 1) - (control sample)Located on the Tigris River just downstream by 150 meters the nature of the rocky bedrock bottom.

Second Station (St. 2): At the downstream area, this plant was sampled from untreated contaminated water.

Third Station (St. 3) : Located 25 meters from the downstream area.

Fourth Station (St. 4) : It is located 75 meters from the downstream area.

Fifth station (St.5): Located 150 meters from the downstream area.

Fourth Station (St. 6) : Located 300 meters from the downstream area

Materials and Methods: Study Area : The study area, which spans about 250 kilometers and consists of a wide variety of natural composition and geomorphology, is situated in the north of Iraqi province of Salah al-Din. This region is bounded north by 33.45° and 35.20°, east by 42° and 45.10', and south by the Tigris river. The Tigris, an important river in Iraq, begins in Turkey and enters Iraq proper. The total distance from the river's source to its final discharge point at gulfis 1,718 kilometers. Through Iraq, the river travels for around 1418 km. As far as river lengths go, it's the 39th longest in the world.

Five sites along the Tigris River were sampled throughout the study period (November 2021–October 2022).

A- C- Evidence gleaned from physical and chemical analysis: Ambient and water temperatures were recorded using mercury thermometers (48.0 to 13.0 C) .

B- 1-Total Dissolved Solid (T.D.S): The measurement was done using the Tester Type T.D.S Meter electronic device.

C- 2-pH: Use an Oyster pH meter

3-Oxygen Concentration (DO) Dissolved Oxygen : Dissolved oxygen was evaluated using the method established by whinkler, 1882 (APHA 1998) and given in mg/ l

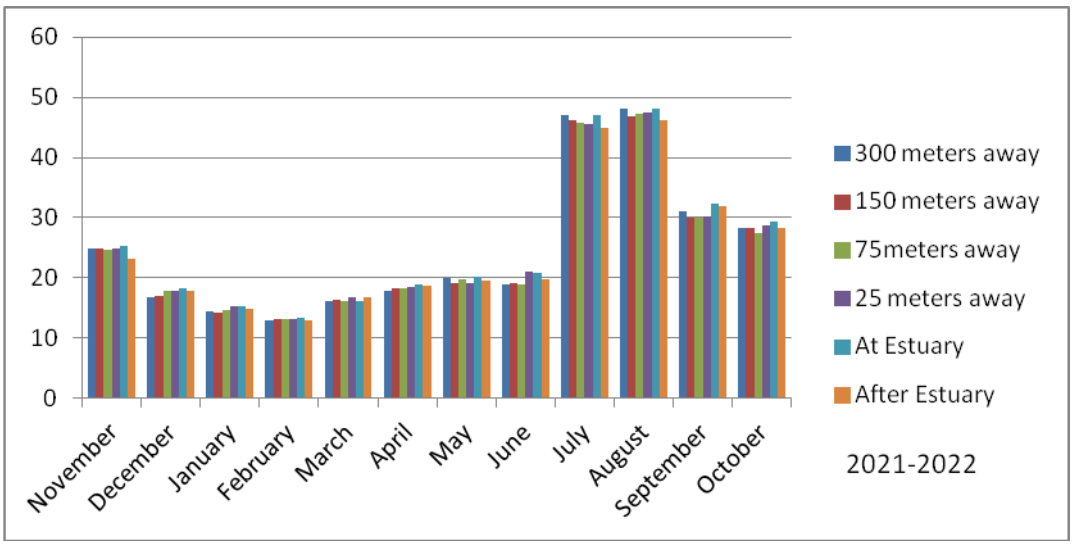
4-Biological Oxygen Demand (BOD): by (APHA,1985)

5-Copper element: - Atomic absorption spectroscopy was used to determine the concentration of copper in the sample. (APHA,2017),

E-Statistical analysis: Anova used statistical data analysis to study the effect of the months of the year on quantitativeproperties.

Results and discussion:

The results of the current study confirmed the existence of discernible monthly differences in air temperatures, which is to be expected given the climatic characteristics of Iraq, which include seasonal and diurnal temperature differences. At the first station before the estuary, the research period's lowest temperature was 13.0 **C**° in November2021 while the study period's greatest temperature was 48.0 degrees Celsius in August of 2022. Despite the fact that there were no geographically significant changes in water temperature amongst the sites, there were temporal variances. the site at a significance level of p≤0.05 (Figure1).



Figure(1):Monthly and locational changes of water **temperature (Celsius) values in the studied site**

Dissolved solids were measured, and the findings indicated a range of 275 to 202 mg/L, with the maximum value recorded in The lowest value was recorded in October at the (after downstream 25 meters) while the highest value was recorded in February at the second plant (downstream). A significant concentration of water-soluble salts was found in this research, which is unacceptable according to the Iraqi Central Standards for Standardization

and Quality Control,It had been decided statistically (P 0.05) that there was both geographical and temporal (Figure2).

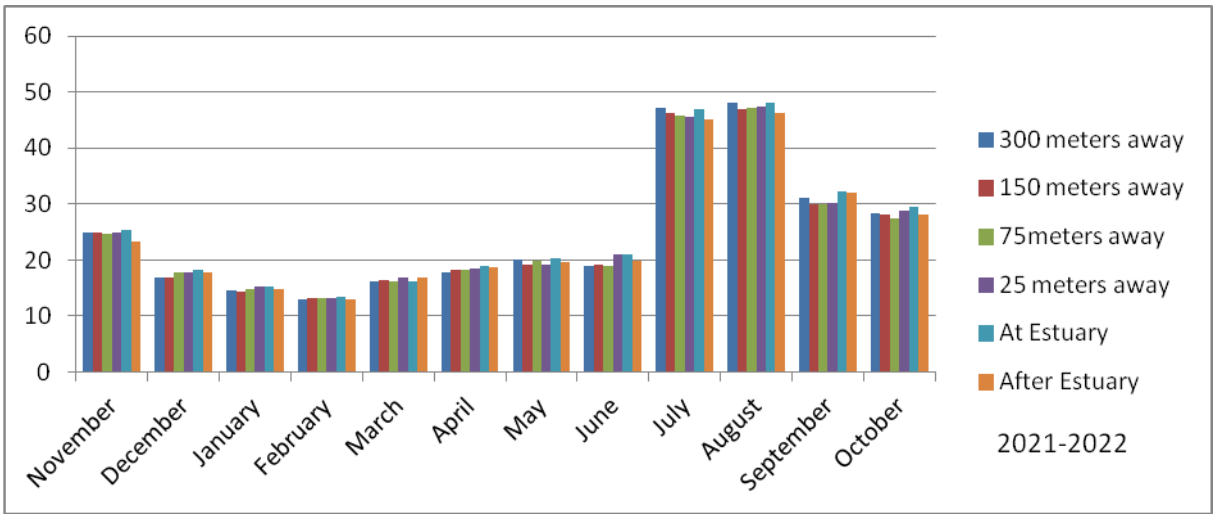


Figure (2): The spatial and temporal variation of Total Dissolved Solids mg/L

The results of the current study showed that pH values ranged between From 8.8 in November at the sixth station (300 meters downstream) to 7.1 in April at the fifth station (150 meters downstream), with the decrease in summer and the increase in winter possibly attributable to lower water levels and increased rainfall, the current study found that the pH ranged from 8.8 to 6.5 Drinking Water Safety According to the Iraqi Drinking Water Specification No. Iraqi water is close to neutrality due to the high cability of Iraqi waters on the acidity equation due to the high concentrations of bicarbonate and carbonate ions that behave in a disorderly manner. and, simultaneously, the World Health Organization, no statistically significant discrepancies were found in the results (WHO, 2004) (Figure3).

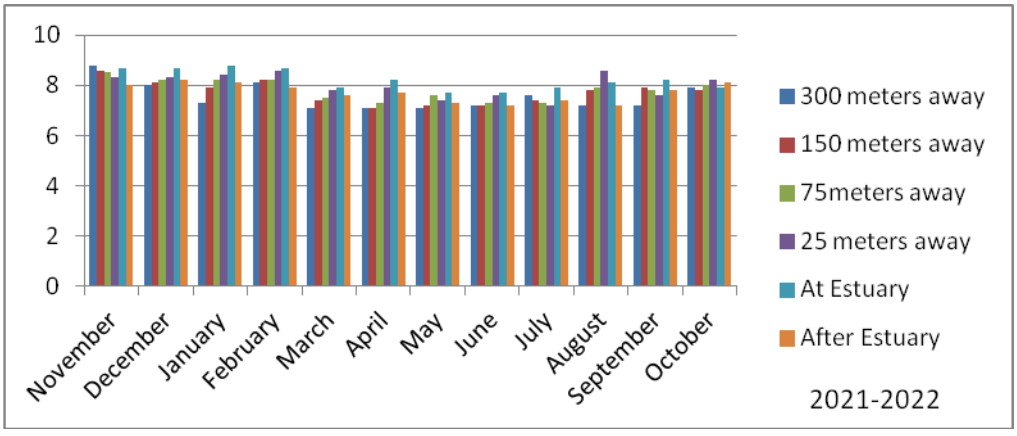


Figure (3) Temporal and spatial pH variation

of dissolved oxygen were found to be higher in the winter and lower in the summer The reason is because the decrease in temperature increases the solubility of oxygen in running water. The second plant (downstream) had dissolved oxygen levels of 8.0 in January

2022 and 5.0 in August 2022, both of which were within the range of the Iraqi water quality guidelines set by the Central Organization for Standardization and Quality Control in 1996 (CEOH, 2003; United States - Environmental Protection Agency, 2003; World HealthOrganization, **1999),** (Figure 4).

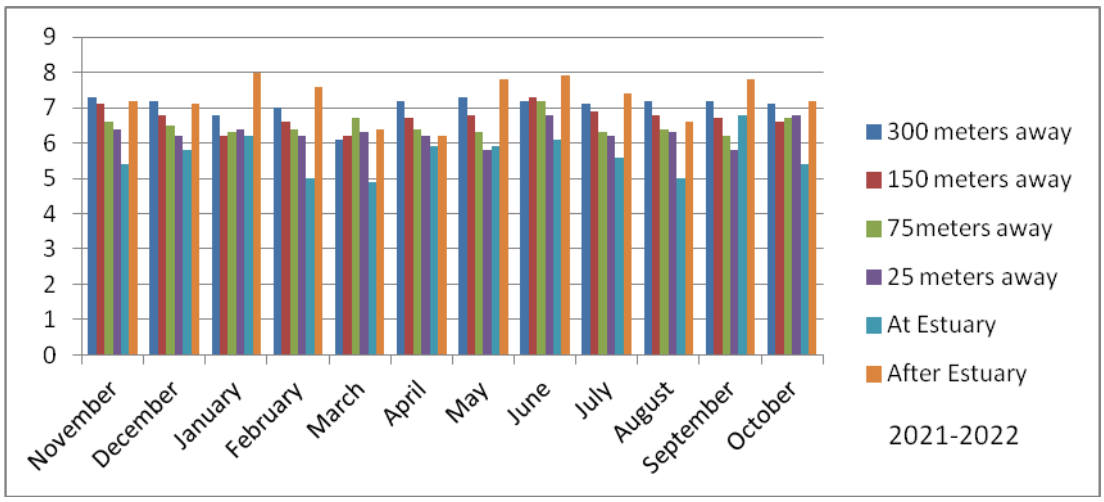


Figure (4) Changes in dissolved oxygen concentrations throughout time and space

For BOD5, also known as the biological oxygen demand, the statistical analysis revealed significant spatial differences and the absence of significant differences conforming to the standards of the Iraqi Central Organization for Standardization and Quality Control. The highest value of 4.2 mg/L was recorded in January 2022 at the second station, and the lowest value of 1.2 mg/L was recorded in March 2022 at the same station (Figure5).

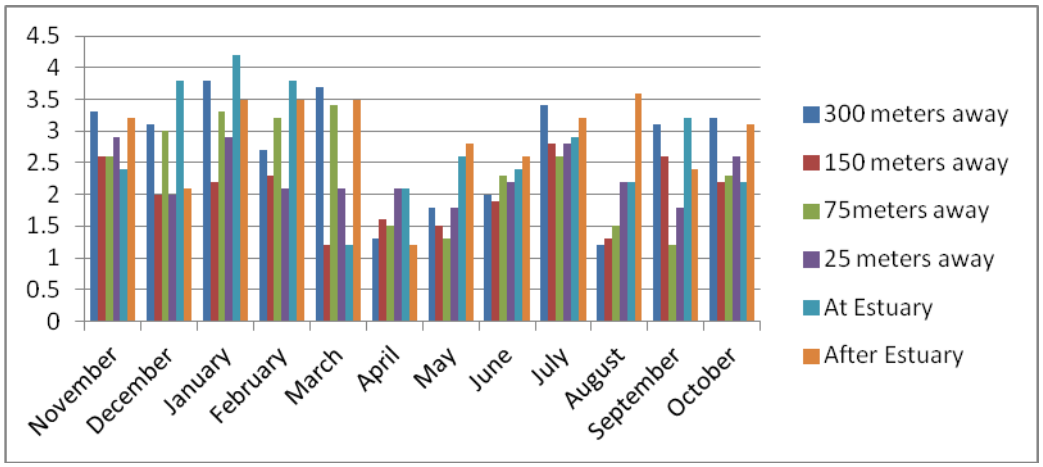


Figure (5) The geographical and temporal variation of the BOD5

Copper : The current study found no detectable value for Copper in the Tigris River's waters at any of the monitored stations.

Table (1): Copper in waters of stations in Tigris River under study

Months	**Site**

	After Estuary	At Estuary	25 meters away	75meters away	150 meters away	300 meters away
November	-	-	-	-	-	-
December	-	-	-	-	-	-
January	-	-	-	-	-	-
February	-	-	-	-	-	-
March	-	-	-	-	-	-
April	-	-	-	-	-	-
May	-	-	-	-	-	-
June	-	-	-	-	-	-
July	-	-	-	-	-	-
August	-	-	-	-	-	-
September	-	-	-	-	-	-
October	-	-	-	-	-	-

- Is Nil

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تحضير وتشخيص بعض مشتقات البريدازين والفثالازين-4,1-دايون وتقييم فعاليتها الحيوية لبعض أنواع البكتريا

Synthesis and characterization of some derivatives of pyridazine and phthalazine-1,4-dione and evaluation of their biological activity for some types of bacteria.

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الملخص

تم في هذه الدراسة تحضير معوضات البريدازين والفثالازين-4,1-دايون عن طريق تفاعل بعض أنواع الاوكسينات (حوامض كاربوكسيلية) والتي تعد من المركبات المهمة كمنظمات نمو في النباتات حيث تم تحويلها الى استرات، ثم مفاعلتها مع الهيدرازين المائي للحصول على هيدرازايد الحامض الذي بدوره يتفاعل مع انهدريد الماليك والفثاليك للوصول الى المركب الهدف، شُخصت المركبات المحضرة بالطرق الفيزيائية البسيطة كاللون ومقدار الاختلاف في درجة الانصهار، كما تم تشخيصها بالطرق الطيفية بواسطة مطيافية FT-IR و ^{1}H-NMR، ثم تقييم فعاليتها البيولوجية ضد بكتريا (Pseudomonas aeruginosa) و (Staphylococcus aureus) حيث أظهر قسم منها تثبيط بشكل كبير واقسم الاخر أظهر تأثيرا متوسطا الى ضعيف نوعا ما.

الكلمات المفتاحية: أوكسين، فثالازين، Pseudomonas aeruginosa

Summary

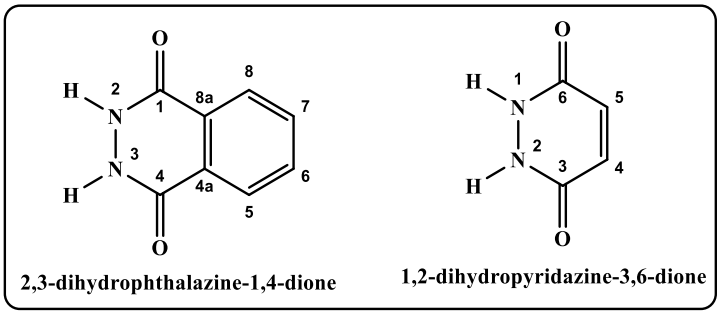
In this study, substitutes for pyridazine and phthalazine-1,4-dione were prepared by reacting some types of auxins (carboxylic acids), which are important compounds as growth regulators in plants, where they were converted into esters, and then reacted with aqueous hydrazine to obtain acid hydrazide, which in turn It reacts with maleic and phthalic anhydrides to reach the target compound. The prepared compounds were identified by simple physical methods such as color and the amount of difference in melting point, and they were also identified by spectroscopic methods by FT-IR and 1H-NMR spectroscopy, and then evaluated their biological activity against bacteria (Pseudomonas aeruginosa) and (

Staphylococcus aureus), where a section of them showed a great inhibition, and the other section showed a moderate to somewhat weak effect.

