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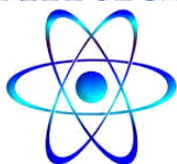
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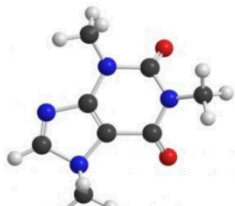
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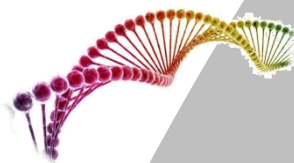
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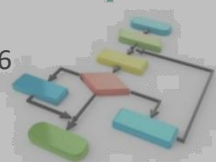


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Synthesis of Iron Oxide Nanoparticles Using Fig (*Ficus macrophylla*) Leaf Extract and Their Application in Treating Contaminated Well Water

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Synthesis of Iron Oxide Nanoparticles Using Fig (*Ficus macrophylla*) Leaf Extract and Their Application in Treating Contaminated Well Water

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Abstract

The current study investigated the water from a well located in the city of Samarra, focusing on the concentrations of toxic heavy metals such as copper, lead, and arsenic. The results were compared with global standards to assess the water's suitability for drinking purposes. Nanotechnology was employed to synthesize iron oxide nanoparticles (FeO-NPs) using aqueous fig leaf extract through both green and chemical methods. The efficiency of these nanoparticles in removing heavy metals from polluted well water was evaluated.

The results revealed that the well water contained high levels of copper, lead, and arsenic, exceeding permissible limits. After treatment with the synthesized iron oxide nanoparticles, copper levels were reduced to within acceptable limits, achieving removal efficiencies of 80% and 83.1% using the green synthesis method, and 76.67% and 80% using the chemical synthesis method. Lead removal efficiencies were 72% and 70.9% for the green method, and 62.2% and 66.5% for the chemical method. Arsenic removal efficiencies were 62.5% and 46.86% for the green method, and 62.2% and 0.0% for the chemical method, depending on the nanoparticle dose.

Keywords: Contamination well waters, Nanotechnology, Fig extract, Heavy metals.

1.INTRODUCTION

Water is a vital resource that sustains all types of plant and animal life. It is often acquired from two main natural sources: surface water, which includes freshwater lakes, rivers, and streams, and groundwater, such as water from boreholes and wells. In nature, water is inherently impure since it absorbs impurities from its surroundings, including those originating from humans, animals, and other biological processes[1]. Groundwater pollution is a significant environmental issue, with a wide range of contaminants harming water supplies. In areas with severe water shortages, the practice of using wells water for agricultural purposes has grown increasingly common. Overall, this resource comprises significant quantities of advantageous nutrients and harmful contaminants like heavy metals, which are presenting potential and challenges for agricultural development. Heavy metals, are a cause for concern due to their high toxicity, even at low quantities [2]. The increase in heavy metal content when irrigating the soil with well water[3]. It will contribute to the biological accumulation of these metals in crops because most of these metals are poorly soluble in water, and thus cause

damage to plants when they reach them, and under certain conditions they become toxic to humans and animals that feed on these plants. The most common heavy metals copper, lead, arsenic, and cadmium. Accumulation of toxic heavy metals in plant living cells results in various deficiencies, reduced cell activity, and inhibition of plant growth. They affect enzymatic action by exchanging metal ions with metallo-enzymes [4].

A range of physical and chemical techniques are commonly employed to eliminate hazardous heavy metals from water solutions. Chemical methods include different methods, ion exchange [5], reverse osmosis [6], membrane filtration [7], etc.. Nevertheless, all of these approaches exhibit low efficacy, high cost, and result in the production of hazardous by-products [8]. The adsorption process is widely regarded as a very effective, safe, straightforward, environmentally friendly, and cost-effective method for eliminating heavy metal ions from contaminated water [9].

This study focuses on the simple and effective use of iron oxide nanoparticles (FeO-NPs) synthesized with fig leaves (*Ficus macrophylla*) as an adsorbent to remove heavy metals from contaminated well water.

2. MATERIALS AND METHODS

2.1 Fig Leaves Extract Preparation

Fig leaves (*Ficus macrophylla*) are collected in the autumn, washed with tap water and distilled water to remove impurities and dust. They are dried for seven days at room temperature, after which these materials are crushed and sieved to (0.125 mm). 10 g of fine fig powder was dispersed in a closed glass beaker filled with 300 ml of deionized water using a magnetic stirrer at 90 °C for 3 h. Then filtration was carried out using white Whatman No.4 filter paper (pore size 20 µm), and the plant extract solution was stored in the refrigerator [10].

2.2 Synthesis of FeO-NPs of plant extract

2.2.1 Green synthesis of FeO-NPs

The preparation process carried out according to [11] with modification. 5 g of iron nitrate was dissolved in 200 ml of deionized water using a magnetic stirrer (800 RPM) at 80 °C for 60 min. Then, 60 ml of plant extract (fig) was added, and the color changed from red to black. After 72 h, 8 ml of ammonia was added, a black precipitate formed. The precipitate is separated by centrifuge, washed with deionized water, and then dried at room temperature.

2.2.2 Chemical Synthesis of FeO-NPs

The work was performed according to the reference [12]. 10 g of iron nitrate was dissolved in 500 ml of deionized water using a magnetic stirrer (800 RPM) at 80 °C for 60 min. Then, 5 g of PVP polymer was added to precursor solution under stirring for 90 min. After homogeneity, 10 ml of ammonia was added and the black precipitate formed. The precipitate is separated by centrifuge, washed with distilled water and ethanol, and then dried in oven at 100 °C for 4 h. After this the black powder was calcination at 500 °C for 2 h.

10 ml of plant extract was centrifuged at 4000 RPM for 10 min, then mixed with 5 mg of iron oxide using Vortex Mixer for 30 min and sonication for 4h. Then, incubator at 37 °C for 48 h. The precipitate is separated by centrifuge, washed with deionized water, and then dried at room temperature.

2.2.3 Characterization of FeO-NPs

The chemical structure and surface morphology of the biosorbent was examined using a scanning electron microscope coupled to an energy dispersive X-ray, JSM 7600F, JEOL Inc., Japan at a working voltage of 5 kV at various magnifications[13].

2.3 Experimental of Water Treatment

2.3.1. Collection of well water

A sample of well water was obtained in the center of the Iraqi city of Tharthar, near the International Paints Factory, and it is highly expected that the water of these wells is polluted with toxic metals. Samples in this study were collected from polluted water in one of the wells located in the city of Samarra. The collected samples were kept in a refrigerated container at approximately 4°C to prevent any biological processes that could affect the analytical results. The samples were then evaluated by the government analytical laboratory in Samarra, Iraq. Contamination levels of well water with copper (Cu), lead (Pb), and arsenic (As) were determined using atomic absorption spectrometry.

2.3.2 Measurement of Heavy metals in well water

This experiment was conducted for various quantities of well water containing copper, lead and arsenic. The concentrations of metal ions at the beginning and end of the experiment were measured using Atomic absorption spectrometer. Hollow cathode lamps were used as radiation sources, with specific wavelengths corresponding to each metal ion. A flame utilizing air acetylene was employed to ascertain the presence of copper, lead and arsenic, and then compared the concentration of these metals within the limits set by the implementing authorities for water quality [14].

Table 1: Characteristics of the well water in present study

Sampling Time		
Param Parameters	Value Values	Permissible limits
Cu Cu	0.3 0.3	0.2 0.2
Pbp Pb	0.927 0.927	0.01 0.01
Asaa AS	0.32 0.32	0.05 [0.05 [14]

2.3.3. Adsorption experiments procedure

The study involved conducting batch tests to examine the impact of the quantity of ingested substance on metal ions using FeO-NPs produced by green method, in addition to FeO-NPs manufactured by the chemical method to remove toxic metals from contamination well water. A solution of well contamination water, consisting of copper, lead and arsenic ions were utilized in batch tests to identify the optimal treatment parameters. The optimal parameter, encompassing the specific dose of the nanoparticles, effectively eliminating the heavy metals present in the well contamination water, were then employed to treat the environmental water sample acquired from well in Samara city. Control tests were conducted in parallel with the batch trials to minimize alterations in all other variables except for the one being examined. The control experiment involved the well water sample containing no FeO-NPs. Adsorbent dosages of 0.015g and 0.03 g were chosen based on previous studies. Every experiment was carried out in a centrifuge tube with a capacity of 50ml, and 25 ml of polluted well water containing varying percentages of copper, lead, and arsenic was added to the

tube, the samples were shaken using an orbital shaker at a speed of 100 revolutions per minute. Offers a high level of interaction between the adsorbent and the adsorbate. Following the shaking of the mixtures, the supernatant liquid was separated by centrifuging them at 5000 revolutions per minute for ten minutes. The removal efficiency of adsorbed metal ions was calculated [15]. According to Equation 1.

$$\text{Removal efficiency (\%)} = \frac{C_0 - C_t}{C_0} \times 100 \dots\dots\dots 1$$

Where C_0 is initial concentrations and C_t final concentration of heavy metal ions

Statistical analysis

The experiments were performed in triplicate, following the same circumstances. The collected data were subsequently analyzed using Duncan Multiple Ranges Test [14].

3. Results and Discussion

3.1 UV –Visible analysis

Figure 1 displays the UV spectra of FeO-NPs nanocomposite synthesized by Fig extract leaf two methods: green and chemical. According to the spectra in Figure 2a, the material has a peak at 375 nm, which characterizes iron oxide synthesized by the green method, and in the figure it peaks in Figure 2b. 290 nm with respect to the approach of FeO-NPs by chemical methods UV/Vis Spectrophotometer.

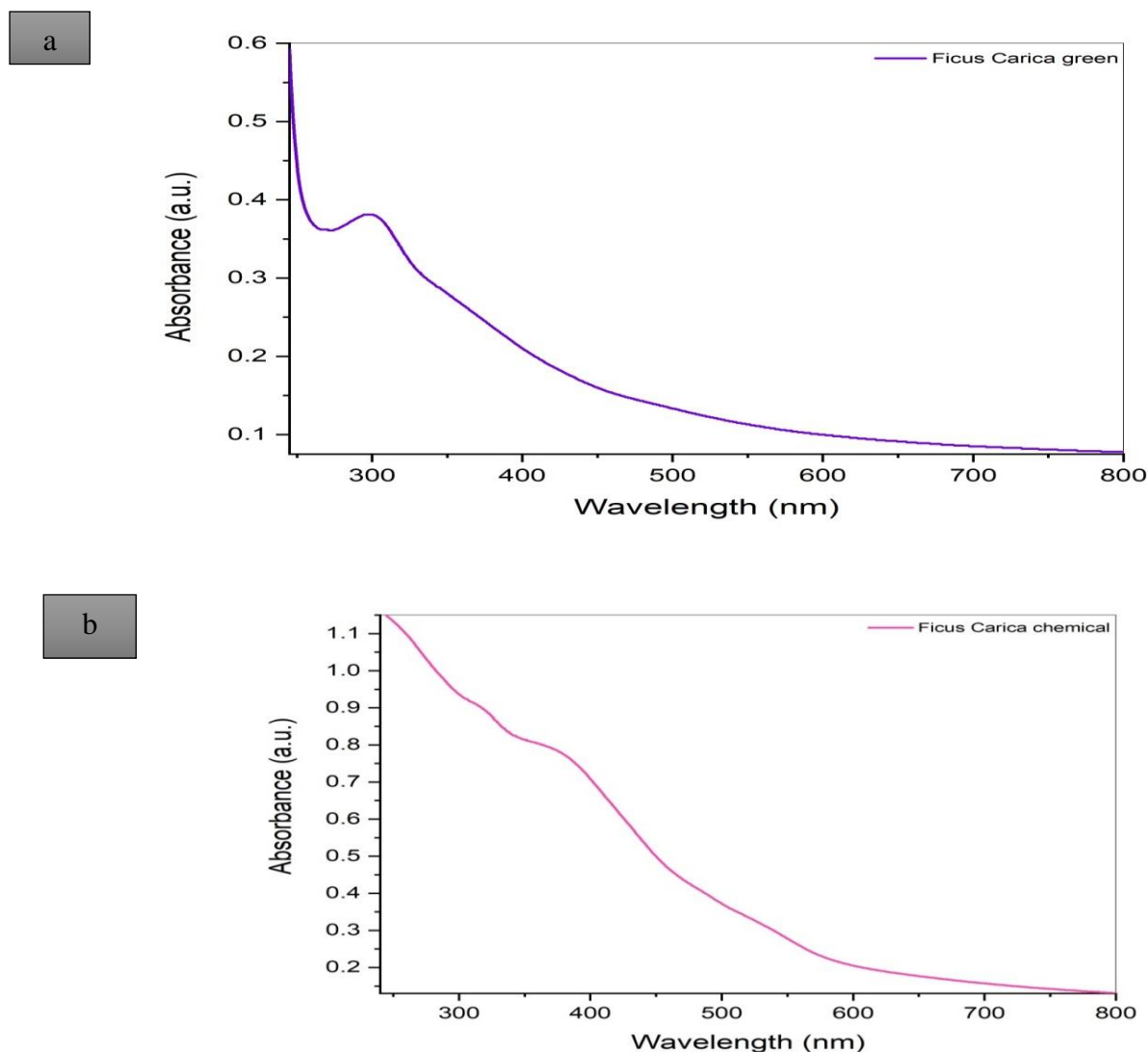


Figure 1: UV spectra of FeO-NPs synthesized by (a) green method and (b) chemical method.