Keywords: Auxin, phthalazine, Pseudomonas aeruginosa.

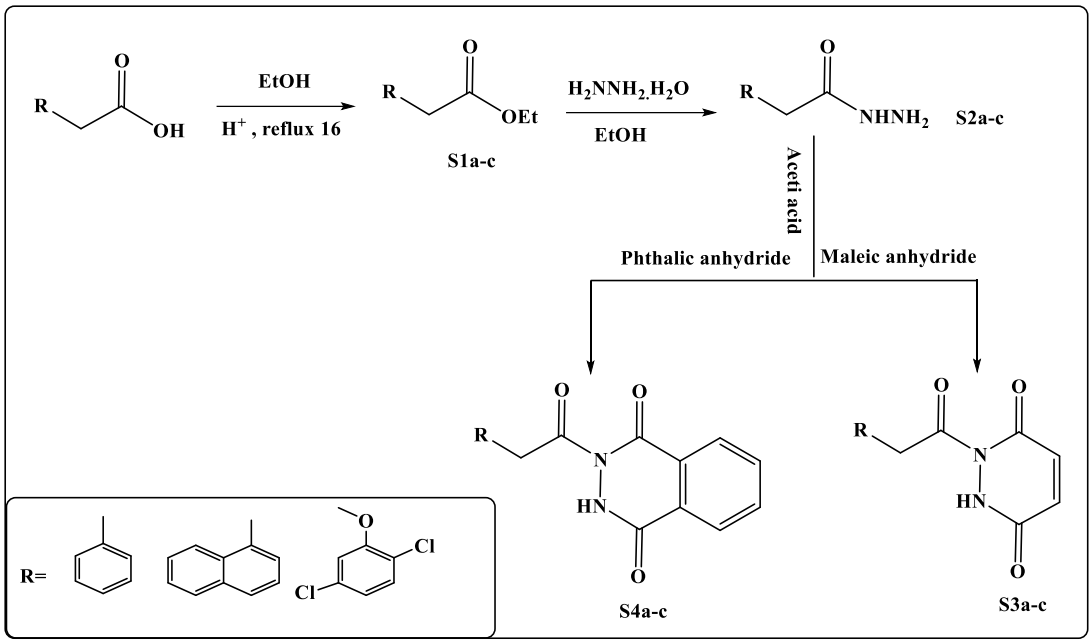
1.المقدمة:

تعتبر مركبات البريدازين والفثالازين من المركبات الحلقية غير المتجانسة سداسية الحلقة حيث تحتوي على ذرتي نيتروجين في الموقعين (2،1) ومجموعتي كاربونيل في الموقعين (6،3) بالنسبة للبريدازين، أما الفثالازين فيكون موقع ذرتي النيتروجين في (3،2) ومجموعتي الكاربونيل في الموقعين (4،1) وتكون ملتحمة مع حلقة بنزين، وتعتبر هذه المركبات مشتقات لحامض الماليك والفثاليك.[1]



إن لهذه المركبات خصائص دوائية واسعة، حيث ان لنواة الفثالازين المجردة (بدون تعويض) استجابة دوائية وتداخل مع البنزودايزيبين (Benzodiazepine) المرتبط مع مستقبلات (GABA-A) وهو المركب المثبط الرئيسي في الجهاز العصبي،[2] كذلك تعمل بعض مشتقات الفثالازين كعلاجات منوّمة ومهدئة للأعصاب[3]، وتوصف ايضاً كونها مضادات للقلق والتشنج العضلي،[4] كما أن لها نشاط كبير في عملية منع تخثر الدم[5]، خفض ضغط الدم،[6] علاج بعض أنواع مرض السكر،[7] والانسداد الرئوي المزمن.[8].

2. التحضير وطرائق العمل.



Scheme 1

2-1: تحضير الاستر: [9]

أُذيب (0.015 mol) من الحامض الكاربوكسيلي في 30mL من الايثانول واضيف اليه 3mL من حامض الكبريتيك المركز تدريجياً على شكل قطرات، صُعّد مزيج التفاعل لمدة 14 ساعة، ثم تُرك ليبرد بدرجة حرارة الغرفة واضيف اليه محلول بارد من 10% بيكاربونات الصوديوم لمعادلة الوسط لحين ظهور الراسب (ملاحظة: ان الاستر النتج من الحامض 2,4-D لم يعطي راسب مباشرة بعد المعادلة إلا بعد استخلاصه بمذيب ثنائي اثيل ايثر). بعدها تم ترشيح وتجفيف الراسب واعيدت بلورته من الإيثانول المطلق.

جدول (1) الخواص الفيزيائية والنسب المئوية للاسترات المحضرة (S1a –c)

Comp. No.	R	Molecular formula	Molecular weight	Color	M.P(^{0}C)	Yield (%)
S_{1a}	phenyl	$C_{10}H_{12}O_2$	164.20	Light yellow	64-66	76
S_{1b}	Naphthalene-1-yl	$C_{14}H_{14}O_2$	214.26	Pink	96-98	78
S_{1c}	2,4-dichloro phenoxy	$C_{10}H_{10}O_3Cl_2$	249.09	Yellow	102-104	60

2-2: تحضير Acid hydrazide: [10]

تمت اذابة (0.015 mol) من الاستر في 25mL من الهيدرازين المائي 80% ثم حُرك المزيج لمدة ربع ساعة بدرجة حرارة الغرفة، ثم أُضيف اليه 10mL ميثانول، صُعّد مزيج التفاعل لمدة 8 ساعات، ثُرك ليبرد قليلا ثم

سكب المزيج على مجروش الثلج حيث تكون راسب اصفر، تم ترشيحه وغسله بالماء المقطر البارد، جُفف الراسب وأعديت بلورته من الايثانول. والجدول أدناه يبين بعض الصفات الفيزيائية للمركبات المحضرة.

جدول (2) الخواص الفيزيائية والنسب المئوية لمشتقات الهيدرازايد المحضرة (S2a –c)

Comp. No.	R	Molecular formula	Molecular weight	Color	M.P(^{0}C)	Yield (%)
S_{2a}	phenyl	$C_8H_{10}N_2O$	150.18	Pale yellow	112-114	71
S_{2b}	Naphthalene-1-yl	$C_{12}H_{12}N_2O$	200.24	Faint yellow	122-124	77
S_{2c}	2,4-dichloro phenoxy	$C_8H_8N_2O_2Cl_2$	235.07	Grey	131-133	68

3-2: تحضير معوضات بريدازين وفثالازين 4,1- دايون:[11]

مُزج (0.003mol) من احد مشتقات هيدرازايد الحامض مع (0.003mol) من انهدريد الماليك أو الفثاليك وأذيب المزيج في 20mL من حامض الخليك مع التحريك لمدة 10 دقائق لأجل تجانس المزيج ثم صُعّد مزيج التفاعل لمدة 6 ساعات، تم تبريد المحلول بواسطة مجروش الثلج، رُشح الراسب وجُفف وتمت إعادة بلورته من ثنائي كلورو ميثان. والجدول أدناه يبين بعض الثوابت الفيزيائية والنسب المئوية للمركبات المحضرة.

جدول (3) الخواص الفيزيائية والنسب المئوية لمشتقات البريدازين والفثالازين (S3a-c)(S4a –c)

No.		R	Molecular formula	Molecular weight	Color	M.P(^{0}C)	Yield (%)
S_{3a}	Malic	Phenyl	$C_{12}H_{10}N_2O_3$	230.22	yellow	85-87	76
S_{3b}		Naphthalene-1-yl	$C_{16}H_{12}N_2O_3$	280.28	Wight	75-77	78
S_{3c}		2,4-dichloro phenoxy	$C_{12}H_8N_2O_4Cl_2$	315.11	Wight	144-146	60
S_{4a}	Phthalic	Phenyl	$C_{16}H_{12}N_2O_3$	280.28	Yellow	85-87	68
S_{4b}		Naphthalene-1-yl	$C_{20}H_{14}N_2O_3$	330.34	Brown	75-77	73
S_{4c}		2,4-dichloro phenoxy	$C_{16}H_{10}N_2O_4Cl_2$	365.17	Purple	144-146	70

4. النتائج والمناقشة:

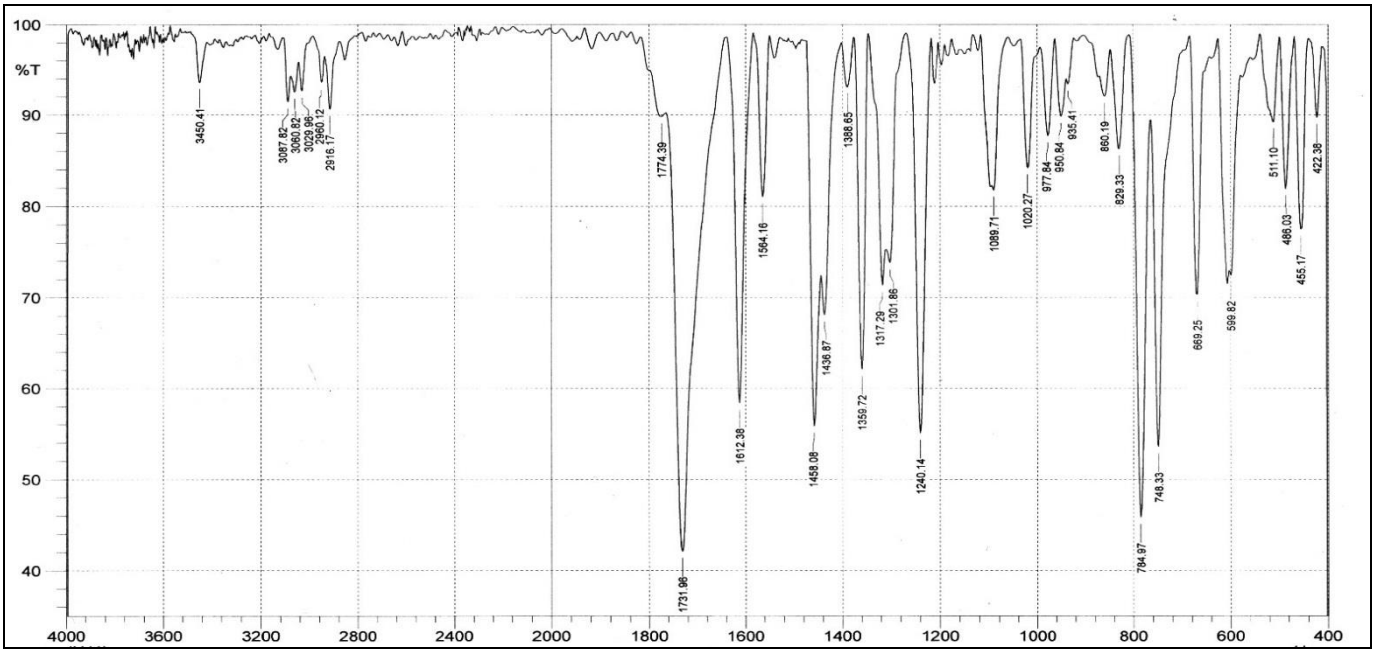
4-1: تشخيص الاسترات:

شُخصت مركبات الاستر الناتجة بواسطة الطرق البسيطة كالانخفاض الواضح في درجة الانصهار عن درجة انصهار الحامض الكاربوكسيلي المحضر منه الاستر، كما تم تشخيصها باستخدام مطيافية الاشعة

الحمراء حيث تبين من الشكل (1) للمركب (S_{1b}) اختفاء حزمة المط العريضة للحامض الكاربوكسيلي (COOH)عند المدى cm^{-1}(υ=3350-2860) دليل على تكون الاستر، كذلك ظهور حزمة امتصاص حادة وقوية عند االتردد cm^{-1}(υ=1731) تعود لاهتزاز المط للكاربونيل الاسترية، أما حزمة الامتصاص الظاهرة عند المدى cm^{-1}(υ=1564 -1458) فهي تعود لاهتزاز الحزم الهيكلية لمجموعة (C=C) الأروماتية. والجدول التالي يوضح قيم الامتصاص للمركبات المحضرة

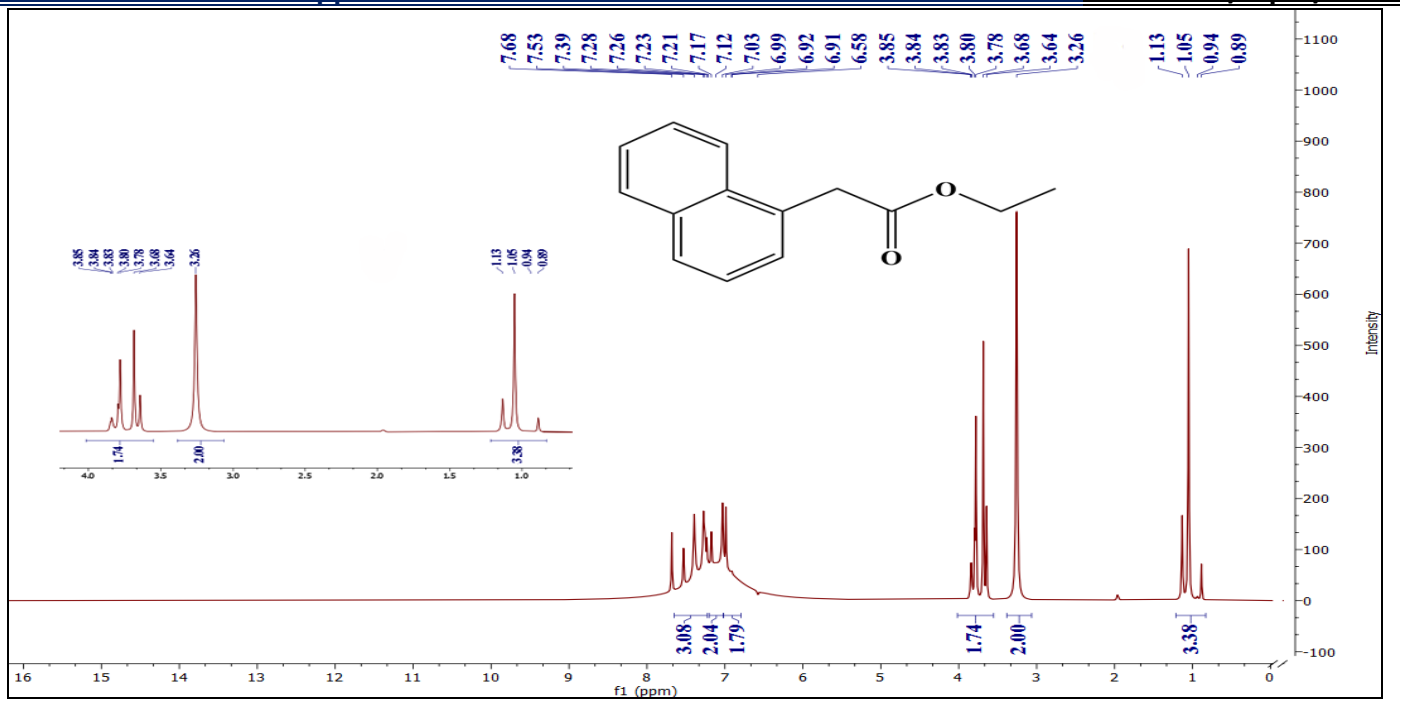
جدول (4) يوضح قيم الامتصاص للمركبين (S1a ,S1c)

No.	R	υ(C-H) aliphatic	υ(C-H) aromatic	υ(C=O)	υ(C=C)	Other
S_{1a}	phenyl	2933	3080	1728	,1475 1552	———
S_{1c}	2,4-dichloro phenoxy	2927	3060	1740	1589 ,1487	870 , 810 (Cl)



شكل (1) طيف الأشعة تحت الحمراء للمركب(S1b)

كذلك تم التأكد من صحة تكون المركب (S_{1b}) من خلال طيف الرنين النووي المغناطيسي ^{1}H-NMR كما في الشكل (2) والذي أظهر إشارة ثلاثية عند الازاحة الكيميائية [(t,3H), ppm (δ=0.89-1.13)] بانشطارات واضحة تعود لمجموعة المثيل كما مبين في الصورة المكبرة ضمن الطيف، كما ظهرت إشارة أحادية عند الازاحة الكيميائية [(s,2H)ppm, (δ=3.26)] والتي تعود لمجموعة المثلين الواقعة بين مجموعة الكاربونيل وحلقة الفنيل، أما الإشارة الرباعية المميزة عند الازاحة الكيميائية [(q,2H)ppm,(δ=3.64-3.85)] فهي تعود لمجموعة المثلين (-O-CH_2-CH_3) الواقعة بين مجموعة المثيل وذرة الاوكسجين ذات التأثير الحثي الساحب مما سبب ازاحتها في هذا الموقع.[12] بينما ظهرت بروتونات حلقتي الفنيل لنواة النفثالين في الازاحة الكيميائية عند المدى [(m, 7H)ppm, (δ=6.58-7.68)].