3.2 SEM and EDS analysis

Figure .2 display the SEM and EDS of iron oxide nanoparticles synthesized by the green and chemical methods, respectively. It appears from the Figures 1a and 1c that some of the particles are clustered together, that the shapes of the iron nanoparticles were spherical and with varying sizes ranging from 30.04 to 40.71 nm by the green method and from 63.14 to 72.45 nm by the chemical method. Also, the surface of FeO-NPs was shown to have a heterogeneous structure, which is characterized by a large number of pores that vary in size and shape. This makes it effective in removing pollutants from contamination well water [16]. Figures 1b and 1d shows the EDS spectra of FeO-NPs prepared by the green and chemical methods, respectively. It can be seen from Figure 1b the presence of a sharp peak belonging to iron atoms in sample at about 6.5 keV, in addition to the presence of oxygen and carbon, similarly in Figure 1d there is a sharp iron peak at 6.3 keV, in addition to oxygen and carbon atoms. The result obtained in this study is agreement with other study [17].

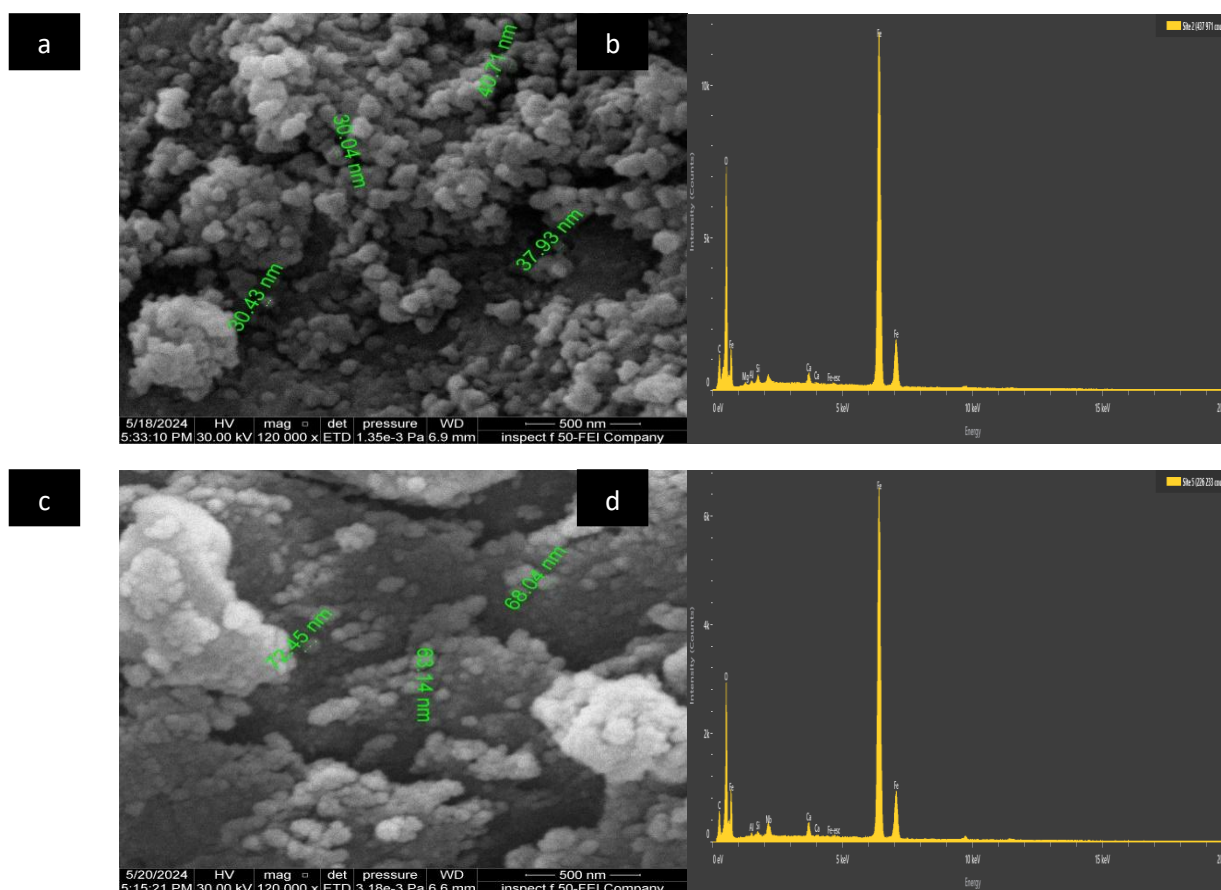


Figure 2: SEM image and EDS of FeO-NPs synthesis using fig leaf extract by (a,b) green methods (G FeO-NPs) and (c,d) chemical method (C FeO-NPs).

3.3 Analysis of wells water

The concentrations of copper, lead, and arsenic in the well water were found to be higher than the maximum allowable limit. The fact that the water of wells contains high quantities of copper, lead, and arsenic metals indicates that these metals pose a significant threat to the state of the ecosystem.

Table 2 shows the concentration of heavy metals in well water after treatment with iron oxide nanoparticles. It has been found that copper is hazardous to plant seedlings. This toxicity is caused by the production of reactive oxygen species, which leads to oxidative stress. Additionally, Cu decreases the activity of catalase. Protein structure oxidation. Under stress conditions, the germination rate is decreased and biomass is mobilized by the release of glucose and fructose. This release inhibits the breakdown of starch and sucrose in reserve tissue by reducing the activities of alpha-amylase and invertase isoenzymes. In the present study results showed a decrease in copper levels in water samples ranging between 0.06-0.05 ppm when treated with 0.015 g and 0.03 g of iron oxide nanoparticles manufactured using green methods and the chemical method, respectively. This decrease was within permissible limits for irrigation water set by the world Health Organization (0.2) ppm [3], the results of the current study are consistent with a study conducted by (Al-Khafaji and Younus 2020) Who found that the determination of lead and heavy elements in nearby wells

Table 2: Characteristics of the well water in present study

Parameters	Concentration/ppm		
	Copper	lead	Arsenic
GFeO-NPs/0.015	0.06 d	0.26 e	0.12 e
GFeO-NPs/0.03	0.05 de	0.27 d	0.17 c
CFeO-NPs/0.015	0.07 c	0.35 b	0.0 f
CFeO-NPs/0.03	0.06 d	0.31 d	0.16 c

Lead toxicity results in the suppression of ATP synthesis, oxidative damage to lipids, and DNA harm due to excessive generation of reactive oxygen species (ROS). Furthermore, lead significantly hinders the process of seed germination. The mentioned factors include root elongation, seedling development, plant growth, transpiration, chlorophyll production, and water and protein content. It is a highly prevalent heavy metal pollutant found in soils. It exhibits a high level of toxicity towards living beings.

Lead (Pb) lacks any biological purpose, although it can induce morphological, physiological, and biochemical impairments in plants.

In the current study, the results shown in Table 2 indicated lead concentrations in well water samples, which ranged from 0.26 to 0.35 ppm, lead concentrations are still higher than the permissible limit for irrigation water set by the world Health Organization (0.01)ppm [17], the reason for the increase in lead levels in well water, especially wells in the city center, is due to the increase in population density and the increase in fumes resulting from car exhausts as a result of increased human activities, as the combustion of gasoline is responsible for the increase in lead levels in the environment [18].

According to equation 1, the efficiency of removing lead from contaminated well water using nano-iron particles manufactured from fig leaf extract using the green method and the chemical method at dose 0.015 g and 0.03 g was 72%, 70.9% and 62.2%, 66.5 % respectively.

The results obtained in this study are similar to the results in the study Al-Khafaji and Younus [19], which estimated lead levels in well water adjacent to oil refineries, where the highest level of lead was 0.9 and the lowest level of lead was 0.2. Al-Tememi indicated that the concentration of lead in groundwater in the Mount Sanab area south of Basra Governorate was 0.28ppm to 2.64ppm, with an

average of 0.98ppm [29]. In a similar study in Nigeria, conducted by (Okparaocha and Oyeleke) noted lead concentrations in water 0.06 -0.96 (mg.L⁻¹)[21].

Arsenic is an element with the chemical formula As and atomic number 33. Arsenic is classified as a metalloid. It possesses multiple allotropes, however, alone the gray variant holds significance in the industrial sector. The plant root cells readily absorb both forms of inorganic arsenic, namely arsenate (AsV) and arsenite (AsIII). Once inside the cell, AsV can be easily transformed into AsIII, which is the more harmful of the two forms [18]. In current study, arsenic levels were higher than the permissible limit of 0.05 ppm. After treatment with different dose of 0.015 and 0.03 g of FeO-NPs manufactured using the green and chemical method. Arsenic levels in well water decreased to 0.0 in some treatments.

According to equation 1, the efficiency of removing arsenic from contaminated well water using FeO-NPs manufactured from fig leaf extract using the green method and the chemical method at dose 0.015 g and 0.03 g was 62.5%, 46.86% and 62.2%, 0.0% respectively. The results of the current study are consistent with many other studies that were conducted to determine arsenic levels in groundwater[22, 23].

4.Conclusion

The results of this study showed that iron oxide nanoparticles can be biosynthesised using the aqueous extract of fig leaves and chemically synthesised and used to purify well water from heavy metals. The results revealed that the well water contained high levels of copper, lead and arsenic that exceeded the permissible limits. After treatment with composite iron oxide nanoparticles, copper levels were reduced to acceptable limits.

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A Review of Video Colorization Recent Techniques and Future Scope

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ABSTRACT

The colorization process attempts to assign colors to grayscale or black-and-white images and videos. While previous image colorization algorithms have produced impressive results and significant improvements, video colorization still faces some challenges due to temporal consistency requirements. This paper addresses the issues specific to the colorization process and outlines possible future research avenues. This review does not aim to cover all specific applications in image and video colorization. Rather, it highlights issues that have seen significant development in recent years, especially those involving significant theoretical research, and points out their strengths and weaknesses. Moreover, there are few detailed studies on image and video colorization methods. This paper focuses on recent and improved colorization techniques, classifies them, and evaluates them based on the results achieved by each technique using fixed metrics, color spaces, and specific training datasets. From a new perspective, we classify colorization techniques into four main techniques: scribble-based colorization, optical flow-based colorization, fully automated colorization, and model-based colorization. Optical flow-based systems require accurate approximations, while scribble-based methods are labor-intensive. Model-based techniques struggle to obtain sufficient reference images, while fully automated systems may struggle to meet the colorization needs. The results shown in this paper show that the best approach for video colorization is the model-based approach which starts with training on a large and diverse set of images to learn color attributes and then moves on to training on videos to train the model to assign colors to their correct positions during transitions between frames.

Keywords: Temporal consistency, Video colorization, Generative Adversarial Networks.