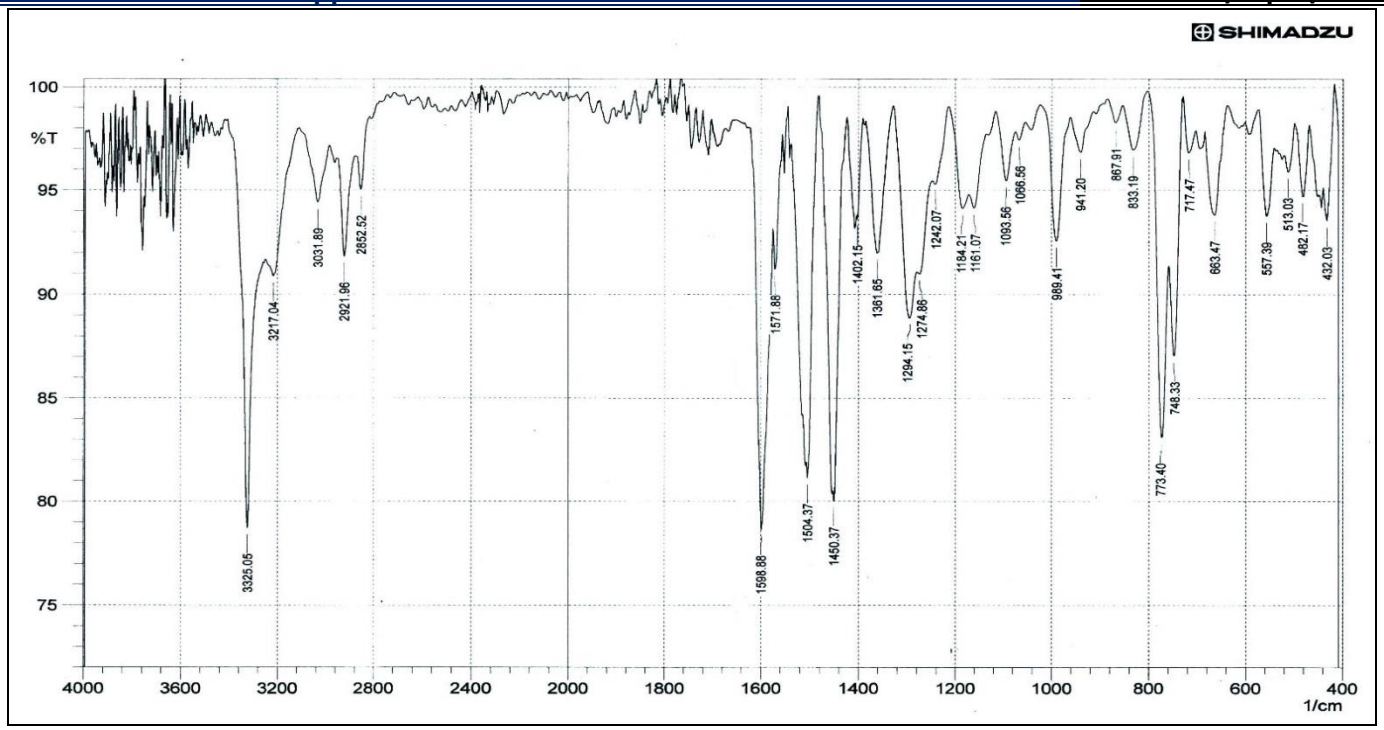
شكل (2) طيف الرنين النووي المغناطيسي للمركب (S1b)

2-4: تشخيص هيدرازايد الحامض:

تم تشخيص المركبات الناتجة من خلال طيف الأشعة تحت الحمراء للمركب (S_{2b}) حيث أظهر الشكل (3) حزمة امتصاص واضحة عند المدى (υ=3325-3217)cm^{-1} تعود لاهتزازات المط المتجانس وغير المتجانس لمجموعة الأمين الاولي (-N-NH_2) كما ظهرت حزم امتصاص عند المدى (υ=3031)cm^{-1} و (υ=2921)cm^{-1} عائدة لاهتزازات المط للاصرة (C-H) الأروماتية والأليفاتية على التوالي، أما الحزمة الحادة الظاهرة عند التردد (υ=1598)cm^{-1} فمن المرجح انها تعود لمجموعة الكاربونيل للأمايد (N-CO)، كذلك ظهرت حزم امتصاص عند المدى (υ=1504-1450)cm^{-1} عائدة لاهتزازات الحزم الهيكلية للأصرة (C=C) الأروماتية. والجدول التالي يوضح بعض قيم الامتصاص للمركبات المحضرة.

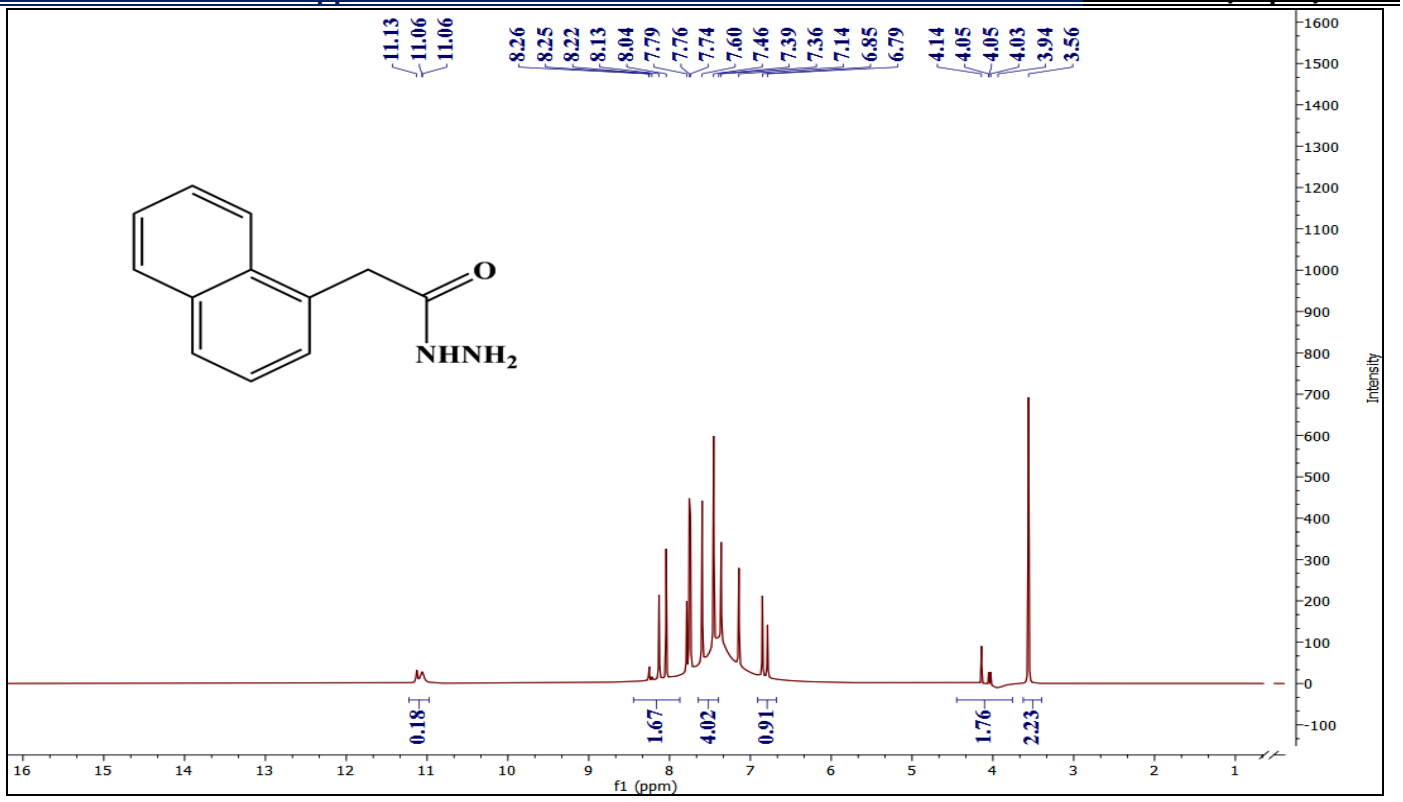
جدول (5) يوضح قيم الامتصاص للمركبين (S2a ,S2c)

No.	R	υ(-NH_2)	υ(C-H) aromatic	υ(C-H) aliphatic	υ(C=O)	υ(C=C)
S_{2a}	phenyl	3353,3298	3097	2901	1612	1478 ,1533
S_{2c}	2,4-dichloro phenoxy	3350,3288	3027	2887	1635	1531 ,1466



شكل (3) طيف الأشعة تحت الحمراء للمركب(S_{2b})

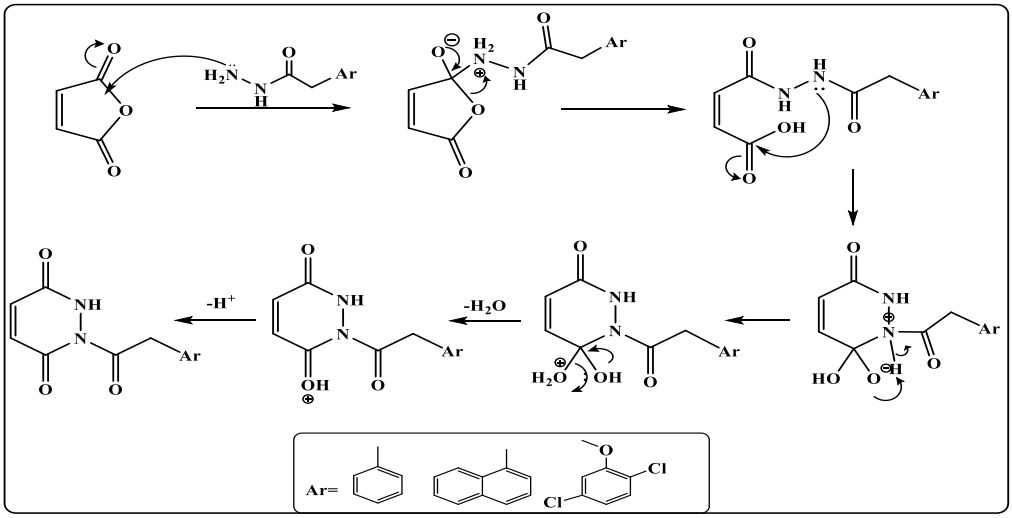
كذلك تم إثبات صحة المركب (S_{2b}) من خلال طيف الرنين النووي المغناطيسي ^{1}H-NMR الموضح في الشكل (4) فقد اظهر إشارة أحادية واضحة عند الازاحة الكيميائية [(δ=3.56) ppm ,(s,2H)] والتي تعود لبروتونات مجموعة المثلين، بينما الإشارة الأحادية الظاهرة عند الازاحة الكيميائية [ppm (δ=4.14) (s,2H),)] فهي عائدة لبروتونات مجموعة الأمين الاولي للهايدرازيد ($NH-NH_2$)،[13] كما ظهرت إشارة متعددة عند الازاحة الكيميائية [(m,7H), ppm (δ=6.79-8.26)] والتي تعود لبروتونات حلقتي البنزين الملتحمتين لنواة النفثالين، بينما بروتون مجموعة الأمين الثانوي فقد ظهر بإشارة واضحة عند الازاحة الكيميائية [(s,1H), ppm (δ=11.06)].



شكل (4) طيف الرنين النووي المغناطيسي للمركب (S_{2b})

3-4: تشخيص مشتقات البريدازين والفثالازين:

حُضرت هذه المركبات من تفاعل هيدرازايد الحامض مع أنهدريد الماليك والفثاليك، حيث تضمن التفاعل هجوم نيوكليوفيلي من قبل مجموعة الأمين الأولي على ذرة كاربون مجموعة الكاربنويل في حلقة أنهدريد الماليك تبعتها عدة خطوات تضمنت حذف جزيئة ماء وصولا للحلقة السداسية كما في الميكانيكية المقترحة التالية[14].



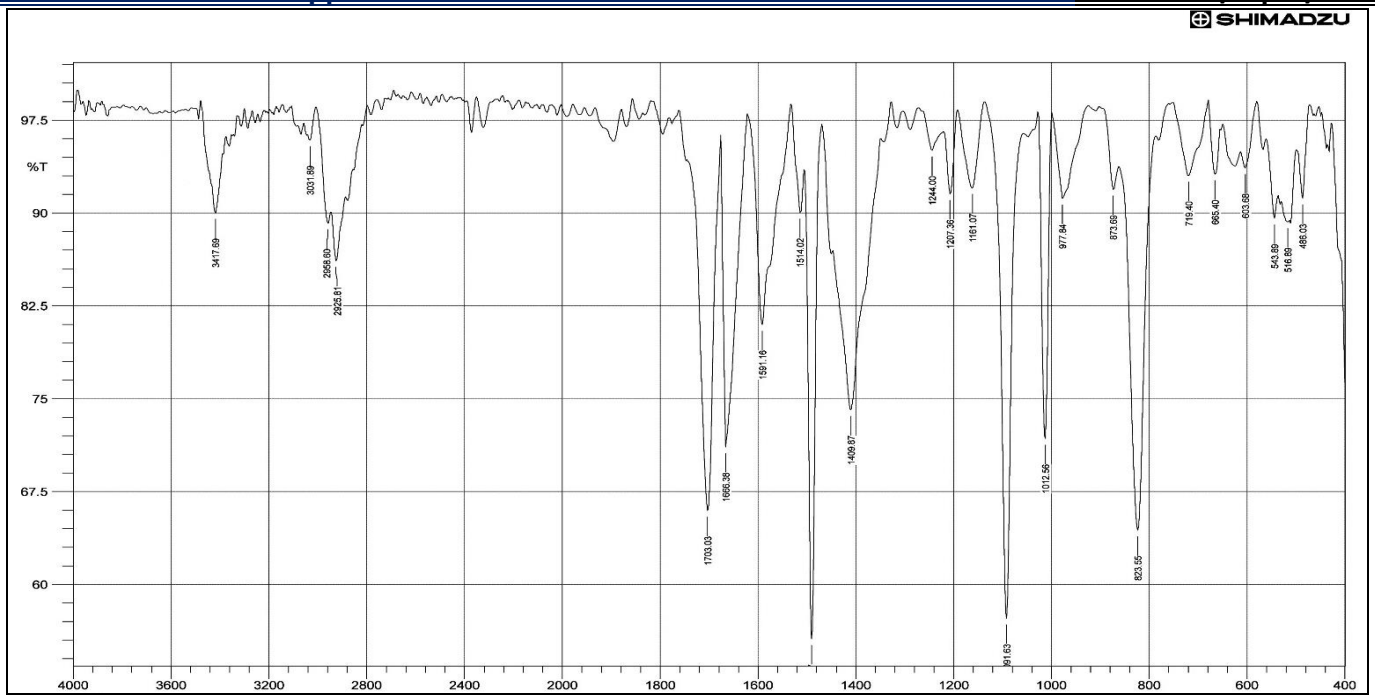
شُخصت هذه المركبات بواسطة الطرق الطيفية حيث أظهر طيف الاشعة تحت الحمراء كما في الشكل (5) للمركب (S_{3b}) حزمة مط عند التردد (υ=3417)cm^{-1} تعود لاهتزاز المط لمجموعة الأمين الثانوي لحلقة البريدازين (N-NH-C) ، كما ظهرت حزم امتصاص عند المدى (υ=3031)cm^{-1} و (υ=2925)cm^{-1} عائدة لاهتزازات المط للاصرة (C-H) الأروماتية والأليفاتية على التوالي، كذلك ظهور حزمتي امتصاص عند

التردد cm^{-1}(υ=1703) و cm^{-1}(υ=1666) يرجح انها تعود لمجموعتي الكاربونيل المتناظرة لحلقة البريدازين، أما الحزمة الظاهرة عند التردد cm^{-1}(υ=1591) فهي تعود لاهتزاز المط لمجموعة الكاربونيل الامايدية (C-N-CO)، وظهور حزم امتصاص عند المدى cm^{-1}(υ=1514-1409) تعود لاهتزازات الحزم الهيكلية للاصرة (C=C) الأروماتية.

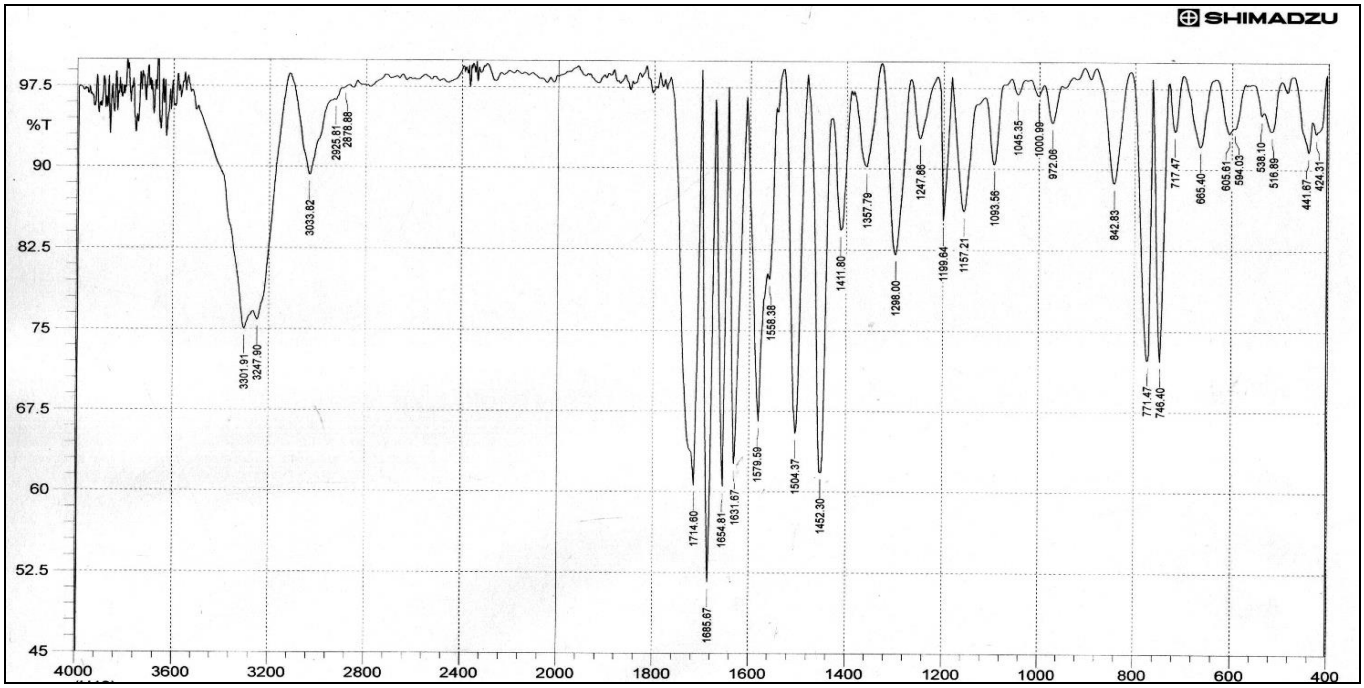
كذلك اظهر الشكل (6) في طيف الأشعة تحت الحمراء للمركب (S_{4c}) حزمة مط عند التردد (υ=3301-3274)cm^{-1} تعود لاهتزاز المط لمجموعة الأمين الثانوي لحلقة الفثالازين (C-NH-N) ، كما ظهرت حزم امتصاص عند المدى cm^{-1}(υ=3033) و cm^{-1}(υ=2925) عائدة لاهتزازات المط للاصرة (C-H) الأروماتية والأليفاتية على التوالي، كذلك ظهور حزمتي امتصاص عند التردد cm^{-1}(υ=1714) و cm^{-1}(υ=1685) عائدتان لمجموعتي الكاربونيل المتناظرة لحلقة الفثالازين، أما الحزمة الظاهرة عند التردد cm^{-1}(υ=1654) فهي تعود لاهتزاز المط لمجموعة الكاربونيل الامايدية (C-CO-N)، بينما ظهرت حزمة امتصاص واضحة عند التردد cm^{-1}(υ=1631) تعود لاهتزاز الاصرة (C=C) لحلقة الفثالازين (الكين حلقي)، كذلك ظهور حزم امتصاص عند المدى cm^{-1}(υ=1579-1452) تعود لاهتزازات الحزم الهيكلية للاصرة (C=C) الأروماتية.

جدول (6) يوضح قيم الامتصاص للمركبات (S_{2a} ,S_{2c})

No.		R	υ(-NH)	υ(C=O) cyclic	υ(C=O) Amide	υ(C=C) Alkene.	υ(C=C) Arom.
S_{3a}	**Malic**	Phenyl	3367	1713-1698	1660	1625	,1488 1540
S_{3c}		2,4-dichloro phenoxy	3294	1707-1701	1680	1633	,1471 1550
S_{4a}	**Phthalic**	Phenyl	3330	1720-1693	1630	1607	,1491 1520
S_{4b}		Naphthalene-1-yl	3301	1702-1698	1671	1650	,1466 1522



شكل (5) طيف الأشعة تحت الحمراء للمركب(S_{3b})

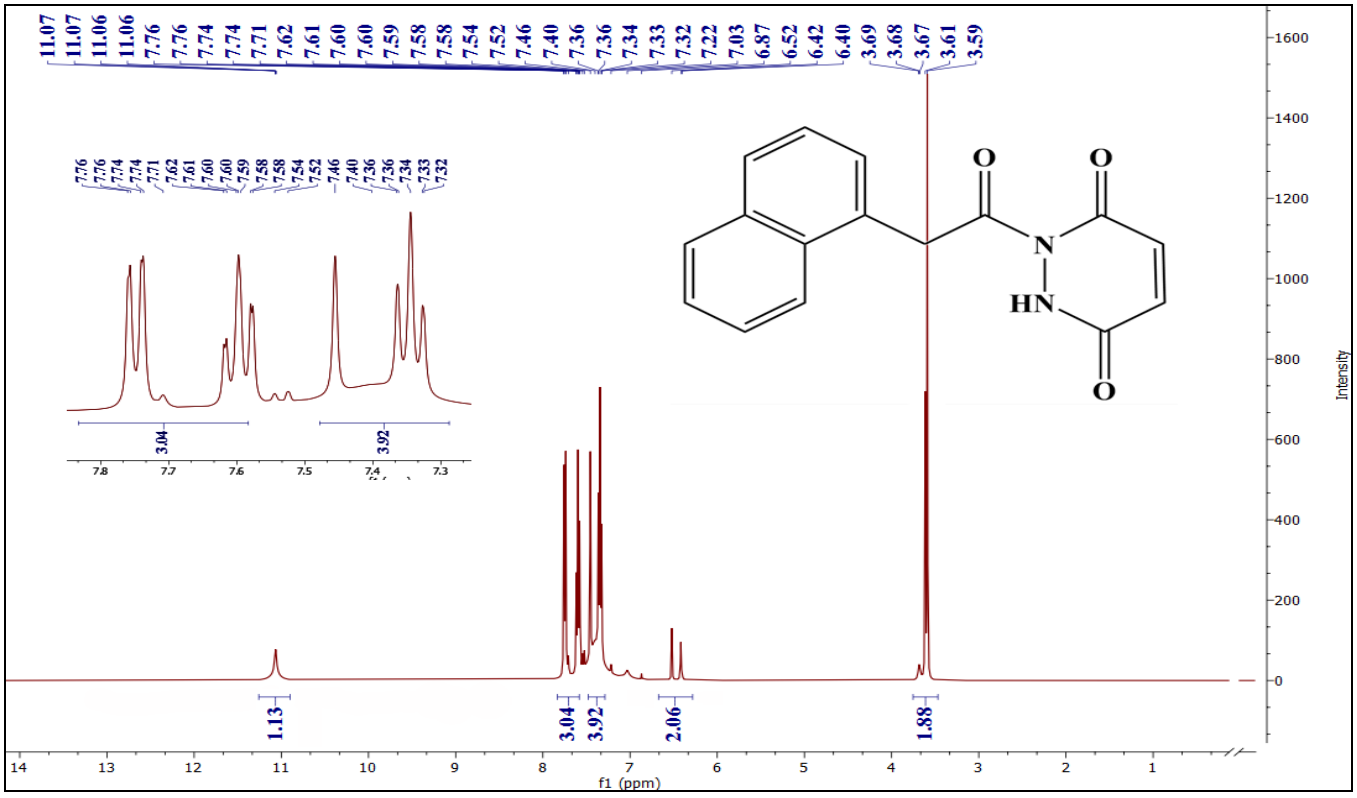


شكل (6) طيف الأشعة تحت الحمراء للمركب(S_{4c})

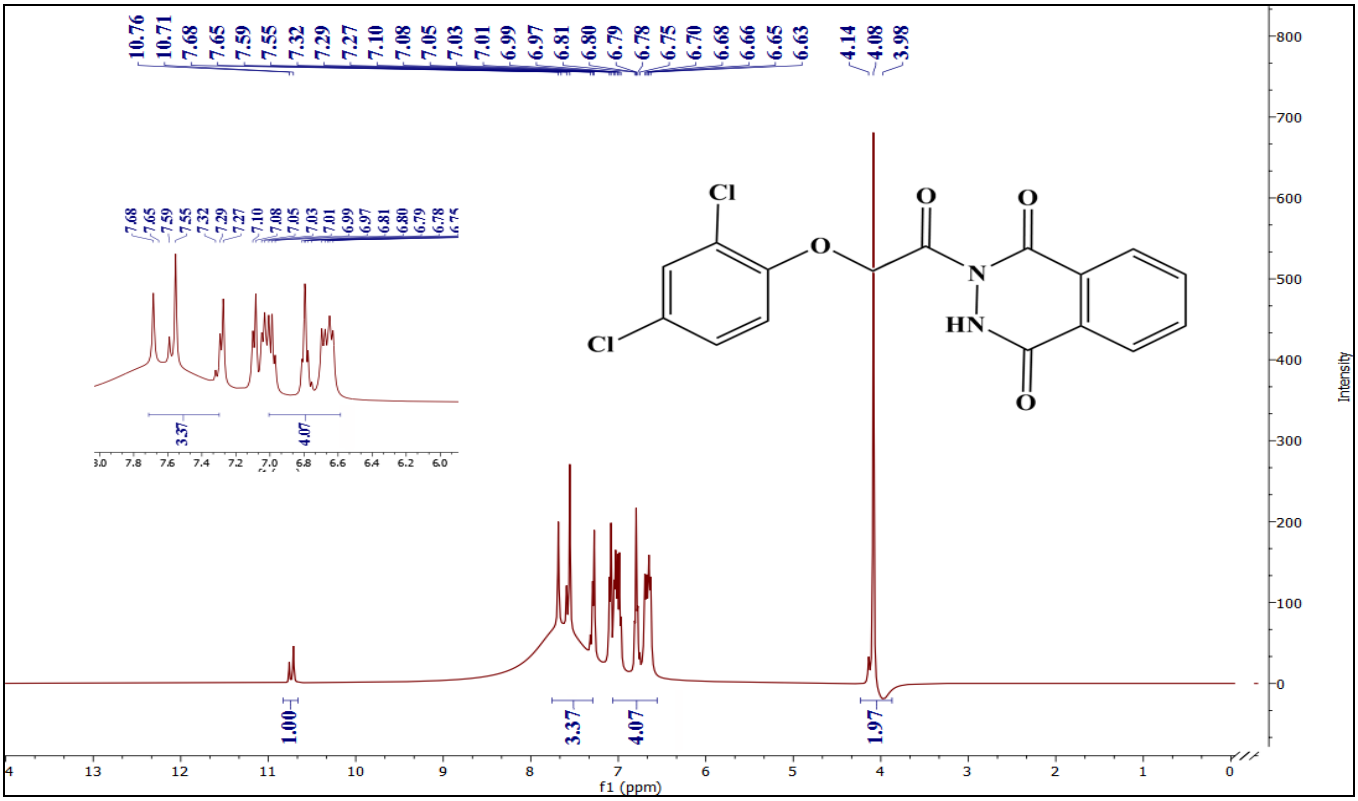
كذلك أظهر طيف الرنين النووي المغناطيسي ^{1}H-NMR إشارات واضحة تؤيد صحة تكوين المركب (S_{3b}) كما في الشكل (7) حيث ظهرت إشارة أحادية مميزة عند الازاحة الكيميائية [(δ=3.61)ppm,(s,2H)] تعود لمجموعة المثيلين الاليفاتية، كما ظهرت اشارتين منفصلتين عند المدى [(δ=6.40-6.42) ppm ,(s,2H)] من المرجح أنهما عائدتان الى بروتونات (-CH=CH-) لحلقة البريدازين، كذلك ظهرت إشارة متعددة وبانشطارات واضحة عند المدى [(δ=7.22-7.76) ppm,(d,4H,t,3H)] عائدة للحلقة الأروماتية الملتحمة، بينما ظهرت

إشارة أحادية عند الازاحة الكيميائية [(s,1H), ppm (δ=11.07)] تعود لبروتون مجموعة الأمين الثانوي لحلقة البريدازين.

أما المركب (S_{4c}) فقد أظهر إشارة أحادية عند الازاحة الكيميائية [(s,2H), ppm (δ=4.08)] تعود لمجموعة المثيلين الاليفاتية، أما الإشارات المتعددة عند المدى [(d,2H,t,2H), ppm (δ=6.63-7.10)] فهي تعود لبروتونات حلقة الفنيل الملتحمة مع حلقة الفثالازين كما هو واضح من الصورة المكبرة داخل الشكل (8) بينما الإشارات عند المدى [(s,1H,d2H), ppm (δ=7.27-7.68)] فهي عائدة لبروتونات حلقة الفنيل الأروماتية المعوضة بمجموعتي كلور وهذا هو السبب في ظهورها أبعد إزاحة نوعا من حلقة الفنيل الملتحمة في نواة الفثالازين، كما أن الإشارة الظاهرة عند الازاحة الكيميائية [(s,1H), ppm (δ=10.71)] تعود لبروتون مجموعة الأمين الثانوي في حلقة الفثالازين (-N-NH-CO).



شكل (7) طيف الرنين النووي المغناطيسي للمركب (S_{3b})



شكل (8) طيف الرنين النووي المغناطيسي للمركب (S_{4c})

4-4:الجزء البيولوجي:

تم اختبار الفعالية البيولوجية لعدد من المركبات المحضرة وذلك باتباع طريقة (Kirby-Bauer) والتي تدعى بطريقة الانتشار بالاقراص (Disk Diffusion Method)[15]، حيث تم تقييم الفعالية البيولوجية في هذه الدراسة ضد نوعين مختلفين من البكتريا هما:

1- الزائفة الزنجارية Pseudomonas aeruginosa.

2- المكورات العنقودية الذهبية Staphylococcus aureus.