1. INTRODUCTION

Video colorization refers to the process of setting realistic and reasonable colors to grayscale or monochrome images and videos [1]. Color is vital for human perception and can enhance the visual experience by conveying additional information and drawing attention to more precise details [2]. We all know that videos are just a series of frames and video colorization is essentially an extension of image colorization. Colorization is a prominent area of research within digital image and video processing, encompassing areas like Pattern Recognition, Computer Graphics, Computer Vision, and Human-Computer Interaction. The problem statement of video colorization is the creation of an automated system that transforms grayscale and monochrome (black-and-white) videos into colored versions while preserving naturalness, consistency, and visual quality [1]. Its applications are diverse, including the coloring of grayscale images, the restoration of colors in classic films, and the automated colorization of cartoons [2]. Traditional colorization techniques often require user input and can be sensitive to parameter selection, which makes them time-consuming and labor-intensive [2]. Even short films may necessitate the processing of hundreds of images, particularly for grayscale images or

cartoon colorization [2]. Recent advancements typically employ Convolutional Neural Networks (CNNs) for model development and Generative Adversarial Networks (GANs) for training, leading to marked improvements in efficiency [2]. As a result, the effectiveness and efficiency of image colorization have seen substantial progress, although deep learning technology also introduces a range of opportunities and challenges for this process [2]. This paper was constructed as follows: Section one, the Introduction, and section two discusses the related work of the colorization process. Section three describes the three types of color spaces that are mainly used in image and video processing, specifically in the process of colorizing images and videos: RGB, YUV, and CILAB. In section four, the four techniques used in the colorization process are classified. This work discusses a review of video coloring. Section five consists of four main techniques: colorization by optical flow approaches, colorization by scribble approaches, colorization by exemplar approaches, and colorization by fully automatic approaches. The results obtained to evaluate the performance of some algorithms that use the same data set and the same metrics but different color spaces and coloring methods are presented in section six. Section seven introduces the analyzation of the research paper. Indeed, Section Eight provides the summary of the review paper.

2. RELATED WORKS

2.1 Based Coloring Methods

Coloring processes can be classified in different ways. Below are some classifications of traditional coloring processes based-colorization semi-automatic [3]:

2.1.1 Luminance Keying: uses a user-defined lookup table to convert gray values into specific hues, brightness levels, and saturation [3]. One challenge with this technique is that pixels with the same intensity in different parts of the image cannot be assigned different colors. Segmentation addresses this issue. In older movies, there can be luminance fluctuations, where different colors are applied to equal intensities. R.C. Gonzalez et al [4]. Reposed a solution to tackle this problem. Luminance keying is frequently employed to colorize videos.

2.1.2 Color-by-Example: Welsh It is proposed to use local textural information to clarify color-to-intensity assignments [5]. This approach might not be suitable for cartoons featuring uniform regions.

2.1.3 Motion Estimation: This process occurs when the alterations among two repeated frames are minimal. These methods use optical flow to estimate pixel-to-pixel correlation, allowing for the direct transmission of chromatic information between pixels. However, when optical flow estimation fails due to significant motion among two repeated frames, keyframes are manually colored to account for changes and ensure accurate colorization [6].

2.1.4 Color Propagation: Consistency in the color domain results in consistency in the grayscale domain, and vice versa. These methods expand color from a few user-defined seed pixels to cover the entire image. An example of these strategies was presented by Levin et al. [7].

2.2 Based on Automatic Coloring Techniques

2.1.1 Therefore, all techniques and algorithms that rely on semi-automatic coloring and require user intervention face significant processing time challenges, particularly when coloring video clips with many frames and scenes. As a result, it is preferable to use automatic coloring techniques when coloring video clips. Here are some classifications of automatic coloring techniques:

- 2.1.2 Reference Color Frames-Based Colorization:** syndicates controllability and user effort by using reference photos with the expected color movement to direct the process. The orientation photos may be supplied by the worker, acquired from the net, or a large dataset. Mentioning an orientation picture improves colorization outcomes and meets user expectations. While the photographs vary significantly, they have commonalities in certain locations. Areas with comparable hue or texture generally have similar structures or lines. We can generate pictures based on similarities between grayscale and reference photographs [8].
- 2.1.3 Deep Learning-Based Colorization Algorithms:** Were recently proposed. These methods require a large number of grayscale and colorful image pairs for training and can automatically colorize [9]. Cheng et al. recommended a colorization method using deep networks. The network uses the semantic feature descriptor as input, outputs the chrominance value, and then refines the output [10]. Iizuka et al. introduce a new CNN technique that integrates universal priors and resident image features [11].
- 2.1.4 Resemblance With CNN Features:** Researchers have introduced a new technique, an automated colorization approach based on reference photos, enabling users to generate various colorization styles. The network structure is separated into two subnetworks: resemblance and colorization. The similarity subnetwork defines similarity mappings. Among the reference and goal pictures. The colorization subnetwork brings in line pixels in the brightness station by the resemblance subnetwork. It then employs large data learning to improve mismatched pixel colors [8].
- 2.1.5 Similarity to Luminosity Features:** methods that color a grayscale image need one or additional reference pictures before applying luminosity channel mapping to the input image. Color data is transferred from comparable areas of the reference picture to the input grayscale picture. Because the grayscale picture has only one dimension, a color position image's brightness channel may be used to compete with the grayscale input [15]. The approach changes the reference picture to RGB color space and chooses a few of pixels for sampling. The grayscale picture pixels are skimmed in raster demand, and the top similar area is selected based on neighborhood numbers. This model may be applied to a solo frame from a video clip. The video was colorized using the same goal swatch as the first frame [15], [16]. This approach effectively handles color discrepancies. After detecting the necessary pixel, the sample model is employed to create a brilliant colorization effect. The fault distance $E(N_g, N_s)$ is calculated using the L2 distance measured among neighborhoods N_g in grayscale and N_s in colorized pictures [2].

2.2 Video Colorization Quality Evaluation Metrics

Current video colorization approaches use various metrics to evaluate the quality of colorization, including subjective and objective assessments, temporal consistency, color diversity, and semantic interpretability.[12]. Here are a few significant metrics.

- 2.2.1 Personal Assessment:** Typically, human spectators rate the excellence of colorized videos. Colorization quality is best measured by human vision, making it the most accurate method. However, it's time-consuming.
- 2.2.2 Neutral Assessment:** Measured and neural network models are utilized to evaluate the quality of colorization by comparing a reference video to a colorized grayscale video. Image quality assessments. Several metrics are usually used, including PSNR (Peak Signal-to-Noise Ratio), MSE (Mean Squared Error)[13]. RMSE (Root Mean Squared Error), SSIM (Structural

Similarity Index), FID (Fréchet Inception Distance) [14]. LPIPS (Learned Perceptual Image Patch Similarity) [15], RA (Reconstruction Accuracy) and CC (Correlation Coefficient) [16].

Temporal consistency calculations. In video colorization, temporal consistency denotes the constancy of color results between frames. To ensure a smooth viewing experience, colorized videos should maintain constant colors through the sequence to prevent lambent or abrupt color shifts. The warp error () introduced in [17]. It is commonly employed in video colorization to ensure temporal consistency. As in equation (1-2).

$$CapE_{warp}(R_t, R_{t+1}) = \frac{1}{\sum_{i=1}^n S_t^{(i)}} \sum_{i=1}^n S_t^{(i)} \|R_t^{(i)} - \hat{R}_{t+1}^{(i)}\|, \quad (1)$$

$$E_{warp} = \frac{1}{T-1} \sum_{t=1}^{T-1} E_{warp}(R_t, R_{t+1}), \quad (2)$$

here, \hat{R}_{t+1} signifies the R_{t+1} ; S_t Uncovered means uncovered areas., by values of 0 or 1; t signifies the time phase, and n signifies total numeral of pixels in the frame.

Though, E_{warp} It is non-related to the color of the video too can be meaningfully changed by the performance of current estimating models used for measurement. Developed the (CDC) metric to assess temporal consistency during warp video colorization activities [18]. CDC, created for movie colorization, it is used to test the temporal consistency of color disseminations, as is unrelated to video colors and can be affected by flow estimation module performance. The CDC measure estimates the (JS) deviation of color deliveries across the following frames [18]. As in equation 3.

$$CDC = \frac{1}{3 \times (N-1)} \sum_{c \in \{R, G, B\}} \sum_{i=1}^{N-t} JS(N_c(l^i), N_c(l^{i+t})), \quad (3)$$

Wherever N signifies the distance of the video sequence, t signifies the time stage, an $N_c(l^i)$ signifies the normalized chance supply of color i picture on channel c , which can stay intended used the picture histogram.

Color Diversity Estimation. Existing video colorization approaches use the colorfulness score (CF) to assess color variety and vividness in created films [19]. The CF can be expressed as in equation (4):

$$CF = \sigma_{rgyb}(\mathcal{Z}_t) + 0.3 \times \mu_{rgyb}(\mathcal{Z}_t), \quad (4)$$

Where, $\sigma_{rgyb}(\mathcal{Z}_t)$ and $\mu_{rgyb}(\mathcal{Z}_t)$ signify the standard deviation and the nasty value of the pixel mist in the color plane, individually, and is the production frame on a time[19]. Semantic Interpretability Estimation. To assess semantic interpretability, existing video colorization algorithms use top-1 and top-5 correctness measures on a pre-trained VGG-16 [20].

2.3 Dataset Used for Video Colorization

To colorize videos, colorization models must be trained on a large dataset of high-quality, colorful videos with good brightness and contrast, as well as multiple scenes. The dataset must be balanced so that the training is thorough and not biased toward a particular category. There are many datasets used to train models to colorize grayscale and monochrome videos and images that have proven effective in obtaining good colorization results [21], including:

2.3.1 DAVIS is a dataset used in video processing tasks that has been segmented into high-quality, high-resolution videos with dense annotation at two determinations, 480p and 1080p. There are

50 video sequences with 3455 frames closely glossed at the pixel level. 30 videos with 2079 frames are used for training and 20 videos with 1376 frames are used for validation [31].

2.3.2 Videvo It is a website that provides a free collection of videos, to create the Videvo dataset, we collected 100 high-resolution movies from Videvo.net. The big and varied content can be used to train and evaluate video processing tasks, such as colorization. The Videvo dataset consists of 80 training videos and 20 testing videos [32].

2.3.3 NTIRE 2023 used a subset of the Large-scale Diverse Video (LDV) dataset as the training and validation sets. The LDV collection covers multiple content categories, motion types, and frame rates. The original LDV collection includes 240 good-quality movies at a resolution of 960×536 . We use 200 as the training set and 15 as validation. The validation set is separated into publicly available video frames to reduce variations caused by different decoding algorithms. Video frames are changed to grayscale with 'cv2.cvtColor()' [22].

3. Color Space Used in Video Colorization

In video colorization, different color spaces are employed based on the objectives of the task, including the need for color accuracy, enhanced computational efficiency, or consistency across video frames.[23]. The primary color spaces typically utilized in video colorization techniques are as follows:

3.1 CILAB Color Space

LAB is intended to be a perceptually uniform color space, with the L channel expressing brightness and the A and B channels representing color components (green to red and blue to yellow)[13]. The characteristics of LAB are comparable to human perception of color differences and color matching. LAB is useful since it is very accurate, and it is frequently a favorite because the perceptual regularity that impacts color transfer between pictures is more accurate [24].

3.2 RGB Color Space

RGB is the most often utilized for digital video and picture processing. Colors are represented by the intensity of the red, green, and blue channels. Many automated video colorization models, such as CNNs or GANs for learning-based systems, can work directly with RGB pictures since they naturally represent color. Colorized frames may also be readily mapped to RGB-enabled monitor and television screen output [25].

3.3 YUV Color Space

YUV distinguishes between the luminance (Y) component (brightness) and color components (U and V or Cb and Cr). Keeping the brightness information in the Y channel (also known as the "Brightness Information Channel") is critical for keeping video clarity and realism because it offers enough brightness and contrast during colorization without sacrificing detail. Optical flow or reference-based approaches establish temporal consistency by segregating color and brightness, which is useful for monitoring color propagation across the optical flow channel [25].

4. Basic Techniques Used in Video Colorization

4.1 Manual Colorization

It is a traditional process that involves physically applying color to each frame of the movie. It had drawbacks, such as being time-consuming and labor-intensive, and was commonly used in

previous films before the advent of modern computing and artificial intelligence technology. This technique employs a variety of methods, including frame-by-frame, layered coloring, and animated cel coloring approaches [26].

4.2 Digital Colorization

It was created as computer technology advanced, and digital tools were available to aid in the colorization process. These approaches basically colored gray-level photos with software, which improves productivity, consistency, and accuracy when compared to human methods. This technique corresponds to several different forms of digital colorization, including region, segmentation, and software tool techniques [27].

4.3 AI-Based Colorization

Recent methods such as machine learning and deep learning algorithms were used. These systems, which frequently include deep learning learned on large datasets, automatically predict and use appropriate colors. The advantages of the AI-based colorization approach over other techniques are that they greatly reduce human labor while increasing the practicality of colorized films. Generative Adversarial Networks (GANs) and Unsupervised Deep Learning Colorization methods [28].

5. Classification of Video Colorization Techniques

Colorization of video involves the addition of color to video frames that are originally in grayscale, preprocessing monochrome content into vibrant and natural representations of color. The technique quality depends on Input data, the colorized output, and performance measures [3]. The technology of video colorization has become indispensable in a variety of fields, such as historical film restoration, medical imaging enhancement, and the development of special effects in the film industry. The timeline for the four broad coloring video techniques is discussed below and shown in Figure (1).

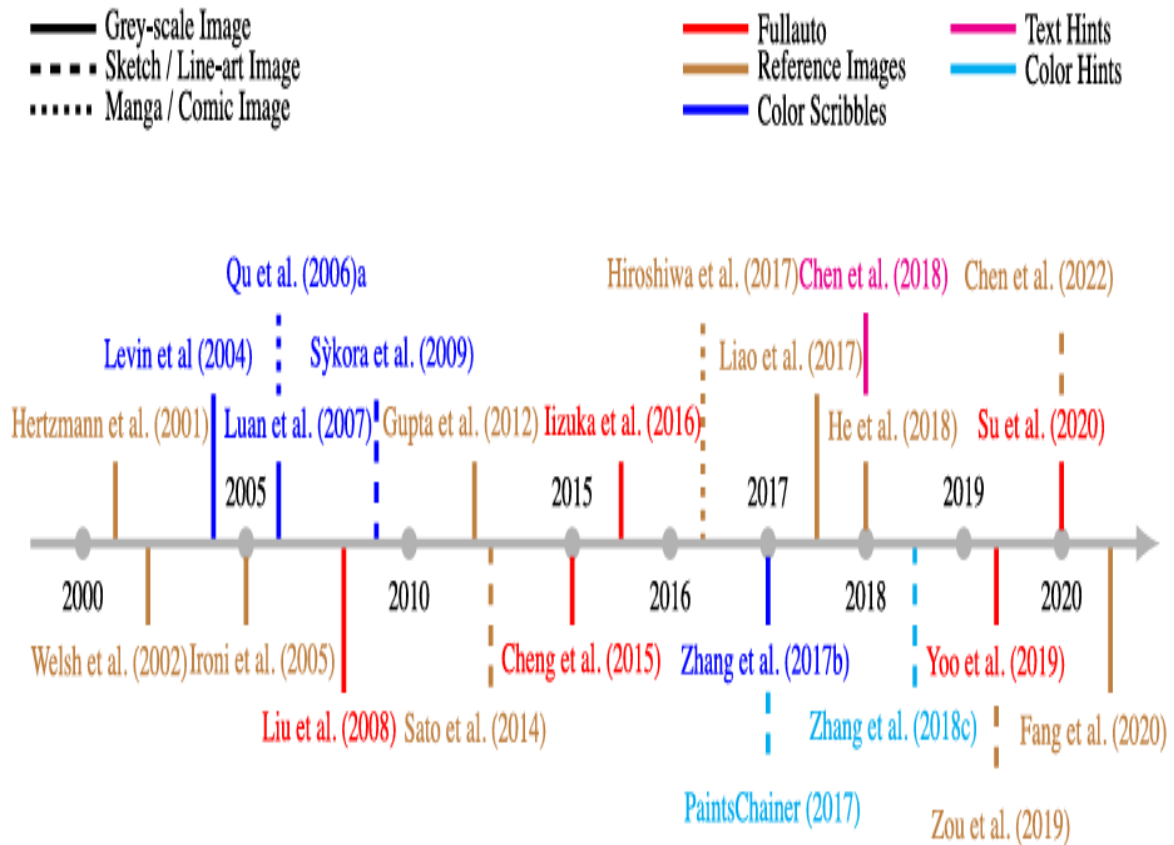


Figure (1) Timeline of Image and Video Colorization Techniques[2].

5.1 Optical-Flow-Based Approaches

The easiest way to colorize video is to start with an image colorization approach and then apply post-processing to maintain temporal consistency. These approaches employ optical flow to convey color features among frames, which produces smoother results. However, optical flow-based approaches have limitations since they depend on picture colorization and estimated optical flow accuracy [29]. As shown in Figure (2).



Figure (2) Optical-Flow-Based Approaches [10].

5.2 Scribble-Based Approaches

This dynamic colorization technique does not need exact manual segmentation or accurate rate monitoring. The approach uses a uniform foundation for both still photos and image sequences. To color each zone, the user scribbles the appropriate color within the region rather than following its exact boundaries. Our approach uses supplied limitations to automatically propagate colors to the remaining pixels in a picture sequence [13]. As shown in Figure (3).



Figure (3) Scribble-Based Methods[30].

5.3 Exemplar-Based Approaches

Scribble-based colorization approaches offer adjustable possibilities, but they necessitate time-consuming human tasks. Researchers have developed model-based colorization strategies to address this issue. The first frame is frequently colored, and the color is subsequently diffused to the following frames. This approach uses colors from reference and preceding frames to colorize the present frame [31]. As shown in Figure (4).



(a)

(b)

(c)

(a) Reference Frame, (b) Preceding Frame, (c) Present Frame

Figure (4) Exemplar-Based Approaches [12].

5.4 Fully Automatic Methods

Exemplar-based colorization approaches can produce acceptable color outcomes with reference photos as guidance. However, obtaining appropriate reference photos is problematic, limiting its utility. Fully automatic video colorization solutions are gaining popularity due to their ease of use and reduced need for gathering relevant references. Fully automated video colorization methods convert grayscale movies to color without using colored images or scribbles during the inference phase [13]. The statistical analysis with all features of the four techniques is shown in Figure (5).

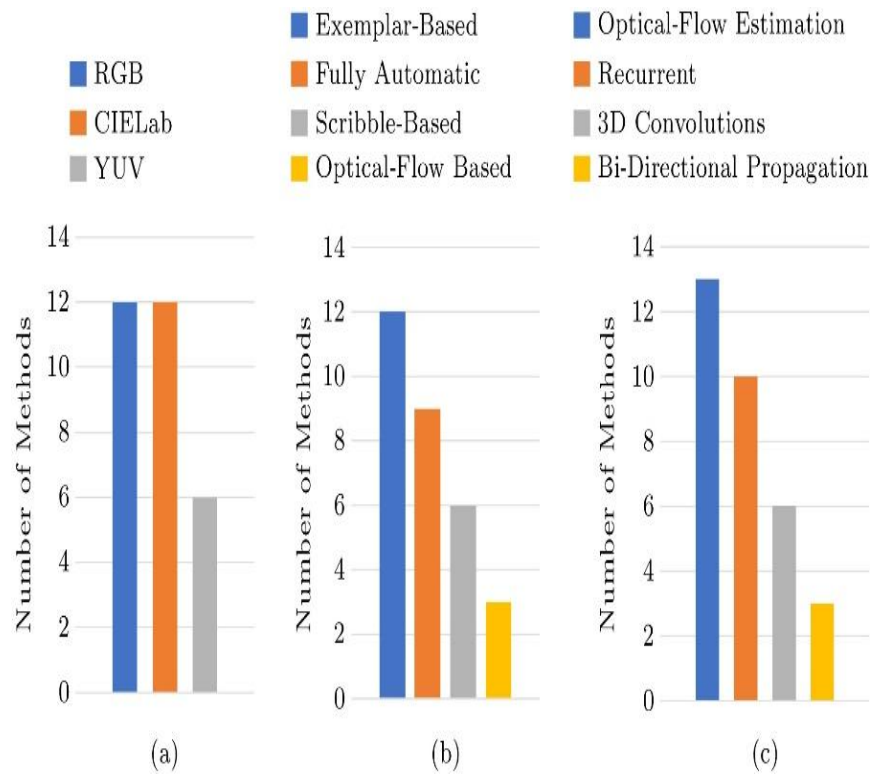


Figure (5) Video Techniques Statistical Analysis (a) Color Space, (b) Class, and (c) Historical Enhancement.[3].

6. PERFORMANCE EVALUATION

Researchers have evaluated the efficacy of video colorization algorithms using deep learning approaches as shown in Table 2. The table records a compression of the performance of several deep learning approaches on three current colorization datasets: Videvo [32], NVCC [22]. It's worth mentioning that all algorithms use a similar training technique. Developed methods for picture colorization. The results show that using image colorization approaches for video colorization is not effective.[33], [34], [35]. Image-based colorization approaches focus solely on individual video frames, disregarding temporal consistency throughout the sequence.[36]. Used automated methods to create video colorization networks, resulting in improved temporal consistency compared to image colorization networks. These approaches use 3D or optical flow convolution to bring into line video frames. Table 1 demonstrates that fully automatic approaches have a tall FID value, indicating deprived colorization performance across all three datasets. Exemplar-based approaches are commonly utilized to create brighter colors in video frames compared to the abovementioned ways. I successfully transferred color since the reference to video frames, achieving a tall FID-measured performance for DAVIS, Videvo, and NVCC datasets. Exemplar-based methods use 3D or optical flow convolutions to improve temporal consistency and colorization performance, surpassing prior completely automated systems [37], [8], [38].

Table 1. Video Colorization Approaches Performance Evaluation Was Showed on Three Communal Datasets

Author	Color space	Class	DAVIS			Videvo			NVCC 2023		
			FID	SSIM	PSNR	FID	SSIM	PSNR	FID	SSIM	PSNR
Zhang et al.[34]	RGB	Based on the image	114.58	0.959	30.90	79.49	0.959	31.18	71.64	0.939	30.64
Kang et al.[33]	RGB	Based on the image	85.43	0.936	30.54	54.91	0.934	30.75	64.08	0.957	30.38
Ji et al.[35]	CIE Lab	Based on the image	92.42	0.942	31.15	67.71	0.953	31.36	62.49	0.933	30.78
Lei al.[39]	RGB	Fully-automatic	110.88	0.951	30.53	82.07	0.949	30.01	81.42	0.927	28.65
Liu et al.[36]	CIE Lab	Fully-automatic	116.64	0.955	31.10	80.71	0.957	31.30	72.73	0.936	30.46
Zhang et al.[8]	CIE Lab	Exemplar-based	70.21	0.951	33.24	55.01	0.957	33.11	35.43	0.930	32.03
Yang et al.[38]	CIE Lab	Exemplar-based	45.18	0.936	33.72	32.32	0.967	34.07	26.63	0.949	33.43

7. THE PROPOSED WORK ANALYSIS

In this work, video and image colorization was analyzed via colorization approaches, taking into account that video is a series of sequential pictures. A variety of old and current colorization procedures were studied and discussed. Also, a selection of techniques was assessed using data sets used in training processes, the metrics utilized, and color spaces. After studying the prior research, it became evident that approaches that rely on training on reference photos before going on to training on video are the best methods utilized in the training process since they produce better outcomes than completely automatic colorization systems.

8. CONCLUSIONS

This paper provides a comprehensive overview of images and video colorization techniques, color spaces, evaluation metrics used to assess colorization quality, and the most commonly used datasets in the training process. By reviewing the literature and timeline of colorization techniques, we categorize them into four types based on user interaction: fully automated colorizing, scribble-based colorizing, optical flow-based colorizing, and example-based colorizing. By examining the current literature and looking at traditional and current colorization techniques, we evaluate their strengths and weaknesses, along with the metrics used to assess colorization quality. We analyze the performance of image and video colorization algorithms using benchmark datasets. Our results indicate that model-based video colorization algorithms generally outperform other techniques. Although deep learning video colorization systems have made important progress and overcome various challenges, they still suffer from some challenges, such as image resolution quality, temporal consistency between frames, and producing different color results due to multiple colors of the same scene; for example, the color of the leaves of the same tree changes with climate and seasonal changes. Traditional methods such as

scribble and optical flow provide realistic colors but are not effective for long movies because they are time-consuming and labor-intensive. Example-based methods are the best based on the results shown in Table 1, but they involve challenges in obtaining suitable images for training.

9. FUTURE WORK

Future ideas will help reduce potential problems in video colorization, making it easier and more effective for a wide range of applications. Below, we present these ideas.