جدول (7) الفعالية التثبيطية للمركبات النهائية المحضرة

No.	Conc. mg/ml	*Pseudomonas aeruginosa*	*Staphylococcus aureus*
S_{3a}	1×10^{-4}	+	-
	1×10^{-3}	+	+
	1×10^{-2}	++	+++

S_{3b}	1×10^{-4}	+	-
	1×10^{-3}	+	+
	1×10^{-2}	++	+
S_{3c}	1×10^{-4}	+++	+++
	1×10^{-3}	+++	+++
	1×10^{-2}	++	+++
S_{4a}	1×10^{-4}	+	+
	1×10^{-3}	++	++
	1×10^{-2}	+++	++
S_{4b}	1×10^{-4}	+	+
	1×10^{-3}	++	+
	1×10^{-2}	++	++
S_{4c}	1×10^{-4}	+++	+++
	1×10^{-3}	+++	+++
	1×10^{-2}	++	++

(-) = لا يوجد تثبيط **(+) = تثبيط بقطر 5 – 10 ملم**

(++) = تثبيط بقطر 15 – 20 ملم **(+++) = تثبيط بقطر 25 – 30 ملم**

تبين من خلال الجدول (7) أن أعلى نسبة تثبيط كانت للمركبين (S_{3c}) و (S_{4c}) ويعود السبب ربما أن الحامض المشتق منه هذين المركبين هو 2,4-dichlorophenoxyacetic acid ويرمز له اختصاراً (2,4-D) حيث يعتبر هذا المركب من المبيدات القوية خصوصا للبكتريا والفطريات بالأضافة الى كونه مبيد عشبي[16] أما بقية المركبات كانت نسبة تثبيطها متوسطة الى ضعيفة نوعاً ما.

Reference:

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Isolation and Identification of *Proteus Mirabilis* Bacteria From Different Pathological Specimen Using Random PCR Method

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Abstract

Background polymerase chain reaction (PCR) is a method used to amplify a specific region of DNA. . which are repeated many times to produce millions of copies of a specific DNA segment In a random PCR, the primers used in the reaction are not designed to bind specifically to a known target sequence, instead they are designed to bind to any region of the genome Random PCR is often used to create basic code of random DNA fragments which can be used as starting material for other techniques such as genomic sequencing, cloning or mutation. **Methodology** Fifty samples of *proteus mirabilis* were isolated from different sources of pathological conditions such as Urinary tract infection, middle ear infections, and wounds infection, after which they were cultured on their selective media and biochemical test were performed to confirm the purity of the samples .**Result** the efficiency of each primer and the differentiating ability of each primer. The efficiency of the OP A-17, OPA-19, OP A-20 primers, respectively, was 39.13, 34.7, 26.08, while the discriminating ability for them was 37.2, 37.2, 25.5. Therefore, through these results, it is clear The best primer is OP A19 and OP A 17, as this study agrees with the study where the results were close to our study. In the context of polymerase chain reaction (PCR), the efficiency of a primer refers to how well it amplifies the target DNA sequence >

Keywords: *proteus miribalis* , randam PCR

Introduction

Proteobacteria involve the use of molecular techniques, such as DNA sequencing and gene expression analysis, to study the genetic Character and functions of these bacteria at the molecular level.(1) Such studies have helped researchers to better understand the genes and pathways involved in important biological processes in Proteobacteria, such as nutrient uptake, metabolism, and virulence.(2) They have also helped to identify potential targets for the development of new antibiotics and other therapies against Proteobacteria infections.(3) In addition to their medical importance, Proteobacteria are also important model organisms for the study of evolution, as they have a large and diverse genome and a long evolutionary history.(4) Molecular genetic studies of Proteobacteria have provided insights into the evolution and adaptation of these bacteria to different environments and hosts.(5) 6) It is often used to generate a diverse set of DNA sequences for further analysis, such as DNA sequencing or gene expression analysis.(7) Proteus is a genus of gram-negative Proteobacteria that includes several species that are important human pathogens, such as Proteus mirabilis and Proteus vulgaris.(8) These bacteria are often associated with urinary tract infections, and they can also cause infections in other parts of the body, such as the respiratory tract and the skin.(9) It is possible to use random PCR to amplify and study DNA from Proteus bacteria.(10) For example, researchers may use random PCR to amplify and sequence DNA fragments from a Proteus bacterial sample to identify genes or pathways that are involved in important biological processes, such as virulence or antibiotic resistance.(11) Alternatively, they may use random PCR to generate a diverse set of DNA fragments for further analysis, such as gene expression analysis or DNA microarray analysis.

Methodology

Collection of specimen

Following the isolation of (50) specimens of *Proteus mirabilis* from various sources of pathological conditions such as urine, middle ear infections, and wounds, the specimen were cultured on their selection medi

DNA extraction of bacteria DNA extraction is the process of isolating DNA from a sample of cells or tissue. In the case of bacteria, DNA extraction is typically performed on a sample of bacterial cells.(12) There are several different methods that can be used to extract DNA from bacteria, including mechanical lysis, chemical lysis, and enzymatic lysisMechanical lysis involves physically breaking open the cells to release the DNA.(13) This can be done using methods such as grinding the cells with a pestle and mortar, or using a bead mill or sonicator to mechanically disrupt the cellsChemical lysis involves using a chemical agent to break open the cells and release the DNA.(14) Common chemicals used for this purpose include detergents, such as sodium dodecyl sulfate (SDS), or enzymes such as lysozyme Enzymatic lysis involves using enzymes, such as proteinases or nucleases, to break down the cell walls and release the DNA Once the cells have been lysed and the DNA has been released, it can be purified using techniques such as centrifugation, filtration, or precipitation to remove contaminants and obtain pure DNA.(15) The purified DNA can then be used for a variety of downstream applications, such as PCR, DNA sequencing, or genetic engineering**.**

Table 1 The list of RAPD primer

NO.	Primer	Sequence
1	OP A-17	5'-AGCCAGCGAA
2	OP A-19	(5'- CAAACGTCGG
3	OPA-20	(5'-GTTGCGATCC

Result and discussion

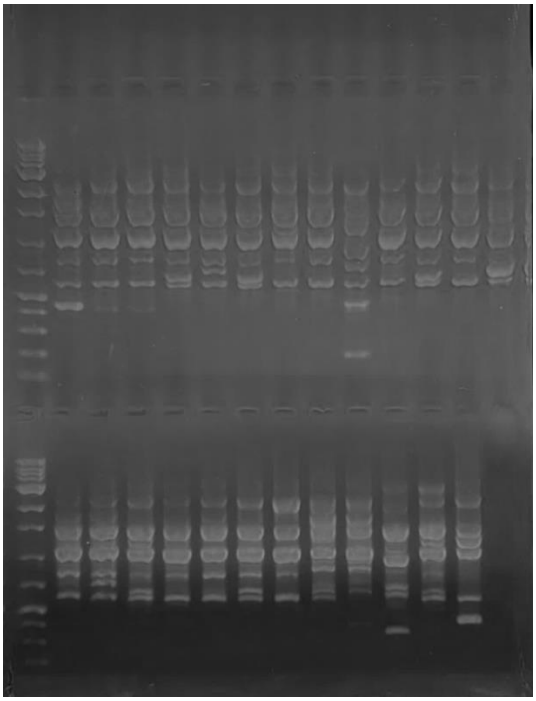

Figure 1 PCR product of primer OP A-17 The product was electrophoresis on 2% agarose at 5 V/cm^2. 1x TBE buffer for 90 min.

Primer OP A-17

For OP A-17 18 bundles were recorded and bundles sizes ranged between 200-3000 bp, featured by the presence of two public bundles 200 and 2000 present and 16 differential bundles; 350, 355 ,400, 425, 600, 625, 650, 700,750 , 800,850, 950, 1000 1300, 1500 and 3000 bp (Figure 1). This figure shows the number of the main bands that were recognized by the primers, where the total number of was 18, and the number of different bands that were recognized was 16. This indicates the efficiency of this primer and its discriminatory ability was high, and this is consistent with the study by [(3)] Polymerase chain reaction (PCR) is a laboratory technique that is used to amplify a specific DNA sequence. It is often used to analyze and identify specific genes or to detect the presence of certain bacteria, such as Proteus. In order to perform PCR on a sample that may contain Proteus, you would need to first obtain a sample of DNA from the sample. Then, you would need to set up the PCR reaction using specific primers (short pieces of DNA) that are designed to bind to the DNA sequence that you are interested in amplifying. You would also need to add enzymes, such as a polymerase, and nucleotides (the building blocks of DNA) to the reaction. The PCR reaction is then run through a series of

temperature cycles, which causes the primers to bind to the target DNA and the polymerase to synthesize new strands of DNA. The result is a large number of copies of the target DNA sequence, which can then be analyzed and identified.

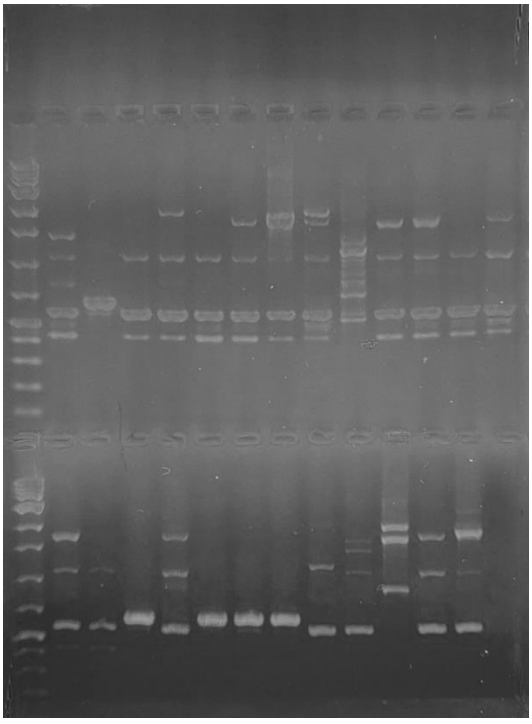

Figure 2 PCR product of primer OP A-19 The product was electrophoresis on 2% agarose at 5 V/cm^2. 1x TBE buffer for 90 min.

primer OP A-19

For OP A-19 primer, 16 bands sizes ranged between 350-2500 bp, all of its differential bundles; 350, 400, 500, 525, 600, 650, 700, 800, 900, 1000, 1100, 1200, 1450, 1550, 1750 and 2500 (Figure 2). In the context of polymerase chain reaction (PCR), primers are short pieces of DNA that are used to amplify a specific DNA sequence. Differential bundles of primers could refer to a set of primers that are designed to amplify different DNA sequences. These primers might be used in a PCR reaction to analyze and identify multiple different DNA sequences in a single sample, or to detect the presence of multiple different target organisms (such as bacteria or viruses) in a sample. Primers are typically used in pairs, with one primer binding to the DNA template at the beginning of the target sequence, and the other primer binding at the end of the target sequence.(5) The polymerase enzyme then synthesizes new strands of DNA between the two primers, amplifying the target sequence. Differential bundles of primers would include a set of primers that are specific to different target sequences.(3) These primers could be used in a single PCR reaction, or in multiple reactions to amplify multiple different sequences.

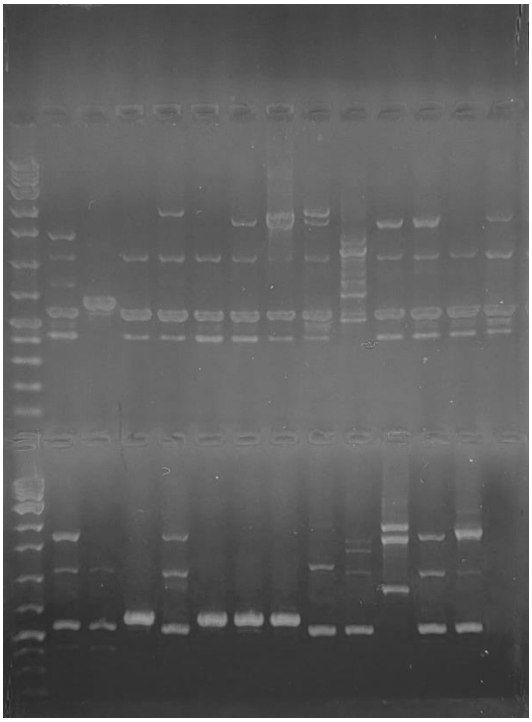

Figure 3 PCR product of primer OPA-20 The product was electrophoresis on 2% agarose at 5 V/cm^2. 1x TBE buffer for 90 min.

For OPA-20 primer, 12 bands sizes ranged between 250-3000 bp, featured by the presence of one public bundles 1200 bp present and 11 differential bundles; 250, 300, 400, 500, 600, 700, 750, 800, 1000, 1200 and 3000 bp (Figure.3). In the random method, each primer recognizes more than one gene piece

Table 2 The prefixes outputs from the total and differentiated bundles with their efficiency ratios and discriminatory ability for the studied samples

Primer	No. of bind	Differentiated Bundles	The Formal Plurality of each Prefix	Efficiency Ratios	Discriminatory Ability
OP A-17	18	16	88.88	39.13	37.2
OP A-19	16	16	100	34.7	37.2
OPA-20	12	11	91.66	26.08	25.5
Total	46	43			

Table 6 shows the efficiency of each primer and the differentiating ability of each primer. The efficiency of the OP A-17, OPA-19, OP A-20 primers, respectively, was 39.13, 34.7, 26.08, while the discriminating ability for them was 37.2, 37.2, 25.5. Therefore, through these results, it is clear The best primer is OP A19 and OP A 17, as this study agrees with the study [6], where the results were close to our study. In the context of polymerase chain reaction (PCR), the efficiency of a primer refers to how well it amplifies the target DNA

sequence. The efficiency of a primer is typically measured by the amount of PCR product that is produced from a given amount of template DNA.(4) A primer with high efficiency will produce a large amount of PCR product from a small amount of template DNA, while a primer with low efficiency will produce less PCR product from the same amount of template DNAThe efficiency ratio of a primer is a measure of how much more efficient it is compared to another primer.(7) For example, if a primer has an efficiency ratio of 2, it is twice as efficient as the other primer. A higher efficiency ratio indicates that the primer is more efficient at amplifying the target DNA sequence There are several factors that can affect the efficiency of a primer, including the length and sequence of the primer, the temperature of the PCR reaction, and the presence of secondary structure or other contaminants in the primer. Optimizing these factors can help improve the efficiency of a primer and increase the yield of PCR product

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The Relationships between $Q_\gamma(G), R_\gamma(G)$ and $K_\gamma(G)$ with Some Other Topological Indices with a Sum Graph of the Group Z_{pq^m}

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Abstract

Suppose G be a graph with the set of vertex V(G) and set of edge E(G), and let du P denote the vertex degree u ∈ V(G). The first general Zagreb index (or general Zeroth-order Randic index) Q_γ is defined as $Q_\gamma(G) = \sum_{i=1}^{k} d_i^\gamma = \sum_{i\sim j} \left(d_i^{\gamma-1} + d_j^{\gamma-1}\right)$, where k is the order of vertices and $\gamma \in \mathbb{R}$.The generalized connectivity index R_γ, is defined as $R_\gamma(G) = \sum_{i\sim j} \left(d_i d_j\right)^\gamma$.The general sum-connectivity index H_μ as $H_\mu(G) = \sum_{i\sim j}\left(d_i + d_j\right)^\mu, \mu \in \mathbb{R}$. .In this paper, we investigate the relationships between generalized $Q_\gamma(G), R_\gamma(G)\ and\ K_\gamma(G)$ and several topological indices with sum graphs of groups Z_{pq^m} .

Keywords: The first general Zagreb index, The generalized Randic index, and the generalized Sum-Connectivity index.

1. Introduction

The graph G is a tuple (V, E) that comprises a finite Vertices set V and finite edges set E; each edge is an unordered pair of vertices. The vertices associated with an edge e are called the end-vertices of e.

We usually refer to (u, v) as an edge between two vertices, namely u and v. In addition, we represent the graph G by V (G) vertices set and the edges of G by E(G) set. Let e = (u, v) be the graph G edge. Then the two vertices u and v are known as adjacent in G, and the edge e is known as incident to the vertices u and v. Also, the vertex, u, is known as a neighbour of v in G and vice versa. The terminologies and notations used but not identified in this article can be found in [1]. The indices' topological are the numerical values that associate with the graph structure.

Over the years, numerous topological indices have been studied and proposed based on the graph's distance, degree, and other parameters. Some of them are mentioned in [7, 11]. Historically, indices of Zagreb can be considered the first degree-based topological indices that are applied to many sciences such as physical, chemical, and economical. [22-25] are numerous studies related to ring or group theory aspects for the Z modulo by graph theory; referer to [3-11, 21, 26] and their references. In 2018, the authors Alwardi A. et al. investigated the entire indices of Zagreb of graphs [2].