- 1- Ensuring Temporal Consistency: Improving methods to ensure smooth color transitions between frames to prevent flickering and maintain consistency over time. Improving the consistency of colorized video frames via Optimizing Temporal Correlation.
- 2- Enhance the perceived quality of colorized videos by using intelligent techniques and hybrid methods.
- 3- Expand Applications: Increasing the use of video colorization across various domains like historical restoration, animation, and multimedia content enhancement.
- 4- Incorporation of Semantic Models: Integrating semantic understanding to better interpret the video content, resulting in more accurate and contextually relevant colorization.

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Modified Taylor Method for solving multi – singularity non - linear Emden – Fowler equation

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ABSTRACT

The types of multi-singularity nonlinear Emden-Fowler equation mentioned above were solved in this paper using Taylor's method which is an improvement of the previous method using the Taylor series so that the numerical results will not be affected. In conclusion, the proposed method prove to be very effective towards solving problems of initial and boundary values because it gives a scientific solution to many nonlinear problems. The results show the work efficiency of the method when solving several nonlinear problems. The proposed algorithm is straightforward, enhances the solution, enhances accuracy of the solution, and provides consistent results. In addition, the method is equally effective for solving different classes of singular nonlinear first-order IVPs and their initial and boundary conditions do not require alteration. The study conducted on proposed new procedure demonstrates its convergence to find the exact solution of the third order singular nonlinear ordinary differential equations in an effective manner.

Keyword: Taylor Method , Emden – Fowler equation, initial value problems.

1- INTRODUCTION

Single ordinary differential equations are similarly commonly applied in a number of fields including mathematics, physics, engineering, economics and few sciences. These equation are used to describe the dynamic systems that is the systems that undergo changes or transformations and are vital in understanding several of natural and artificial processes.

Analysis of single ordinary differential equations is essential for an awareness of temporal changes of parameters and prediction of further evolution of the system. Besides, it is used to describe matters concerning motion, development, several chemical processes and other matters of change that are indexed at time. The importance of each individual ordinary differential equation is in the capability of presenting various dramatic experiences in the various fields of science and engineering, as well as the provision of fundamental mathematical tools for solving dynamic equations and predicting future phenomena [1,2].

Some of the techniques in solving single ordinary differential equations encompass the Adomian decomposition method, the homotopic analysis method, the variable iteration method and others [3]. An optimal homotopy method of the single nonlinear Blasius problem is described in reference [4]. The modified heterogeneity iteration approach was used to solve the fourth order singular equivalent partial differential equations. The obtained algorithm is very fast and it is most effective for application purposes [5]. For determining the coefficients of a polynomial the method of sequential approximation is offered to consider a single tenth-order linear differential equation [6]. Furthermore, the Laplace transform was adopted to obtain the solution in the power series in the forms of singular, non-singular, linear, and non-linear differential equations [7].

The Emden-Fowler equation is a singular second-order nonlinear differential equation governing what lights as the fundamental pressure, density, and temperature in the atmosphere of calm stars. These include the first-step, the integrated, second-step and the Fowler type successively applied to

second-order ODEs on both linear and nonlinear single first – value problem (FVPs) [8]. It has been established that the solutions of both linear and nonlinear Lane-Emden differential equations can be completely handled by the symmetric analysis method with Laplace transform [9]. There is provided a single starting value polynomial solution to the Lane-Emden equations [10]. By means of the second transformed solution of the same IVPs the solutions of the individual IVP of Lane-Emden type equations can be obtained. [11]. The nonlinear ordinary Lane-Emden differential equations were accessed employing the GFCF clustering approach [12].

In this paper, the Emden-Fowler equation will be solved using the New Taylor method, and the individual points in the equation will be treated. Section 2, the nonlinear Emden-Fowler differential equation and its cases are described. Section 3, an explanation of the proposed Tyler method for solving the nonlinear Emden-Fowler differential equation in its general form. Section Four: Applying the method to a number of examples to prove the accuracy and efficiency of the proposed method. The fifth chapter presents conclusions and approximate solutions to the examples and summarize the results of this study compared to other methods.

2. Emden – Fowler Equation singular differential equations

An Emden-Fowler type singular differential equation (EFSDE) of non-linear type may be expressed in a general form as follows [13]:

$$x^{-r} \frac{d^n}{dx^n} \left(x^r \frac{d^m}{dx^m} u \right) + f(x)g(u) = 0, \quad (1)$$

Here $f(x)$ and $g(u)$ be distinguished functions of x and u , respectively, Instances of non-linear singular problems with various values of $m, n, f(x)$ and $g(u)$:

- (i) When $n = m = 1$, equation (1) is referred to as a two-order Emden-Fowler-type equation and finds utility in the fields of fluid mechanics, pattern generation, and population distribution. Assuming $f(x) = 1$ and $g(u) = u^n$, equation (1) provides the first form of the Lane-Emden equation. Conversely, if $f(x) = 1$ and $g(u) = e^u$, equation (1) provides the second form of the Lane-Emden equation.
- (ii) When $n = 1$, and $m = 2$, or $n = 2$, and $m = 1$, equation (1) is referred to as the first and second types of third-order Emden-Fowler singular differential equations, respectively. These equations find utility in the fields of fluid mechanics, pattern formation, and population distribution. Assuming $f(x) = 1$ and $g(u) = u^n$, equation (1) provides the first form of the Lane-Emden equation. Conversely, if $f(x) = 1$ and $g(u) = e^u$, equation (1) provides the second form of the Lane-Emden equation.

The applications of the third-order Emden-Fowler equations in several sectors of applied sciences and engineering have recently attracted significant interest from scholars. The existence of discontinuity and significant nonlinearity in the Emden-Fowler equations becomes the process of determining the analytical solution quite challenging. Hence, it is crucial to enhance the precision, computational dependability, and efficiency of the numerical method used to solve the nonlinear Emden-Fowler equations. Many scientists have proposed different methods of algorithms (numerical, semi-analytical and analytical) for different classes of Emden-Fowler equations. [14,15,16].

3. Proposed method

Lane-Emden type equations appear in many mathematical physical phenomena such as convection currents, radiative cooling, etc. Their general form is:

$$x^{-r} \frac{d^n}{dx^n} \left(x^r \frac{d^m}{dx^m} \right) u + f(x)g(u) = 0, \quad 0 \leq x \leq 1 \quad (2)$$

with the initial conditions:

$$y(x_0) = \alpha, y'(x_0) = \beta, \dots, y^{n+m-1}(x_0) = \gamma \quad (3)$$

The proposed method is based on rearranging the equation (2) in a way that allows it to be differentiable according to the Taylor series method.

Suppose the solution to equation (2) is in the form of a Taylor series about the initial value x_0 :

$$u_{tay}(x) \approx \sum_{i=0}^k \frac{u^i(x_0)}{i!} (x - x_0)^i, \quad k = 0, 1, 2, \dots \quad (4)$$

The unknown coefficients of equation (4), denoted as $u^i(x_0)$. The values $u^i(x_0)$ should be determined by differentiating equation (1) k times with relationship to x , after expressing it in the following format:

$$\frac{d^i}{dx^i} \left(x^{-r} \frac{d^n}{dx^n} \left(x^r \frac{d^m}{dx^m} \right) u + f(x)g(u) \right) = 0, \quad \text{for } i = 0, 1, 2, \dots, k \quad (5)$$

By substituting $x = 0$, in every step of derivation, with initial conditions (3) in equation (5), unknown constants $u^i(0)$ for all $i = 0, 1, 2, \dots, n$ can be readily determined. At last, the approximate solution of equation (1) may be expressed as:

$$u(x) = \lim_{k \rightarrow \infty} u_{tay}(x) \quad (6)$$

The proposed approach has a high convergence rate in comparison to the rates documented in prior literature. To validate this and assess the efficacy of the approach, a number of hypothetical scenarios derived from contemporary sources were solved. It has been determined that the proposed method exhibits both separate and efficient theoretical and computational convergence rates. The Taylor series algorithm, designed for efficient and rapid development and calculation of solutions, is as summarized below:

Algorithm

Step 1. Input the values and given functions $k, n, m, f(x), g(u)$.

Step 2. Input the initial conditions α, β , and γ .

Step 3: Applying equation (5) and substituting x equal to zero.

Step 4: Determine the unknown constants $u^i(0)$ for all values of i ranging from 0 to k .

Step 5. The solution can be obtained by substituting all values of $u^i(0)$ for all $i = 0, 1, 2, \dots, k$ in equation (4).

4. Application

Several cases of the Emden-Fowler will be solved to showcase the efficiency of the algorithm in comparison to various prior approaches. The efficiency will be evaluated using the absolute error and the E_2 norm error E_∞ [13,17]:

$$E_2 = \sqrt{h \sum_{i=1}^{n-1} |u(x_i) - R(x_i)|^2} \quad (7)$$

$$E_\infty = \max_{0 \leq i \leq N} |u(x_i) - R(x_i)| \quad (8)$$

Problem 1: Suppose the nonlinear Emden-Fowler equation as follows: [13]

$$u'''(x) + \frac{3}{x} u''(x) - u^3(x) = 24e^x + 36xe^x + 12x^2e^x + x^3e^x - x^9e^{3x}, \quad x \in [0,1] \quad (9)$$

With initial conditions:

$$u(0) = 0, u'(0) = 0, u''(0) = 0 \quad (10)$$

To solve equation (9) in the proposed method, The equation (9) is rewritten as a function F in the form::

$$F = x u'''(x) + 3 u''(x) - x u^3(x) - x (24e^x + 36xe^x + 12x^2e^x + x^3e^x - x^9e^{3x}) \quad (11)$$

Now the above equation is derived with respect to x to get:

$$F'(x) = 4 \left(\frac{d^3}{dx^3} u(x) \right) + x \left(\frac{d^4}{dx^4} u(x) \right) - u(x)^3 - 3xu(x)^2 \left(\frac{d}{dx} u(x) \right) - 24e^x - 96xe^x - 72x^2e^x - 16x^3e^x - x^4e^x + 10x^9e^{3x} + 3x^{10}e^{3x}$$

Substituting the initial conditions (10) and put $x = 0$, we get:

$$u'''(0) = 6$$

In the same method, the second derivative of the equation (11) is found:

$$F''(x) = 5 \left(\frac{d^4}{dx^4} u(x) \right) + x \left(\frac{d^5}{dx^5} u(x) \right) - 6u(x)^2 \left(\frac{d}{dx} u(x) \right) - 6xu(x) \left(\frac{d}{dx} u(x) \right)^2 - 3xu(x)^2 \left(\frac{d^2}{dx^2} u(x) \right) - 120e^x - 240xe^x - 120x^2e^x - 20x^3e^x - x^4e^x + 60x^9e^{3x} + 9x^{10}e^{3x} + 90x^8e^{3x}$$

Substituting the initial conditions (10) and put $x = 0$, we get:

$$u^{(4)}(0) = 24$$

Also

$$F'''(x) = -360e^x - 480xe^x - 180x^2e^x - 24x^3e^x - x^4e^x + 27x^{10}e^{3x} + 270x^9e^{3x} + 810x^8e^{3x} + 6 \left(\frac{d^5}{dx^5} u(x) \right) - 3xu(x)^2 \left(\frac{d^3}{dx^3} u(x) \right) + 720x^7e^{3x} - 6x \left(\frac{d}{dx} u(x) \right)^3 - 18u(x) \left(\frac{d}{dx} u(x) \right)^2 - 18xu(x) \left(\frac{d}{dx} u(x) \right) \left(\frac{d^2}{dx^2} u(x) \right) + x \left(\frac{d^6}{dx^6} u(x) \right) - 9u(x)^2 \left(\frac{d^2}{dx^2} u(x) \right)$$

Then

$$u^{(5)}(0) = 60$$

By repeating the steps for k times and using equation (4), then the approximate solution:

$$u_{tay}(x) \approx \sum_{i=0}^k \frac{u^{(i)}(0)}{i!} x^i$$

$$u_{tay}(x) = x^3 + x^4 + \frac{1}{2}x^5 + \frac{1}{6}x^6 + \frac{1}{24}x^7 + \dots$$

Where the exact solution: [13]

$$u(x) = x^3 e^x$$

Table 1: Analytic and Numerical solution of equation (9) with $k = 8$.

x	Analytical Solution $h=0.02$	Quintic trigonometric B-spline collocation method [13] $N=16, h=0.02$	Proposed Method $h=0.02, k=8$	Absolute Error Proposed Method $h=0.02$
0	0	0	0	0
0.1	0.001105170918	0.0011051717098	0.001105170918	0
0.2	0.009771222065	0.0097712234899	0.009771222065	0
0.3	0.036446187804	0.364461895845	0.03644618780	$2.0000000 \times 10^{-11}$

0.4	0.095476780649	0.095476782577	0.09547678064	$3.0000000 \times 10^{-11}$
0.5	0.206090158837	0.206090160655	0.2060901588	$1.0000000 \times 10^{-10}$
0.6	0.393577660884	0.393577662466	0.3935776604	$4.0000000 \times 10^{-10}$
0.7	0.690717186623	0.690717179812	0.6907171760	2.5000000×10^{-9}
0.8	1.139476955388	1.139476956120	1.139476939	1.6000000×10^{-8}
0.9	1.793050668033	1.793050668282	1.793050591	7.7000000×10^{-8}
E_2		1.9287×10^{-7}	$4.41006555 \times 10^{-8}$	

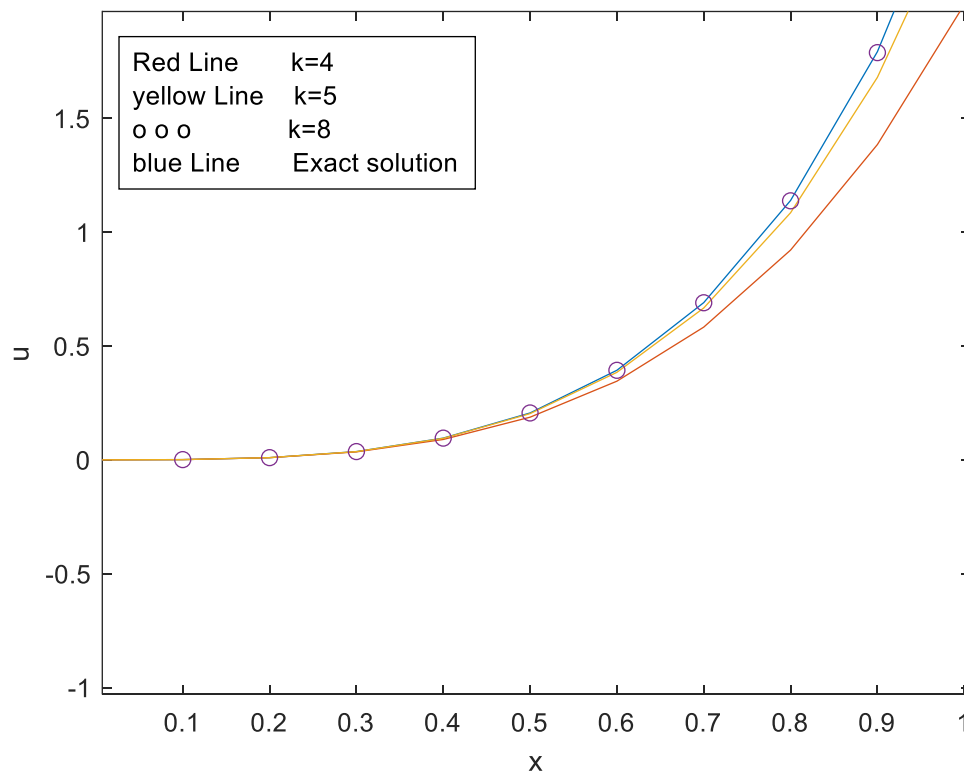


Figure 1: Comparison analytic and numerical solution of equation (9) With different values for k .

It is noted from Table (1) that The new style gives best results than the Quintic trigonometric B-spline collocation method [13] based on norm error E_2 in equation (7) and it also improves with increasing repetitions k as shown in the figure (1).

Problem 2: Let the nonlinear Emden-Fowler equation as follows:

$$u'''(x) + \frac{2}{x}u''(x) = \frac{9}{8}(x^6 + 8)u^{-5}(x), \quad x \in [0,1] \quad (12)$$

with ICs

$$u(0) = 1, \quad u'(0) = 0, \quad u''(0) = 0. \quad x \in [0,1] \quad (13)$$

To solve equation (12) with conditions (13), we assume the function:

$$F = x u'''(x) u^5(x) + 2 u''(x) u^5(x) - x \left(\frac{9}{8}(x^6 + 8) \right), \quad (14)$$

Find the derivative of the equation (14) with respect to x to get:

$$F'(x) = 3 \left(\frac{d^3}{dx^3} u(x) \right) * u(x)^5 + x \left(\frac{d^4}{dx^4} u(x) \right) * u(x)^5 + 5x \left(\frac{d^3}{dx^3} u(x) \right) * u(x)^4 * \left(\frac{d}{dx} u(x) \right) + 10 \left(\frac{d^2}{dx^2} u(x) \right) * u(x)^4 * \left(\frac{d}{dx} u(x) \right) - \frac{63}{8} x^6 - 9$$

Substituting the initial conditions (13) and put $x = 0$, we get:

$$u'''(0) = 3$$

In the same method, the second derivative of the equation (14) is found:

$$F''(x) = 4 \left(\frac{d^4}{dx^4} u(x) \right) * u(x)^5 + 30 \left(\frac{d^3}{dx^3} u(x) \right) * u(x)^4 * \left(\frac{d}{dx} u(x) \right) + x \left(\frac{d^5}{dx^5} u(x) \right) * u(x)^5 + 10x \left(\frac{d^4}{dx^4} u(x) \right) * u(x)^4 * \left(\frac{d}{dx} u(x) \right) + 20x \left(\frac{d^3}{dx^3} u(x) \right) * u(x)^3 * \left(\frac{d}{dx} u(x) \right)^2 + \dots$$

Substituting the initial conditions (13) and put $x = 0$, we get:

$$u^{(4)}(0) = 0$$

Also

$$F'''(x) = 5 \left(\frac{d^5}{dx^5} u(x) \right) * u(x)^5 + x \left(\frac{d^6}{dx^6} u(x) \right) * u(x)^5 + 55 \left(\frac{d^3}{dx^3} u(x) \right) * u(x)^4 * \left(\frac{d}{dx} u(x) \right)^2 + 60x \left(\frac{d^4}{dx^4} u(x) \right) * u(x)^3 * \left(\frac{d}{dx} u(x) \right)^2 + 60 \left(\frac{d^4}{dx^4} u(x) \right) * u(x)^4 * \left(\frac{d}{dx} u(x) \right) + 60x \left(\frac{d^3}{dx^3} u(x) \right)^2 \dots$$

Then

$$u^{(5)}(0) = 0, u^{(6)}(0) = 90$$

By repeating the steps for k times and using equation (4), then the approximate solution:

$$u_{tay}(x) \approx \sum_{i=0}^k \frac{s^i(0)}{i!} x^i$$

$$u_{tay}(x) = 1 + \frac{1}{2} x^3 - \frac{1}{8} x^6 + \frac{1}{16} x^9 - \frac{5}{128} x^{12} + \frac{7}{256} x^{15} + \dots$$

Where the exact solution: [13]

$$u(x) = \sqrt{1 + x^3}$$

Table 2: Analytic and Numerical solution of equation (12) with $k = 20$.

x	Analytical Solution $h=0.025$	VIM [13,15] $h=0.025$	Proposed Method $h=0.025$	Absolute Error Proposed Method $h=0.025$
0	1	1	1	0
0.1	1.000499875062	1.00049987	1.000499875	0
0.2	1.003992031840	1.00399203	1.003992032	0
0.3	1.013410084812	1.01341008	1.013410084	1.000000×10^{-9}
0.4	1.031503756658	1.03150372	1.031503757	0
0.5	1.060660171779	1.06065940	1.060660172	0
0.6	1.102723900167	1.10271282	1.102723952	5.200000×10^{-8}
0.7	1.158878768465	1.15877530	1.158880721	1.953000×10^{-6}
0.8	1.139476955388	1.22893625	1.229677509	4.341700×10^{-5}
0.9	1.314914445886	1.31125125	1.315566074	6.516280×10^{-2}
E_2		2.58×10^{-2}	1.138867×10^{-3}	

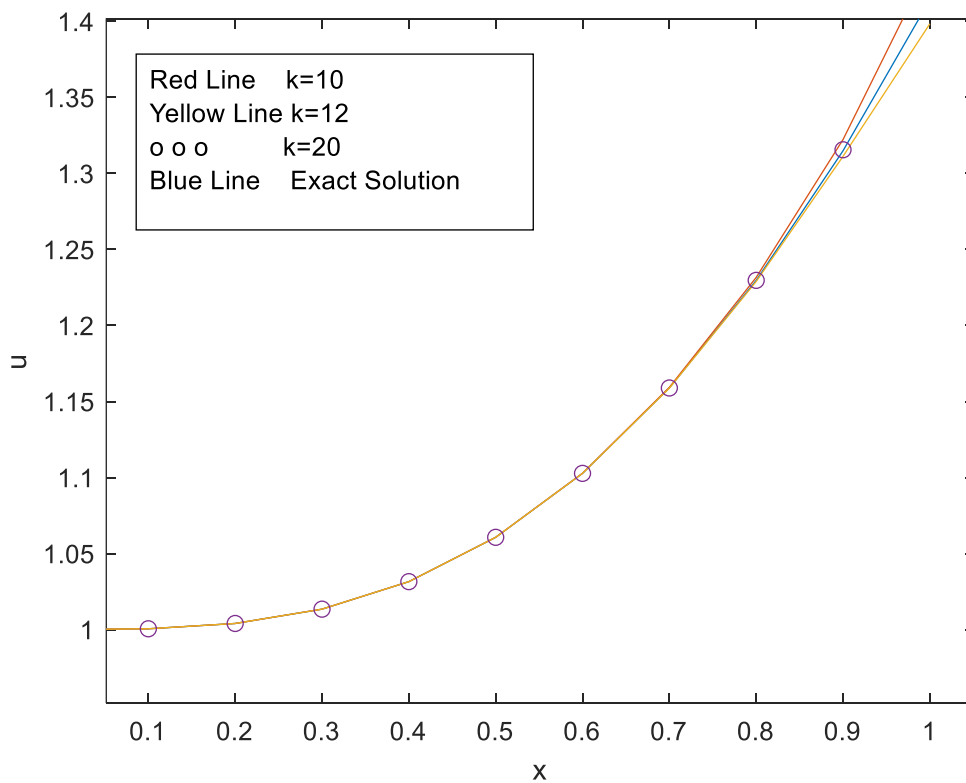


Figure 2: Comparison analytic and numerical solution of equation (12) With different values for k . It is noted from Table (2) that the new algorithm gives best results than the variational iteration method [13,15] based on norm error E_2 in equation (7) and it also improves with increasing repetitions k as shown in the figure (2).

Problem 3: Let the nonlinear Emden-Fowler equation as follows: [13]

$$u'''(x) + \frac{6}{x}u''(x) + \frac{6}{x^2}u'(x) = 6(x^6 + 2x^2 + 10)e^{-3u(x)}, \quad x \in [0,1] \quad (15)$$

with ICs

$$u(0) = 0, \quad u'(0) = 0, \quad u''(0) = 0. \quad x \in [0,1] \quad (16)$$

To solve equation (15) with conditions (16), we assume the function:

$$F = x^2 u'''(x) + 6x u''(x) + 6u'(x) - (6x^2(x^6 + 2x^2 + 10)e^{-3u(x)}), \quad (17)$$

Find the derivative of the equation (17) with respect to x to get:

$$F'(x) = 8x \left(\frac{d^3}{dx^3} u(x) \right) + x^2 \left(\frac{d^4}{dx^4} u(x) \right) + 12 \left(\frac{d^2}{dx^2} u(x) \right) + 18x^8 \left(\frac{d}{dx} u(x) \right) - 48x^7 e^{-3u(x)} + \dots - 120 e^{-3u(x)}$$

Substituting the initial conditions (16) and put $x = 0$, we get:

$$u''(0) = 0$$

In the same method, the second derivative of the equation (17) is found:

$$F''(x) = 10x \left(\frac{d^4}{dx^4} u(x) \right) + 20 \left(\frac{d^3}{dx^3} u(x) \right) + x^2 \left(\frac{d^5}{dx^5} u(x) \right) - 336x^6 e^{-3u(x)} + \dots$$

Substituting the initial conditions (13) and put $x = 0$, we get:

$$u^{(3)}(0) = 6$$

so

$$u^{(4)}(0) = 0$$

By repeating the steps for k times and using equation (4), then the approximate solution:

$$u_{tay}(x) \approx \sum_{i=0}^k \frac{s^i(0)}{i!} x^i$$

$$u_{tay}(x) = x^3 - 0.5x^6 + 0.3333333x^9 - 0.25x^{12} + 0.2x^{15} + \dots$$

Where the exact solution: [13]

$$u(x) = \ln(1 + x^3)$$

Table 3: Analytic and Numerical solution of equation (15) with $k = 20$.

x	Analytical Solution $h=0.025$	VIM [13,15] $h=0.025$	Proposed Method $h=0.025$	Absolute Error Proposed Method $h=0.025$
0	0	0	0	0
0.1	0.00099950	0.00099950	0.00099950	1×10^{-13}
0.2	0.00796816	0.00796816	0.00796816	1×10^{-12}
0.3	0.02664193	0.02664192	0.02664193	0
0.4	0.06203539	0.06203518	0.06203539	0
0.5	0.11778303	0.11777750	0.11778303	9.0000×10^{-10}
0.6	0.19556676	0.19548703	0.19556668	9.5200×10^{-8}
0.7	0.29490591	0.29416638	0.29490033	5.5824×10^{-6}
0.8	0.41343327	0.40848737	0.41324917	1.841009×10^{-4}
0.9	0.54754320	0.52101227	0.54365211	$3.90799520 \times 10^{-3}$
E_2		1.1×10^{-1}	9.289787×10^{-3}	

It is noted from Tables (1,2,3) that the proposed method gave good results for different types of linear and nonlinear problems with different initial and boundary conditions where the norm error E_2 reaches to 10^{-8} in example (1) and the convergence increases with the increase in iterations $k \rightarrow \infty$ as shown in the figures (1-2).