The sum graph of the group (Z_n **)** is a finite group of order n; the graph vertices represent (Z_n**)** elements such that there is an edge between the vertices *x* and *y* if and only if $\mathcal{O}(x) + \mathcal{O}(y) > \mathcal{O}(\mathrm{Zn})$ denoted by $G_+(\mathrm{Zn})$. The Topological indices have been studied in the last few years; they may be found in [12-16]. In 2013, the index of Zagreb was re-defined as first, second, and third indices of Zagreb by Ranjini et al. [3-4].

In this paper, we investigate the relationships between generalized $Q_\gamma(G), R_\gamma(G)$ and $K_\gamma(G)$ and several topological indices with a sum graph of group $G = Z_{pq}{}^m$.

2. Basic Concepts and Terminology

In what follows, we focus on a topological index that will need in the subsequent considerations.

The first general Zagreb index (or general Zeroth-order Randic index) Q_γ is defined as

$Q_\gamma = Q_\gamma(G) = \sum_{i=1}^{k} d_i^\gamma = \sum_{i\sim j}\left(d_i^{\gamma-1} + d_j^{\gamma-1}\right)$, Where k is the order of vertices and $\gamma \in \mathbb{R}$ [14].

The generalized Randic index (or Connectivity index) R_γ, is defined as

$R_\gamma = R_\gamma(G) = \sum_{i\sim j}\left(d_i d_j\right)^\gamma$, And $\gamma \in \mathbb{R}$ [14].

The general Sum-Connectivity index H_μ as

$H_\mu = H_\mu(G) = \sum_{i\sim j}\left(d_i + d_j\right)^\mu$, And $\mu \in \mathbb{R}$ [15].

3. The Sum Graphs of Groups Z_{pq^m} and $Z_{p^nq^m}$

The following section will investigate the concept of Z_n Group via graph theory by defining that by order laws. Moreover, we found the degree of the exceptional value of Z_n at pq^m and p^nq^m where p , q are prime numbers to power the positive integers numbers n and m.

Definition 3.1 [26]: "Let G be a finite cyclic group. The group sum graph, denoted by $G_+(V,E)$ of G_+ is a graph with $V(G_+) = \cup_{x\in G}\langle x\rangle$ and two distinct vertices x and y are adjacent in G_+, denoted by $\langle x\rangle\sim\langle y\rangle$ if and only if $\mathcal{O}(x) + \mathcal{O}(y) > \mathcal{O}(G)$, where $\mathcal{O}(G)$ is the order of the group G. (i.e.)."

$V(G_+) = \bigcup_{x\in G}\langle x\rangle$,

$E(G_+) = \{xy|\ \langle x\rangle\sim\langle y\rangle$ if and only if $\mathcal{O}(x) + \mathcal{O}(y) > \mathcal{O}(G)$, where $x, y \in G$ and $x \neq y\}$.

Remark 3.2: "Every finite cyclic group holds the group sum graph are connected and cyclic graphs."

Theorem 3.3: If $G_+\left(V\left(Z_{pq^m}\right), E\left(Z_{pq^m}\right)\right)$, then

$$\underset{u\in V(Z_{pq^m})}{\deg(u)} = \begin{cases} q^{m-1}(pq-(p+q)+1) & \text{if } \mathcal{O}(u) \neq pq^m \\ pq^m - 1 & \text{if } \mathcal{O}(u) = pq^m \end{cases},$$

where $2 < P < q$ are distinct prime numbers, $n \geq 1, n \in \mathbb{Z}^+$.

$2 < p < q$ are distinct prime numbers and $n, m \geq 1$ are positive integer numbers.

4. $Q_\gamma(G), R_\gamma(G)$ and $K_\gamma(G)$ Topological Indices of $G_+\left(V\left(Z_{pq^m}\right), E\left(Z_{pq^m}\right)\right)$

Notice in this section that we will compute the $Q_\gamma(G), R_\gamma(G)\ and\ K_\gamma(G)$ Topological indices.

Theorem 4.1: If $G_+\left(V(Z_{pq^m}),E(Z_{pq^m})\right)$, then the first general Zagreb index is Q_γ where $\gamma \in R$, $Q_\gamma(Z_{pq^m}) = \sum_{i=1}^{pq^m} d_i^\gamma$

$$= q^{m-1}(pq-(p+q)+1)\left[q^{m-1}(p+q-1)\left(q^{m-1}(pq-(p+q)+1)\right)^{\gamma-1} + (pq^m-1)^\gamma\right]$$

.

Proof: The vertices of $G_+\left(V(Z_{pq^m}),E(Z_{pq^m})\right)$ have a degree by theorem 3.3, where $2 < p < q$, and m is a positive integer number $m \geq 1$, we

$$: Q_\gamma = Q_\gamma(Z_{pq^m}) = \sum_{i=1}^{pq^m} d_i^\gamma = \underbrace{\left(q^{m-1}(pq-(p+q)+1)\right)^\gamma + \left(q^{m-1}(pq-(p+q)+1)\right)^\gamma + \ldots + \left(q^{m-1}(pq-(p+q)+1)\right)^\gamma}_{q^{m-1}(p+q-1)-terms}$$

get

$$+\underbrace{(pq^m-1)^\gamma + (pq^m-1)^\gamma + \cdots + (pq^m-1)^\gamma}_{q^{m-1}(pq-(p+q)+1)-terms}.$$

$$= q^{m-1}(p+q-1)\left(q^{m-1}(pq-(p+q)+1)\right)^\gamma + \left(q^{m-1}(pq-(p+q)+1)\right)(pq^m-1)^\gamma$$

.

$$Q_\gamma(Z_{pq^m}) = q^{m-1}(pq-(p+q)+1)\left[q^{m-1}(p+q-1)\left(q^{m-1}(pq-(p+q)+1)\right)^{\gamma-1} + (pq^m-1)^\gamma\right].$$

Remark 4.2: The generalized Randic index (or Connectivity index) R_γ, is defined as

$$R_\gamma = R_\gamma(G) = \sum_{i\sim j} (d_i d_j)^\gamma$$

Theorem4.3: If $G_+\left(V(Z_{pq^m}),E(Z_{pq^m})\right)$, then $M_2(Z_{pq^m}) = R_1 = \sum_{i\sim j} d_i d_j$

$$= q^{m-1}(pq-(p+q)+1)(pq^m-1)\frac{\left(q^{m-1}(pq-(p+q)+1)+1\right)}{2}$$

Proof: Suppose that $S=\{a_1,a_2,\cdots,a_\alpha\},\alpha=q^{m-1}(pq-(p+q)+1),S\subseteq Z_{pq^m}$, where $a_i\in Z_{pq^m}$, $O(a_i)=pq^m,1\leq i\leq\alpha$, we see that, every $a_i\in S,1\leq i\leq\alpha$ is adjacent to all vertices that belong to Z_{pq^m} except itself. (The graph is considered a simple graph).

$$R_1=d(a_1)\left[\sum_{i=1}^{pq^m} d_i-d(a_1)\right]+d(a_2)\left[\sum_{i=1}^{pq^m} d_i-\left(d(a_1)+d(a_2)\right)\right]+$$
$$\cdots+d(a_\alpha)\left[\sum_{i=1}^{pq^m} d_i-\sum_{i=1}^{\alpha} d(a_i)\right]$$

Now, since $d(a_1)=d(a_2)=\cdots=d(a_\alpha)=d(a)=pq^m-1$ and

$\sum_{i=1}^{pq^m} d_i=2\ell$ (ℓ is the size of the graph).

$$\Rightarrow R_1=d(a)[(2\ell-d(a))+(2\ell-2d(a))+\cdots+(2\ell-\alpha d(a)]\ldots(1)$$

$$=d(a)\left[\alpha\cdot 2\ell-\sum_{i=1}^{\alpha} id(a)\right]$$

$$=d(a)\left[2\alpha\ell-d(a)\frac{\alpha(\alpha-1)}{2}\right],\quad\left[\sum_{i=1}^{\alpha} i=\frac{\alpha(\alpha-1)}{2}\right]$$

$$=\alpha d(a)\left[2\ell-\frac{(\alpha+1)}{2}d(a)\right]$$

$$=q^{m-1}(pq-(p+q)+1)(pq^m-1)\frac{\left(q^{m-1}(pq-(p+q)+1)+1\right)}{2}$$

So, in general, we get by eq (1).

$$R_\gamma=(d(a))^\gamma[(2\ell-d(a))^\gamma+(2\ell-2d(a))^\gamma+\cdots+(2\ell-\alpha d(a))^\gamma]$$

$$=(d(a))^\gamma\sum_{i=1}^{\alpha}(2\ell-id(a))^\gamma$$

$$R_\gamma=\left(pq^{m-1}\right)^\gamma\sum_{i=1}^{\alpha}\left[\left(q^{m-1}(pq-(p+q)+1)\left(q^{m-1}(pq+p+\right.\right.\right.$$
$$\left.\left.\left.q-1)-1\right)-i(pq^m-1)\right]^\gamma$$

Remark 4.4: In [15], Zhou defined the general Sum-Connectivity index k_μ as

$$k_\mu=k_\mu(G)=\sum_{i\sim j}(d_i+d_j)^\mu$$

Theorem 4.5: If $G_+\left(V(Z_{pq^m}),E(Z_{pq^m})\right)$, then the general sum-connectivity index of G+(Z_{pq^m})

$$k_\mu=\sum_{i\sim j}(d_i+d_j)^\mu$$

$$= q^{m-1}(pq-(p+q)+1)\left[\frac{\left(q^{m-1}(pq-(p+q)+1)-1\right)}{2}\left(2(pq^m-1)\right)^{\mu}+\right.$$

$$\left.q^{m-1}(p+q-1)\left(pq^{m-1}+q^{m-1}(pq-(p+q)+1)\right)^{\mu}\right]$$

Proof: Suppose that $S=\{a_1,a_2,\dots,a_\alpha\}, \alpha = q^{m-1}(pq-(p+q)+1)$

$S \subseteq Z_{pq^m}, a_i \in Z_{pq^m}$, where $\mathcal{O}(a_i)=pq^m, \forall\ 1 \le i \le \alpha$.

Since every a_i adjacent to all vertices to $Z_{pq^m}, \forall\ 1 \le i \le \alpha$ except itself, then

$$k_\mu$$
$$= \left(d(a_1)+d(a_1)\right)^{\mu}+d(a_1)+d(a_2)^{\mu}+\cdots+\left(d(a_1)+d(a_\alpha)\right)^{\mu}$$
$$-\left(d(a_1)+d(a_1)\right)^{\mu}$$
$$+\underbrace{\left(d(a_1)+d(0)\right)^{\mu}+\cdots+\left(d(a_1)+d(a_{2m}pq^{m-1})\right)^{\mu}}_{q^{m-1}(p+q-1)\text{-times}}+\cdots$$
$$+\left(d(a_\alpha)+d(a_1)\right)^{\mu}+\cdots+\left(d(a_\alpha)+d(a_\alpha)\right)^{\mu}$$
$$-\left[\left(d(a_\alpha)+d(a_1)\right)^{\mu}+\left(d(a_\alpha)+d(a_2)\right)^{\mu}+\cdots+\left(d(a_\alpha)+d(a_\alpha)\right)^{\mu}\right]$$
$$+\left[\underbrace{(d(a_\alpha)+d(0))^{\mu}+(d(a_\alpha)+d(p))^{\mu}+\cdots+\left(d(a_\alpha)+d(\alpha_{2m}pq^{m-1})\right)^{\mu}}_{q^{m-1}(p+q-1)\text{-times}}\right]$$

Now, since $d(a_1)=d(a_2)=\cdots=d(a_\alpha)=d(a)=pq^m-1$ and

$d(0)=d(p)=\cdots=d(\alpha_{2m}pq^{m-1})=q^{m-1}(pq-(p+q)+1)$

$k_\mu=[(\alpha-1)(2d(a))^{\mu}+\beta(d(a)+\alpha)^{\mu}+[(\alpha-2)(2d(a)]^{\mu}+$
$\beta(d(a)+\alpha)^{\mu}]+\cdots+[(\alpha-\alpha)(2d(a))^{\mu}+\beta(d(a)+\alpha)^{\mu}]$,

where $\beta=q^{m-1}(p+q-1)$

$$k_\mu=(2d(a))^{\mu}\sum_{i=1}^{\alpha}(\alpha-i)+\alpha\beta(d(a)+\alpha)^{\mu}$$

since $\sum_{i=1}^{\alpha}(\alpha-i)=\frac{\alpha(\alpha-1)}{2}$

$$k_\mu=\alpha\left[\frac{(\alpha-1)}{2}(2d(a))^{\mu}+\beta(d(a)+\alpha)^{\mu}\right]$$

$$= q^{m-1}(pq-(p+q)+1)\left[\frac{\left(q^{m-1}(pq-(p+q)+1)-1\right)}{2}\left(2(pq^m-1)\right)^{\mu}+\right.$$

$$\left.q^{m-1}(p+q-1)\left(pq^{m-1}+q^{m-1}(pq-(p+q)+1)\right)^{\mu}\right].$$

5. The Relationship between $Q_\gamma(G), R_\gamma(G)$ and $K_\gamma(G)$ with Some Other Topological Indices with a Sum Graph of the Group Z_{pq^m}.

Some of the standard topological indices for particular values of γ can be considered a specific case of this index.

This chapter introduces the relationships between the first general index of Zagreb $Q_\gamma(G)$, the second general index of Zagreb $R_\gamma(G)$, and the general sum-connectivity index $K_\gamma(G)$, with some other topological indices, with a sum graph of the group $Z_{p\,q^m}$. The relationships are shown in the tables below:

Table (1): Relationships between the first general index of Zagreb $Q_\gamma(Z_{pq^m})$ and some other topological indices.

Topological indices	Corresponding $Q_\gamma(Z_{pq^m})$index		Formula
	γ	$Q_\gamma(G)$	$q^{m-1}(pq-(p+q)+1)\left[q^{m-1}(p+q-1)\left(q^{m-1}(pq-(p+q)+1)\right)^{\gamma-1}+\right.$
Topological indices	**Corresponding $R_\gamma(Z_{pq^m})$index**		**Formula**
general Randi´c index	γ	$R_\gamma(G)$	$(pq^m-1)^\gamma \sum_{i=1}^{\alpha}[(q^{m-1}(pq-(p+q)+1)(q^{m-1}(pq+p+q-1)-1)-i(pq^m-1)]^\gamma, \alpha = q^{m-1}(pq-(p+q)+1)$
modified second Zagreb index $MO_2(G)$	-1	$R_{-1}(G)$	$\frac{1}{(pq^m-1)}\cdot\sum_{i=1}^{\alpha}[\left(q^{m-1}(pq-(p+q)+1)\right)(q^{m-1}(pq+p+q-1)-1)-i(pq^m-1)]^{-1}$
connectivity index (or Randi´c index)$\lambda(G)$	-1/2	$R_{-1/_2}(G)$	$\frac{1}{\sqrt{(pq^m-1)}}\sum_{i=1}^{\alpha}[\left(q^{m-1}(pq-(p+q)+1)\right)(q^{m-1}(pq+p+q-1)-1)-i(pq^m-1)]^{-1l2}$
Second Zagreb index, $M_2(G)$	1	$R_1(G)$	$(pq^m-1)\sum_{i=1}^{\alpha}[(q^{m-1}(pq-(p+q)+1)(q^{m-1}(pq+p+q-1)-1)-i(pq^m-1)]$

Table (2): Relationships between the second general index of Zagreb $R_\gamma(Z_{pq^m})$ and some other topological indices.