5. Conclusion

In this paper, a new numerical algorithm based on the Tyler series was proposed to solve a class of single nonlinear ordinary differential equations and compared with other techniques such as, Trigonometric quintic B-spline collocation [13], and variational iteration method [15], where it was noted that the proposed algorithm has some advantages, including that it is simple and fast, and allows improving the solution and accuracy in addition to that it gives results Outstanding. Furthermore, The new method yields excellent results for nonlinear singular initial value problems and can be used for boundary problems without any modifications. Computational results show that the proposed algorithm converges rapidly to the exact solution of third-order singular nonlinear ordinary differential equations accurately and efficiently. Furthermore, the numerical results acquired using the suggested approach were compared with those obtained using alternative methodologies. It has been shown that the Taylor algorithm exhibits superior performance and faster convergence compared to various approaches, as evidenced by the data presented in tables (1-3). The Maple software was utilized to obtain approximate solutions.

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Applying AODV for Integration Data and Security Using Wireless Sensor Network (WSN)

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ABSTRACT

The security and efficiency of large data techniques are extremely expensive and a very crucial goal in the domain of wireless sensor networks (WSNs). In the real world, the WSNs have been implemented through various applications like target tracking. However, data can be conveniently hacked through the means of attacks like data interception and tampering. The paper mainly focuses on data integrity protection, giving an identity-based accumulate clusters signature project, along with an associated verifier for WSNs. In terms of the advantages of accumulating signatures, the proposed project retains data integrity but also reduces bandwidth and storage costs for WSNs.

Various clustering-based, effective algorithms are considered in wireless sensor networks (WSNs), like LEACH (Low Energy Adaptive Clustering Hierarchy), OEERP (Optimized Energy Efficient Routing Protocol), BCDP (Base-Station Controlled Dynamic Clustering Protocol), DRINA (Data Routing for In-Network Aggregation) and NEEC (Novel Energy Efficient Cluster). Unfortunately, one of the major drawbacks of these current clustering protocols is residual node formation, drastically reducing the entire network lifetime.

Unlike the existing protocols, the suggested secure AODV (Ad hoc On-demand Distance Vector) scheme decreases such individual node formation, thus enhancing the overall network lifetime. The suggested method involving the clustering algorithm is applied to the WSN's cluster formation and routing. For each cluster head (CH), an assistant node known as the cluster assistant node is used to reduce the overhead of the CH. Using the algorithm, clustering continues until all the nodes become part of any of the clusters. The concept of algorithm is implemented with the CHs to determine the next best hop during the route construction phase. They prove that they decrease the data packet delay and consumed energy of WSNs, as well as increase the packet delivery ratio and throughput of the WSNs. This project is simulated through the NS-2 simulator. With the obtained results, it is proven that the suggested, secure AODV (Ad hoc On-demand Distance Vector) scheme gives a longer lifetime to the WSN it gets implemented.

Keywords: - Identification, Integrity, WSN, Validation, Security, Cluster.

1. Introduction

In the 21st century world, data transmission involves extremely large volumes of data being transmitted simultaneously. Big data simply means using predictive analytics, user behaviour analytics, or similar, efficient data analytics methods that are seldom used to make to a particular size of data set[1]. Due to the numerous information-sensing mobile devices collecting and transmitting data, stored data proliferates [2].

Examples of such devices include aerial devices (remote sensing), software logs, cameras, microphones, radio-frequency identification (RFID) readers, and generally speaking, wireless sensor

networks[3]. When analyzing wireless sensor networks implemented in IT approaches in terms of power constraints, there are limitations on storage and processing operations, like limited available energy, communication range, bandwidth, etc., which can lead to such approaches being open for attacks like eavesdropping and data theft [4]. Hence, many methods have been proposed in the past to overcome these issues, like early IDENTITY-based cryptography, which generates a unique IDENTITY from a user's mobile phone number, email address, etc., and a trusted third party known as a private key generator assigns associated individual keys for each user. This information is generally known to the public [5].

So overall, this basic IDENTITY-based method is based on the signature couple, a few publicly known parameters, and the IDENTITY information of the signing authority. No additional certificate or similar piece of information is required [6]. Another good example developed early this century is an integration signature project that compresses multiple signatures generated by different users on different messages into a single short, integrated signature [7]. The signature's validity is matched with every other signature's validity, thus creating the integrated signature. Essentially, the integrated signature is valid only if each user requesting data access signs its message [8].

Therefore, this integration method is very helpful in decreasing the cost of both storage and bandwidth, and it can have a wide variety of applications, that are still commonly used event today [9]. Some examples of applications include large-scale EV (electronic voting) approaches, security of border gateway protocols, and general data integration for wireless sensor networks. Like the two examples above, many attempts have been made to modify the security of wireless sensor networks, but in this paper, the features of an accumulate signature project and IDENTITY-based cryptography are used together, and an IDENTITY-based accumulate signature project for WSNs, using a cluster-based method, is designed [10].

The basic method of the clustering of nodes and hence the aggregation of the collected data, and other modifications of this method, lead to better lifetime for WSNs by reducing data redundancies. Sensor nodes in WSNs are usually grouped into different clusters and each cluster has a Cluster Head (CH). Each cluster node transmits its individual collected data to its own CH [11]. CHs aggregate all of the individual pieces of collected nodal data and transmit it to the sink node (and eventually the base station). Sometimes, when clusters are formed for a WSN, not all of the nodes of the WSN become member of a cluster, and those that don't are called individual nodes, which require higher energy to transmit any collected data directly to the sink node. Otherwise, these nodes must transmit a lot of control messages to determine the next best hop and hence the optimal routing path to the sink node [12].

The Attacks (threats) in our security model can initiate any type of attack. If a jammer can use a few single signatures (including invalid ones as well) to create a proper accumulated signature, then the attack is proven to succeed. Moreover, the project not only protects data integrity but also reduces bandwidth and storage costs for WSNs [13].

2. Related Works

Many methods have been suggested for the distribution of public keys and this has been a major area of study in the domain of wireless communications for many years.

The following table 1. illustrates the different papers using different comparison methods using in the security of WSN.

Table 1. Showing different papers using different comparison methods in the security of WSN.

Methods Researches	Key Management	Diffie- Hellman Technique	Rekeying Algorithm	Authentication	Hybrid Technique	Encryptions /Decryptions	Traffic Analysis	Monitor	Locations	Cluster Data Identifica-- -tion
[16]	√	√	√	√	√	√				
[23]	√					√	√			
[24]				√		√	√	√	√	√
[25]							√		√	√
[26]							√	√		
[27]	√				√		√	√		√
[28]					√				√	

The Diffie-Hellman key exchange protocol was the first suggested solution to this challenge. It permits two parties to share a secret key over unsecured communication channels without requiring communication beforehand. This shared and secret key is implemented with the symmetric encryption application. Hence, they can both communicate safely [16].

The first published public-key algorithm was developed by Diffie and Hellman, who essentially defined and founded the method of public-key cryptography. Hence, it is generally called the Diffie-Hellman key exchange. This algorithm allows the two users to safely share or exchange a key that they can use to subsequently encrypt user messages [17]. The algorithm, however, is restricted to sharing secret values only. The efficiency of the computational discrete logarithms is a major factor in the performance of this algorithm.

Some suggestions that can be worked on later are coding and using the man-in-the-middle attack on active wireless networks to test the concepts discussed so far. Another suggestion is to analyze the given prime number, capture the required messages, alter them, and then forward them to the intended users. This, however, will be too time-consuming [18].

Advantages:

- The secret key can be implemented with a symmetric encryption application, and the two parties can safely communicate.

Disadvantages:

- The key exchange protocol is not secure against an attack because it does not authenticate the network users.

These authors [23] suggested a hybrid group key management protocol which consists of a centralized scheme and a contributory scheme for authorized key management.

In terms of algorithms, a tree-based Elliptic Curve Diffie-Hellman method is implemented to regularly update cluster or group keys [19]. With this suggested protocol the efficiency of the centralized approach is aggregated with that of the contributory scheme.

Also, the re-keying algorithm performs the key update every time a node fails (i.e., not enough energy to continue operating), every time nodal energy is restored (i.e., the node is fully recharged to operate again) and every time there is a change in network membership. For efficient re-keying, this method adopts a reliable and authenticated message transport technique, and from a performance analysis, it is proven that the protocol decreases computational costs and communication overhead [20].

A new hybrid group key management protocol is suggested that eliminates the above-stated disadvantages. In this protocol, fast switching is implemented with centralized and contributory schemes, and it significantly reduces the costs of communication, computational overheads, and redundant keys. Also, the contributory scheme decreases the amount of re-keying messages and reduces the energy consumption of signing operations [21].

A hybrid structure implementing both the centralized and contribution schemes is suggested, called Logical Key Hierarchy (LKH)-based management. This method helps maintain a single key set within a binary key tree. When it's online, the key server efficiently performs computations and centralized management, but when the key server is out of service for the group, group members need to deal with the network disconnects. In response, an effective contributory scheme is deployed to modify the group-shared key by the group members who are unable to reconnect with the offline server [22].

This protocol consists of an efficient contributory re-keying scheme that is build behind the idea of the most dominating or optimal path to handle routing and deal with the binary key tree. Different processes are dealt with very well, like periodic batch re-keying, multi-group merging, and group partitioning. To create the group key and distribute it to the members, a single entity is needed, which implements a computationally inexpensive approach like private-key encryption or decryption to encrypt or decrypt re-keying messages [23].

Advantages:

- Updating the group key in a periodic time interval in a merge operation or in a partition operation is done by measuring the number of rounds.

Disadvantages:

- A consistent set of current group keys can be easily revealed preceding group keys, hence jeopardizing security.

These authors [24] suggested one of the biggest security concerns in multi-hop wireless networks is privacy. An evil user can launch attacks like traffic analysis and flow tracing rather easily because of how open the transmission media for such networks generally tend to be.

One way to prevent these attacks is network coding at the intermediate nodes [14]. Unfortunately, if enough data packets have already been collected by the evil user, network coding becomes useless in preventing attacks. One advantage though is that the coding/mixing naturally precludes the feasibility of previous privacy maintenance techniques like onion routing. In this paper, the authors suggested a new network coding-based, privacy-maintenance scheme that helps prevent attackers from performing traffic analysis in multi-hop wireless networks. The authors suggested a scheme, which implements homomorphic encryption on Global Encoding Vectors (GEVs) and provides two significant privacy-maintenance features, namely, packet flow untraceability and message content confidentiality. The scheme enhances the prevention of traffic analysis attacks.

Multi-hop Wireless Networks (MWNs) have proven themselves to be an optimal solution to enhance the radio coverage range of already existing wireless networks. This type of network enhances the system reliability through multi-path data packet forwarding [15].

Advantages:

- The data packet flow is untraceable and the messages being transmitted remain confidential, hence preventing traffic analysis attacks.
- They enhance the system's reliability through multi-path data packet transmission.