Table (3): The relationships between general Sum-Connectivity index $K_\gamma(Z_{pq^m})$ and some

Topological indices	Corresponding $K_\gamma(Z_{pq^m})$index		Formula
	γ	$K_\gamma(G)$	$q^{m-1}(pq-(p+q)+1)\left[\frac{(q^{m-1}(pq-(p+q)+1)-1)}{2}(2(pq^m-1))^\gamma+q^{m-1}(p+q-1)(pq^m-1+q^{m-1}(pq-(p+q)+1))^\gamma\right]$
First Zagreb index, $M_1(G)$	1	$K_{1(G)}$	$q^{m-1}(pq-(p+q)+1)[(pq^m-1)(q^{m-1}(pq-(p+q)+1)-1)+q^{m-1}(p+q-1)(pq^m-1+q^{m-1}(pq-(p+q)+1))]$
Harmonic index, $H(G)$	-1	$2K_{-1(G)}$	$2q^{m-1}(pq-(p+q)+1)\left[\frac{(q^{m-1}(pq-(p+q)+1)-1)}{4(pq^m-1)}+\frac{q^{m-1}(p+q-1)}{(pq^m-1+q^{m-1}(pq-(p+q)+1)}\right]$
Sum connectivity index $\chi(G)$	-1/2	$K_{-1/2}(G)$	$q^{m-1}(pq-(p+q)+1)\cdot\left[\frac{q^{m-1}(pq-(p+q)+1)-1)}{(\sqrt{2})^3\sqrt{pq^m-1}}+\frac{q^{m-1}(p+q-1)}{\sqrt{(pq^m-1+q^{m-1}(pq-(p+q)+1)}}\right]$
Hyper-Zagreb index χ_2(G)	2	$K_2(G)$	$q^{m-1}(pq-(p+q)+1)\left[\frac{(q^{m-1}(pq-(p+q)+1)-1)}{2}(2(pq^m-1))^2+q^{m-1}(p+q-1)(pq^m-1+q^{m-1}(pq-(p+q)+1))^2\right]$

ther topological indices.

6. Conclusions

In this paper, we investigate the relationships between generalized $Q_\gamma(G)$, $R_\gamma(G)$ and $K_\gamma(G)$ and several topological indices with sum graphs of groups Z_{pq^m} .

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Contra-*iβ*-continuous and Almost Contra-*iβ*-Continuous Functions

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Abstract

In this study, using the concept of *iβ*-open sets in topological space to study and develop a novel class of functions called contra *iβ*-continuous and almost contra *iβ*-continuous functions as a new expansion of contra continuity and almost contra continuity. The basic properties of both these types of functions are developed. Several theorem and characterizations of these functions are given , Relationships between these functions and other related classes of functions are also discussed. Also ,on the other hand, some examples to show that the converse may not be true have been introduced.

Key words : *iβ*-closed set , contra *iβ*-continuous , almost contra *iβ*-continuous**.**

1.Introduction and Preliminaries

Contra and Almost Contra Continuous Mappings in topological spaces (P, τ) were introduced in [1]. They defined a function $f : P \to K$ is contra continuous if $f^{-1}(U) \in C(P)$, $\forall\ U \in \sigma$ in K . Numerous types of contra continuity have since been researched. Dontchev and Noiri [2] develop and study the contra semi-continuous functions a new weaker form of these functions . Caldas and Jafari introduced and studied the contra-continuous function [3]. By Jafri and Noiri [4,5], contra super continuous, contra pre-continuous and contra α-continuous functions were introduced and studied. Contra γ-continuous & almost contra γ-continuous function have been presented and investigated by Nasef [6]. Many different types of almost contra continuity have recently been studied. Ekici [7,8] proposed the ideas of an almost contra pre-continuous function and an almost contra super continuous function. Baker [9] introduced the concept of contra almost-*β*-continuous functions. In 2012 Mohammed and Khattab [10] presented the idea of contra *i*-continuous functions and contra *iα*-continuous functions and study the concept *is*-contra-continuous in[18]. Shaheen in 2022 introduced the concept of contra *iR*-continuous functions [11] and study the concept *iβ*-open set[12] in topological spaces. In this direction, We continue to investigate some characteristics of contra *iβ*-continuous and almost contra *iβ*-continuous functions. We denoted the topological spaces (P, τ) , (K, σ), (H, θ) simply by P , K , H , respectively. Let E be a subset of a space P. The closure and interior of E in relation to P are denoted by $Cl_P(E)$ and $Int_P(E)$ respectively or simply by $Cl(E)$ $Int(E)$ respectively and the complement of E is denoted by P-E. Open sets (resp. closed sets) by (*OS*), (*CS*), topological spaces by *T.S* .

Let's study the definitions below because they will be helpful in the sequel:

Definition 2.1 In *T.S* *P*, a subset *E* is defined as *θ*-open set[13] denoted by *θ-OS*, if $E = Int\theta(E)$ where $Int\theta(E) = \bigcup\{U: U \in \tau , cl(U) \subseteq E\}$.

Definition 2.2 In *T.S* P, a subset *E* is defined as *δ*-open set[14] denoted by *δ-OS*, if $E = Int\delta(E)$ where $Int\delta(E) = \bigcup\{U: U \in RO(P), U \subseteq E\}$.

Definition 2.3 In *T.S* *P*, a subset *E* is defined as semi-open set[9] denoted by semi -*OS* , if $E \subseteq (cl(Int(E))$.

Definition 2.4 In *T.S P*, a subset *E* is defined as regular open sets [9]denoted by regular-*OS* , if $E = Int(cl(E))$, and regular closed[21] denoted by (regular-*CS*), if $E = cl(Int(E))$.

Definition 2.5 In *T.S P*, a subset *E* is defined as pre-open denoted by *pre-OS* , if $E \subseteq Int(cl(E))$[9]

Definition 2.6 In T.S P, a subset E is defined as α-open set[9] denoted by α-*OS*, if $E \subseteq Int\ (cl(Int(E\ ))$.

Definition 2.7 In *T.S P*, a subset *E* is defined as β-open set (= semi- preopen)[9] denoted by β-*OS*, if $E \subseteq (cl(Int(cl(E))$[9]

Definition 2.8 In *T.S P*, a subset *E* is defined as *b*-open set (= γ -open)[15] denoted by *b-OS*, if $E \subseteq (cl(Int(E)) \cup (Int(cl(E))$.

Definition 2.9 In T.S P, a subset E is defined as *i*-open set [16]denoted by *i-OS* , If $E \subseteq cl(E \cap U)$, $and\ \exists\ U\ \in \tau\ and\ U \neq P, \emptyset$ [16]

Definition 2.10 In T.S P, a subset E is defined as $i\alpha$-open set[9] denoted by $i\alpha$-*OS*, if $E \subseteq cl(E \cap U)$,where $\exists\ U \in \alpha O(P)$ and $U \neq \emptyset, P$ [9]

Definition 2.11 In *T.S P*, a subset *E* is defined as clopen set, if *E* is both *OS* and *CS* .

Definition 2.1 In *T.S P*, a subset *E* is defined as *iR*-open set[17] denoted by *iR-OS* , if $E \subseteq cl(E \cap U)$, where $\exists\ U \in RO(P)$ and $U \neq P, \emptyset$.

Definition 2.12 In *T.S P,* a subset *E* is defined as *is* -open set[18] , denoted by is-*OS* , if $E \subseteq cl(E \cap U)$, where $\exists\ U \in SO(P)$ and $U \neq \emptyset, P$.

Definition 2.13 In *T.S P*, a subset *E* is defined as $i\beta$-open set[12] denoted by $i\beta$-*OS* , if $E \subseteq cl(E \cap U)$, where $\exists\ U \in \beta O(P)$ and $U \neq P, \emptyset$.

The family of all (open θ-open, δ-open, regular open. ,regular closed , semi-open , β-open , *b*-open, *pre*-open, α-open , *i*-open, $i\alpha$-open , clopen , *iR*-open, is-open) sets of *T.S* (P, τ) is denoted by τ , $\theta O(P)$, $\delta O(P)$, $RO(P)$,$RC(P)$, $SO(P)$, , $\beta O(P)$, $BO(P)$, $PrO(P)$, $\alpha O(P)$, $iO(P)$, $i\alpha O(P)$,$CO(P)$, $iRO(P)$, $USO(P)$, $i\beta O(P)$ respectively. The complement of the above open sets are (closed , θ- closed, δ- closed, regular open , regular closed , semi-closed, β- closed, *b*-closed, pre-closed , α- closed , *i*-closed, $i\alpha$- closed , clopen , *iR*-closed , is- closed) sets respectively .The family of all the above closed set denoted by $C(P)$ ($\theta C(P)$, $\delta C(P)$, $RO(P)$, $RC(P)$, $SC(P)$, , $\beta C(P)$, $BC(P)$, $PrC(P)$, $\alpha C(P)$, $iC(P)$, $i\alpha C(P)$,$CO(P)$, $iRC(P)$, $USC(P)$, $i\beta C(P)$ respectively.

Let *P* and *K* be *T.S.*

Definition 2.2A function $f : P \longrightarrow K$ is said to be Contra-continuous [19,23] , denoted by (C. continuous) if $f^{-1}(U) \in C(P), \forall\ U \in \tau$ in *K* .

Definition 2.3 A function $f : P \longrightarrow K$ is said to be Contra-*i*-continuous [10], denoted by (C. *i*- continuous.) if $f^{-1}(U) \in iC(P), \forall\ U \in \tau$ in *K* .

Definition 2.4 A function $f : P \longrightarrow K$ is said to be Contra -$i\alpha$-continuous [10] denoted by (C. $i\alpha$- continuous) if $f^{-1}(U) \in i\alpha C(P), \forall\ U \in \tau$ in *K* .

Definition 2.5 A function $f : P \longrightarrow K$ is said to be Contra-*iR*-continuous [11]denoted by (C. *iR*- continuous) if $f^{-1}(U) \in iRC(P), \forall\ U \in \tau$ in *K* .

Definition 2.6 A function $f : P \longrightarrow K$ is said to be is- contra-continuous[18]denoted by (C. is-continuous) if $f^{-1}(U) \in USC(P), \forall\ U \in \tau$ in *K* .

Definition 2.7 A function $f : P \longrightarrow K$ is said to be Contra-α-continuous[19]denoted by (C. α- continuous) if $f^{-1}(U) \in \alpha O(P), \forall\ U \in \tau$ in *K* .

Definition 2.8 A function $f : P \longrightarrow K$ is said to be Contra-β-continuous[19] denoted by (C. β- continuous) if $f^{-1}(U) \in \beta C(P), \forall\ U \in \tau$ in *K*.

Definition 2.9 A function $f: P \longrightarrow K$ is said to be Contra semi continuous[2][19] denoted by (C. semi-continuous) if $f^{-1}(U) \in SC(P), \ \forall\ U \in \tau$ in K.

Definition 2.10 A function $f: P \longrightarrow K$ is said to be Contra-pre-continuous[19] denoted by (C. pre-continuous) if $f^{-1}(U) \in PrC(P), \forall\ U \in \tau$ in K.

Definition 2.11 A function $f: P \longrightarrow K$ is said to be Contra-*b*-continuous, denoted by (C. *b*- continuous) if $f^{-1}(U) \in BC(P)$, $\forall\ U \in \tau$ in K [15]

Definition 2.12 A function $f: P \longrightarrow K$ is said to be Contra-Completely continuous =(*RC*-continuous)if $f^{-1}(F) \in RO(P), \forall\ F \in (CS)$ in K [20]

Definition 2.13 A function $f: P \longrightarrow K$ is said to be Contra *R*-map, denoted by (C. *R*-map) if $f^{-1}(U) \in RC(P), \forall\ U \in RO(K)$ [21]

Definition 2.14 A function $f: P \longrightarrow K$ is said to be *R*-map [21] denoted by (*R*-map) if $f^{-1}(U) \in RO(P), \forall\ U \in RO(K)$.

Definition 2.15 A function $f: P \longrightarrow K$ is said to be Strongly continuous, denoted by (Str. continuous) if $f^{-1}(U) \in CO(P), \forall\ U \in K$ [22]

Definition 2.16 A *T.S* P is said to be Locally indiscrete if every (*OS*) is (*CS*) in P [22]

Definition 2.17 A function $f: P \rightarrow K$ is defined as Almost contra θ-continuous[14]denoted by (Al. C. θ-continuous) if $f^{-1}(U)$ is θ-*CS* in P, $\forall\ U \in RO(K)$.

Definition 2.18 A function $f: P \rightarrow K$ is defined as Almost Contra super-continuous[14] denoted by (Al. C. super- continuous) if $f^{-1}(U)$ is δ-*CS* in P, $\forall\ U \in RO(K)$.

Definition 2.19 A function $f: P \rightarrow K$ is defined as Almost Contra *b*-continuous [14]denoted by (Al. C. *b*-continuous) if $f^{-1}(U) \in BC(P), \forall\ U \in RO(K)$.

Definition 2.20 A function $f: P \rightarrow K$ is defined as Almost Contra β-continuous[9] denoted by (Al. C. β-continuous) if $f^{-1}(U) \in \beta C(P)$, $\forall\ U \in RO(K)$.

Definition 2.21 A function $f: P \rightarrow K$ is defined as Almost Contra-pre-continuous[19] denoted by (Al.C. pre-continuous) if $f^{-1}(S) \in PrC(P), \forall\ S \in RO(K)$.

Definition 2.22 A function $f: P \rightarrow K$ is defined as Almost perfectly [23] (=regularset-connected) denoted by (Al.per- continuous) if $f^{-1}(U) \in CO(P)\ \forall\ U \in RO(K)$.

Definition 2.23 A function $f: P \rightarrow K$ is defined as Perfectly (totally)-continuous[23] denoted by (Per-continuous) if $f^{-1}(U) \in CO(P), \forall\ U \in \tau$ in K.

Definition 2.24 A function $f: P \rightarrow K$ is defined as Almost-Contra continuous [24] denoted by (Al. C. continuous) if $f^{-1}(U) \in (CS)$ in P, $\forall\ U \in RO(K)$.

Definition 2.25 A function $f: P \rightarrow K$ is defined as *RC*-continuous [19] if $f^{-1}(V) \in RC(P)$, $\forall\ U \in \tau$ in K.

Lemma 2.26 Every (*CS*), regular-*OS*, regular-*CS*, δ- *CS*, α- *CS*, semi- *CS*, θ- *CS*, pre- *CS*, *b*- *CS* , *i*- *CS*, *i*α-*CS*, β- *CS* , clopen set in *T.S* is *i*β- *CS* [12].

Lemma 2.27 Every regular-*OS* in *T.S* is (*OS*)[25]

3.Contra *i*β-continuous functions

Definition 3.1 If $f^{-1}(S) \in iBC(P)$, $\forall S \in \sigma$ in K, a function $f : P \to K$ is said to be **contra-$i\beta$-continuous** denoted as (C. $i\beta$- continuous) .

Proposition 3.2 Every (C. i- continuous) is (C. $i\beta$-continuous).

Proof : Let $f : P \to K$ be (C. i- continuous) and S be any (OS) in K . Because f is (C. i- continuous) $f^{-1}(S)$ is i-CS in P , moreover every i-CS is $i\beta$-CS in P by lemma 2.5 . So , $f^{-1}(S)$ is $i\beta$-CS in P. finally , f is (C. $i\beta$-continuous)■

Proposition 3.3 Every (C.$i\alpha$-continuous) (resp.(C.iR-continuous),(C.α- continuous) ,(C.β-continuous),(C.pre-continuous),(C.semi-continuous),(C.b-continuous) is (C.$i\beta$- continuous.)

Proof : Same the proof of Proposition 3. 2, and since every ($i\alpha$-CS, iR- CS , α- CS, β- CS , pre- CS, semi - CS , b- CS and CS) is an $i\beta$-CS by lemma 2.5 ■ The example 3.4 that follows explains that (C. $i\alpha$- continuous) are not required (C. $i\beta$- continuous).

Example 3.4 Let P={d,b,c,a}, τ={P,{a,d},{b,c}, Ø},σ={Ø,{b,a},P}, $i\alpha C(P)$ ={Ø,{b,c,d} ,{c,a,d},{a,b,d},{a,c,b},{a,d},{c,b}, P}. $i\beta C(P) = \mathcal{P}(P)$ (power set). The identity map $f : P \to K$ is (C. $i\alpha$- continuous) but f is not (C. $i\beta$- continuous)

The example 3.5 that follows proves that (C. $i\beta$- continuous) need not be (C. continuous.) (resp. C. β-continuous), (C. semi- continuous), (C. α- continuous), (C. i- continuous), (C. pre- continuous) and (C. b-continuous.)

Example 3.5 Let P ={a, b, c}, τ ={Ø,{b},{b, c}, P }, σ ={Ø ,{b,c}, P } , $\beta C(P)$ ={Ø,{a,c},{c},{a}, P}, $SC(P)$= $\alpha C(P)$ ={ Ø,{a,c},{c} ,{a}, P }, $i\beta C(P) = \mathcal{P}(P)$, $iC(P)$ ={Ø,{a,c},{a,b},{c},{b},{a},P}, $PrC(P)$={Ø,{a,c},{a,b},{c},{b},{a},P},$BC(P)$={Ø,{a,c,{a,b},{c},{b},{a}, P }. The identity map $f : P \to K$ is (C. $i\beta$- continuous) but f is not (C continuous), (C. β- continuous), (C. semi-continuous), (C. α- continuous), (C. i- continuous), (C. pre- continuous) and (C. b- continuous) .

Proposition 3.6 Every (RC- continuous) is (C. $i\beta$- continuous).

Proof : Let $f : P \to K$ be (RC- continuous)and S be any (OS) in K . Since f is (RC. continuous) then $f^{-1}(S)$ is regular -CS in P. Since , every regular- CS is an $i\beta$-CS via lemma 2.26 So , $f^{-1}(S)$ is an $i\beta$-CS in P . Hence f is (C. $i\beta$- continuous) ■

Proposition 3.7 Every (Per. continuous) (resp. Str. continuous) is (C. $i\beta$- continuous).

Proof : Let $f : P \to K$ be (Per. continuous) and S be any (OS) in K . Since f is (Per. continuous) then $f^{-1}(S)$ is clopen set in P . Because , every clopen set is an $i\beta$ -CS via lemma 2.26 consequently , $f^{-1}(S)$ is an $i\beta$-CS in P. Finally , f is (C. $i\beta$- continuous) ■

The example 3.8 that follows explains that (Per. cont. fun.), (RC- continuous), (Str. continuous) and (C. iR-continuous) are not required (C. $i\beta$- continuous).

Example 3.8 Let P ={a, b, c ,d }, τ ={Ø, {a},{b, c, d}, P }, σ ={Ø ,{b,c}, P }, $CO(P)$ ={Ø , {b, c, d},{a}, P }, $RC(P)$ ={ Ø, {a} ,{b, c, d}, P }, $iRC(P)$ ={ Ø {a} ,{a,c,d} ,{a,b,d} ,{a,b,c} ,{a , d} ,{a , b} ,{a , c} ,{b , c , d}, P}. , $i\beta O(P) = \mathcal{P}(P)$. The identity map is (C. $i\beta$- continuous) for $f : P \to K$ but f not (Per. cont. fun.),(RC-continuous), (Str. continuous) and (C.iR- continuous)

Remark 3.9 : The composition of two (C. $i\beta$- continuous) as notes need not be (C. $i\beta$- continuous) in the example below :

Example 3.10 Let $P = K = H = \{c, b, a\}$, $\tau = \{\emptyset, \{c\}, \{b,c\}, \{a,c\}, P\}$, $\sigma = \{\emptyset, \{b\}, K\} = \theta$, $i\beta O(P) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,c\}, \{b,c\}, P\}$, $i\beta O(K) = \{\emptyset, \{a\}, \{b\}, \{a,c\}, \{a,b\}, \{c\}, \{b,c\}, K\}$. Define $f : P \to K$ by $f(a) = b$ $f(b) = c$ and $f(c) = a$, $g : K \to H$ by $g(a) = b$, $g(b) = c$ and $g(c) = a$. Consequently, f and g are (C. $i\beta$-continuous) but $gof : P \to H$ is not (C. $i\beta$- continuous) since $(gof)^{-1}\{b\} = f^{-1}(g^{-1}\{b\}) = f^{-1}\{a\} = \{c\}$ in P is not $i\beta$-CS.

Theorem 3.11. If $f : P \to K$ is a function, $g : K \to H$ is injective and closed, and if $g \circ f : P \to H$ is (C. $i\beta$-continuous) then f is (C. $i\beta$- continuous).

Proof. Let A be any (OS) in K. Since g is closed, $g(A)$ is closed in H. Since $g \circ f$ is (C. $i\beta$- continuous) and g is injective, we see that $f^{-1}(A) = f^{-1}(g^{-1}(g(A)))$ is $i\beta$ -CS in P, which proves that f is (C. $i\beta$- continuous) ■

As a result, we have the diagram with implications below :

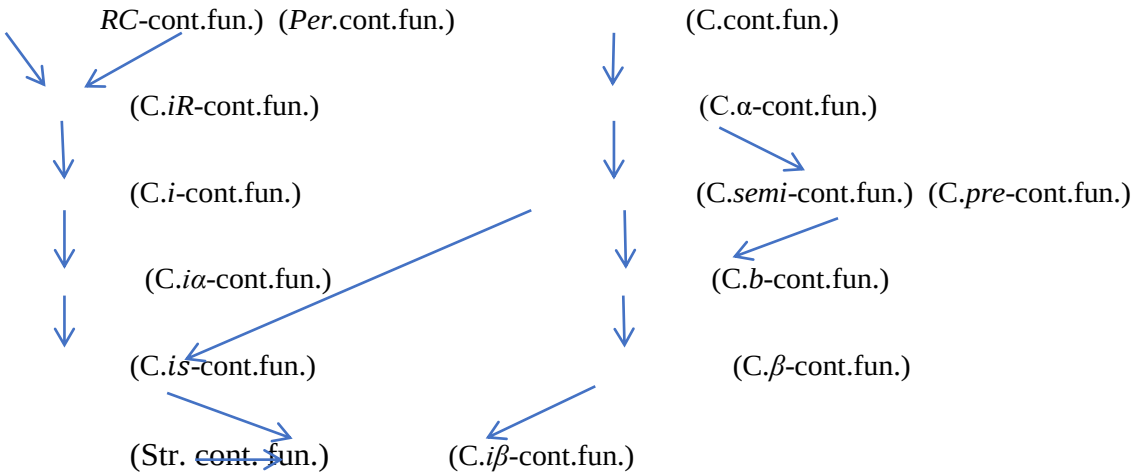

Diagram1

Remark : (cont.fun.) represent the continuous function

4. Almost Contra *iβ*-continuous functions

Definition 4.1 If $f^{-1}(S) \in iBC(P), \forall S \in RO(K)$ **,** a function $f : P \to K$ is called **almost contra *iβ*-continuous** denoted by (Al. C.$i\beta$- continuous)

Proposition 4.2 Every (Al. C. continuous) is (Al. C. $i\beta$- continuous)

Proof : Let $f : P \to K$ be (Al. C. continuous) and S be any regular-OS in K. Since f is (Al. C. continuous) then $f^{-1}(S)$ is (CS) in P and because every (CS) is $i\beta$-CS, $f^{-1}(S)$ is $i\beta$-CS in P. So f is (Al. C. $i\beta$-continuous) ■

Proposition 4.3 Every (C. R-map), (resp. R-map) is (Al. C. $i\beta$- continuous)

Proof : Let $f : P \to K$ be (C. R-map) ,(resp. R-map) and S be any regular- OS in K. Since f is (C. R-map),(resp. R-map) then $f^{-1}(S)$ is (regular- CS) (resp. regular- OS) in P. Since every regular-CS and regular-OS is $i\beta$-CS, $f^{-1}(S)$ is $i\beta$-CS in P. Therefore f is (Al. C. $i\beta$- continuous) ■

Proposition 4.4 Every (Al .per- continuous) is (Al. C. $i\beta$- continuous).

Proof: Let $f: P \to K$ be (Al.per- continuous) and S be any regular- OS in K. Since f is (Al.per-continuous) then $f^{-1}(S)$ is clopen and since every clopen is $i\beta$-CS ,Therefore f is (Al. C. $i\beta$- continuous) ■.

Proposition 4.5 Every (Al. C.pre- continuous), (resp., (Al. C. b- continuous) and (Al. C. β- continuous) is (Al. C. $i\beta$- continuous)

Proof: Clear , since every pre-CS, β-CS and b-CS is an $i\beta$-CS .

Remark 4.6 The example below shown that (Al. C. $i\beta$- continuous) need not be (Al. C.pre-continuous),(Al.per-continuous),(Al.C.b-continuous) and (Al. C. β- continuous).

Example 4.7 Let P=K= {1,2,3 }, τ ={∅,{2},{2,3}, P}, σ = {∅,{1},{3,2},K}, $RO(K)$ ={ ∅,{3,2},{1}, K }. $PrC(P)$={∅,{1,3},{1,2},{3},{2},{1}, P }, $CO(P)$={∅ , P } , $BC(P)$ ={∅,{1,3},{2,1},{3},{2},{1}, P}, $\beta C(P)$ ={∅,{1,3},{3},{1}, P }. $i\beta C(P) = \mathcal{P}(P)$. Clearly , the identity mapping $f: P \to K$ is (Al. C. $i\beta$-cont. fun.) but f is not (Al. C. pre- continuous) and (Al. C. b- continuous) and (Al. C. β- continuous).

Proposition 4.8 Every (Al. C. super-continuous) (resp. (Al.C. θ- continuous)is (Al.C. $i\beta$- continuous) .

Proof : Clear, since every δ-CS and θ-CS is an $i\beta$-CS.

The following examples prove that (Al. C. $i\beta$- continuous) does not have to be (Al. C. super- continuous) and (Al. C. θ- continuous)

Example 4.9 Let $P = \{a, b, c\}, \tau = \{\emptyset, \{a\}, \{a, b\}, P\}, i\beta O(P) = \mathcal{P}(P)$ and the set $\{a, b\} \neq Int\theta(\{a, b\}) = \emptyset$,

Example 4.10 Let P ={a, b, c}, τ ={ ∅,{a},{a ,c},P }, $RO(P)$={∅, P} We note The set {a,c} is not δ-OS since $Int\delta$ ({a,c})= ∅. $i\beta O(P)$= $\mathcal{P}(P)$ (power set).

Corollary 4.11 Every (C. $i\beta$-continuous) is (Al. C. $i\beta$-continuous)

Proof : Let $f: P \to K$ be (C. $i\beta$-continuous) and S be any regular-OS in K , then by lemma 2.27 S is (OS) in K . Because f is (C. $i\beta$-continuous) , then $f^{-1}(S)$ is an $i\beta$-CS in P. Hence f is (Al. C. $i\beta$-continuous) ■
The following examples prove that (Al. C. $i\beta$- continuous) does not have to be (C. $i\beta$-continuous)

Example 4.12 P={a,b,c} with τ ={ ∅ ,{a},{b},{a,b},{b,c} , P } , σ ={∅ , {b }, P}, $RO(K)$ ={∅, P}, $i\beta C(P)$ ={ ∅, {a,b},{b,c},{a,c},{c},{a},P } Clearly , the identity mapping $f: P \to K$ is (Al. C. $i\beta$-continuous) but f is not (C. $i\beta$- continuous)

As a result, we have the diagram with implications below :

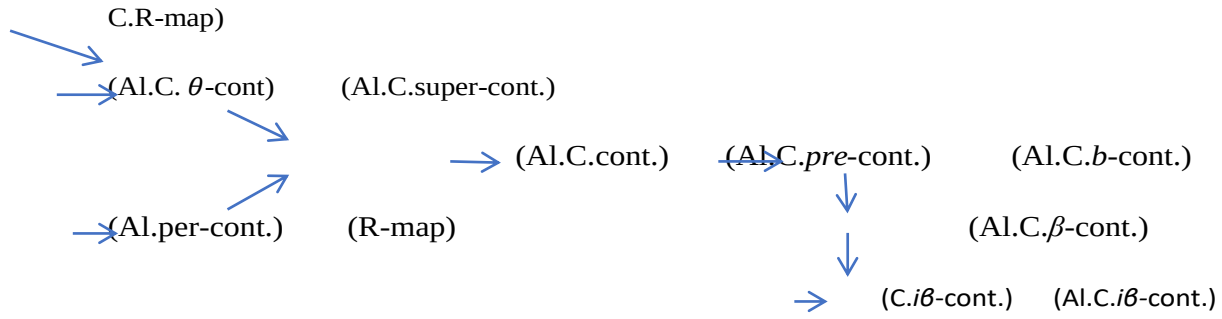

Diagram 2

Remark : (cont.) represent the continuous function

Theorem 4.13 If $f: P \to K$ is a (continuous) and P is locally indiscrete then $f: P \to K$ is (Al. C. $i\beta$-continuous).

Proof. Suppose f is (continuous) and V be any regular-OS of K then V is (OS) in K. Since f is continuous and P is locally indiscrete , then $f^{-1}(V)$ is open and closed in P. Hence $f^{-1}(V)$ is clopen in P and by lemma 2.26 $f^{-1}(V)$ is $i\beta$-CS in P. Therefore, f is (Al. C. $i\beta$- continuous) ■

Theorem 4.14 If $f: P \to K$ is (Al. C. pre- continuous) and A is a semi- OS of P, then the restriction $f \mid A : A \to K$ is (Al. C. $i\beta$- continuous).

Proof. Let $S \in RO(K)$ Since f is contra-pre continuous ,then $f^{-1}(S) \in PrC(P)$ and Since A is semi open in P, by ([7], Lemma 2.1) we get $(f \mid A)^{-1}(S) = A \cap f^{-1}(S) \in PO(A)$ so, $f \mid A$ is (Al. C. pre- continuous) .Since every (Al. C. pre- continuous) is (Al. C. $i\beta$- continuous) by Proposition 4.5 therefore, f is (Al. C. $i\beta$- continuous) ■

Theorem 4.15 If $f: P \to K$ is (Al. C. $i\beta$- continuous) and $g : K \to H$ is (C. continuous) , then $g \circ f: P \to H$ is (Al. C. $i\beta$- continuous).

Proof : Let $f : P \to K$ be (Al. C. $i\beta$- continuous) and S be any regular-OS in H ,then S is (OS) in H . Since g is (C. continuous) then $f^{-1}(S)$ is (CS), hence $f^{-1}(S)$ is an $i\beta$-CS by lemma 2.26 in P. Therefore, f is (Al. C. $i\beta$- continuous) ■

Remark 4.16 : The composition of two (Al. C. $i\beta$- continuous) need not be(Al. C. $i\beta$- continuous) as shown in the example below :

Example 4.17 Let $P = K = H = \{a, b, c\}, \tau = \{\emptyset, \{c\}, \{b,c\}, \{a,c\}, P\}$, $\sigma = \{\emptyset, \{b\}, \{a,c\}, K\} = \theta$, $RO(K) = \{\emptyset, \{a,c\}, \{b\}, K\}$. $RO(H) = \{\emptyset, \{a,c\}, \{b\}, H\}$. $i\beta O(P) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a,c\}, \{b,c\}, P\}$ $i\beta O(Y) = \mathcal{P}(P)$. Define $f: P \to K$ by $f(a) = b$ $f(b) = c$ and $f(c) = a$, $g : K \to H$ by $g(a) = b$, $g(b) = c$ and $g(c) = a$,Then f and g are (Al. C. $i\beta$- continuous) but $gof : P \to H$ is not (Al. C. $i\beta$- continuous) since $(gof)^{-1}\{b\} = f^{-1}(g^{-1}\{b\}) = f^{-1}\{a\} = \{c\}$ is still not an $i\beta$-CS in P.

Corollary 4.18 : For two functions $f: P \to K$ and $g : K \to H$, let $g \circ f$ be that the composition function $P \to H$. The following are hold :

(1) If f is(Al.C.$i\beta$-continuous) and $g$is (Per- continuous), then g is (C. $i\beta$- continuous). (2) If f is (Al.C.$i\beta$-continuous) and g is an R-map, then $g \circ f$ is (Al.C. $i\beta$- continuous). (3) If f is (C. $i\beta$-continuous) and g is (Al. continuous),then $g \circ f$ is (Al.C.$i\beta$- continuous).

Proof:(1) Let V be (OS) in H . Since g is (per-continuous) , $g^{-1}(V)$ is clopen in K. Since f is (Al. C. $i\beta$-continuous), $f^{-1}g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $i\beta$ -OS and $i\beta$-CS in P . Therefore $g \circ f$ is (C. $i\beta$- continuous)■

(2) Let V be any regular- OS in H . Since g is R-map , $g^{-1}(V)$ is regular-OS in K . Since f is (C. $i\beta$-continuous) $f^{-1}g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $i\beta$- CS in P . Therefore, $g \circ f$ is (Al. C. $i\beta$- continuous) ■

(3) Let V be a regular-OS in H . Since g is (Al. continuous), $g^{-1}(V)$ is (OS) in K. Since f is (C. $i\beta$-continuous) $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is $i\beta$ -CS in P . Finally $g \circ f$ is (Al. C. $i\beta$- continuous)■

Conclusions

In this paper, our attention is on the contra $i\beta$-continuity and its properties and their relationship to other as shown in the diagram1. Moreover with the help of these known applications in topological spaces functions there has been research on the almost contra $i\beta$-continuous functions as shown in the diagram 2.

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