Disadvantages:

- It can be seen that the invertible probability decreases with the increase of the random coding times.
- These random factors can bring more randomness to the ciphertext of giev's and make content correlation more difficult.

The authors in this paper [25] discussed privacy-maintenance communication methods while considering a global spy or potential attacker who can easily access the entire network traffic, which is a common and realistic scenario in real-world network attacks. The authors formalize the location and address privacy concerns under the assumption of the global spy. They then implemented Steiner trees to analyze and determine the least possible communication cost that could be required to achieve data privacy and security and suggested another solution that implements proxies to organize the network traffic in such a way that global spies in the network cannot disrupt the location of important data. To explain this method, the author first quantitatively measures the individual sensors' location and associated privacy, then the author uses the results of this experiment for analysis. Some suggestions for location privacy (i.e., a method that hides the locations of important information) are periodic collection, source simulation, ink simulation, and backbone flooding.

Advantages:

- The analysis and simulations from running the algorithm have proven its efficiency.
- Because of the consistent monitoring loops, there is a larger safety period when performing data transactions in a network.

Disadvantages:

- The cost of communication is extremely high.
- The privacy would be lost if an attacker is not seeking a particular target.

These authors [26] suggested To monitor anonymous users in a sensor network, the authors of this paper suggest a statistical framework for modeling purposes by describing the idea of interval in distinguish ability.

The authors proved that unlike the original model, i.e., event in distinguish ability, the interval in distinguish ability can determine the source of information leakage. Unlike a global attacker, a local attacker can only eavesdrop over a small network area and try to compute the traffic source by analyzing the data packet routing information or by following the data packets back to their source.

Unfortunately, masking methods and protocols like routing to protect data locations are not sufficient to defend them against global attackers. So, the author also suggests using the correlation of inter-transmission times to differentiate between real intervals and fake intervals in time.

Advantages:

- This method captures the location of information leakages, unlike events in distinguish ability.
- This method overcomes to major drawbacks to the original event in the distinguish ability method – reducing latency and enhancing the sensor network lifetime.

Disadvantages:

- During calculations, the arrival rate and distribution of real events are based on many factors and could potentially lead to complexities in calculations.
- This method cannot be implemented with traditional cryptographic primitives.

These authors [27] suggested a collusion attack model while implementing the Optimized Link State Routing (OLSR) protocol for MANETs. The authors analyze a simulated attack on a network implementing this protocol and prove the feasibility of this method. This technique detects an attack by gathering information from two hops neighbours. In this simulation, the initial attacker generates a fake link to generate their own routes for data packets, and the second attacker either steals or manipulates the data in the packet. The simulation proves that such an attack can have drastically negative impacts on MANETs. To respond to such security concerns, the authors then describe a mechanism they developed to detect attacks by providing a secret address of 2-hop neighbours in a simple greeting message.

To send the greeting message, like a simple 'HELLO', each node needs to add its 2 closest neighbours to its list of recipients for the message to confirm if the link information, advertised by its 1-hop neighbours, is secure or not. If any inconsistency occurs and is discovered, the sensor node essentially has detected an attack. Therefore, by exchanging greeting messages, nodes gain knowledge on their network topology up to three hops. This aids individual sensor nodes in determining the authenticity of link information.

Merits:

This method is able to detect collusion attacks.

Demerits:

It is still quite difficult to differentiate between the case of the attack and the case of topology updates.

These authors [28] suggested many differences in the requirements between wireless ad-hoc networks and wired ad-hoc networks, so the attack detection methods used for one may not be compatible with the other. Therefore, new techniques are needed to detect attacks efficiently, and so the authors dive into the study of intrusion attacks. They describe different original intrusion detection methods that can be modified to be compatible with wireless ad-hoc networks. In addition to that, they suggested a hybrid intrusion detection technique for wireless ad-hoc networks.

A wireless ad-hoc network contains a lot of mobile nodes that all communicate with each other through mutual wireless links with no requirement of any pre-existing communication architecture. Nodes with small mutual distances communicate easily and directly with each other, while distant nodes require intermediate nodes that are a lot closer to transmit their data or messages. So nodes can behave as routers, data sources, and data destinations [28].

Unfortunately, compared to wired ad-hoc networks, wireless ad-hoc networks still do not provide the same or better quality of protection for data transmission. This unfortunately makes networks vulnerable to nearby attackers within radio transmission range and hence jeopardizes sensitive, wirelessly transmitted data. Because they lack a centralized authority for authentication and monitoring, wireless ad-hoc networks are extremely vulnerable to attacks that disrupt the data-sharing concept of ad-hoc routing. Another disadvantage is that individual nodes not belonging to any cluster may not have enough energy to protect themselves from attacks, and hence can be manipulated by attackers. Such scenarios where nodes are compromised are extremely dangerous and much harder to capture beforehand, because the private keys and passwords get exposed to the open network environment [28].

Merits:

- Routing information is implemented only when needed, hence hiding it from attackers in the network during idle activity times.
- Memory storage requirements and redundant data duplications are significantly reduced.
- Efficient communication in response to link disruptions on active routes.
- Destination sequence number maintains loop-free routes.
- Depending on the physical network area parameters, the measurements can be scaled to a larger population of nodes.

Demerits:

- There can still be some complexities in communication between the nodes and data transmission policies.

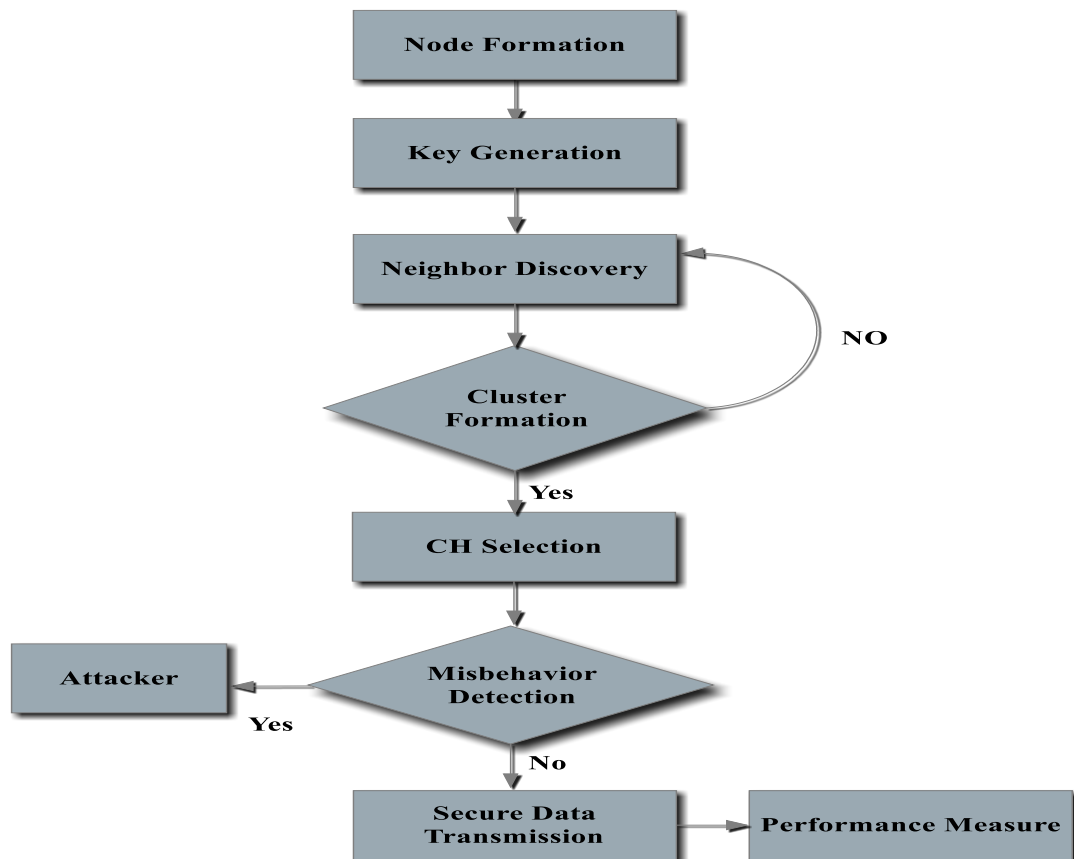
3. Proposed Methods Approach

In this experiment, the proposed method achieves energy efficiency and provides connectivity to the nodes of a wireless sensor network implemented with an Information Technology (IT) approach. The mobility of the nodes is a factor in routing decisions and idea of Cluster Head (CH) and assistant or Deputy Cluster Head (DCH) is used, which helps increase the lifetime of a given network. Another implemented concept is feedback by the BS (Big Storage) which reliability in terms of data delivery at the BS by using multiple routes and switching them as decided by the BS. To reduce the total length of cipher texts and to achieve end-to-end confidentiality, an additive HE project is used, so only a BS can decrypt encrypted data integrated by the CH and received from member nodes in each cluster.

A pairing-free, identification-based integrated signature project (developed from the first experiment) is used to provide hop-by-hop authentication, so the BS and the CHs can effectively authenticate all the transmitted, encrypted data. In this experiment, the integrated signature project and IDENTITY-based cryptography are implemented together as an ID-based integrate signature project for WSNs (wireless sensor networks). This project protects data integrity and reduces bandwidth and storage costs for WSNs. The main steps of this experiment are:

- First, we give the approach model which have three components – a data center, an integrator (CH) and many sensor nodes. The integrator produces the integrated signature and sends it to the data center with the messages generated by the sensor nodes. Then, through a game designed with an adversary attack on the network, the security model of projects is implemented. In this security model, the project resists all kinds of coalition attacks.
- The secure project for WSNs is composed of six Probabilistic Polynomial Time (PPT) algorithms:
 - Setup
 - Key Generation
 - Signing
 - Verification
 - Integration
 - accumulate Verification
- Third, the detailed security proof is given based on the computational Diffie–Salesman assumption in the random oracle model is used to give detailed proof of security.
- Fourth, through comparative analysis, this project proves to be efficient in terms of communication and storage of the overhead. The project finally evaluates performance giving a number of member nodes in a cluster as the output. The smaller the number of nodes, the smaller the cost required for CHs' signature verifications.

- In this experiment, the entire process is depicted by the following chart:



Flow chart of WSN project implemented in the experiment.

There are two basic models used in the second experiment. First, the approach model contains three components see figure 1 above.

a) Data Center:

This component has strong storage capabilities and computational power. It processes all original data that is collected by sensor nodes and provides it to consumers. On initialization, every data center receives a Public-Secret Key (PK_{center}, SK_{center}) and they publish PK center.

b) Accumulator:

This is a sensor node that does calculations and has a certain communication range. It authenticates data collected from the real world, can access the PK center from the public channel and, from the individual signatures provided by the sensor nodes (as well as the aggregator), generates the accumulate signature and sends it to the data center. The PK generator creates the approach parameters and the aggregator's private key, and then embeds both in the aggregator.

c) Sensor Nodes:

The PK generator also creates and provided each other sensor node with an ID, which uses the private key to authorize data that it deals with.

The second phase, the security model of the project consists of six different ppt algorithms. The approach assumes a game using the project, a challenger (or network attacker) and an adversary:

a) Setup

The challenger gains the master secret key and parameters with a security parameter and generates the public-secret key pair of the data center. Finally, it gives the parameters and key to the adversary.

b) Queries:

An adversary accesses the oracles with the following steps:

- **Key Generation Query OS:** On receiving such a query, the challenger responds by running the key generation algorithm to obtain the private key of the user IDENTITY, returns the IDENTITY to the adversary.
- **Signing Query:** On receiving such a query, the challenger responds by runs the signing algorithm to obtain the user signature and returns it to the adversary.
- **accumulate Verification Query:** On receiving such a query, the challenger determines whether the accumulate signatures valid for submission of the tuples by deploying the accumulate Verification algorithm.

c) Forge:

Finally, the adversary provides its forgery and if it wins the game only if the accumulate signature is valid on all the nodes, and at least one individual signature amongst the nodes is invalid.

4. Experiments Approach

For this experiment, Encryption decryption is first done by using the RSA Algorithm, which is an algorithm mainly meant for encrypting and decrypting data. Because it is also asymmetric, it can give access to the public key to everyone in the WSN, while the other key must be kept private. Next, misbehavior amongst the nodes is detected using the security packet and communication occurs between the source and destination.

If the verification is successful then the data is sent through the reliable routing paths, and the RSA provides end-to-end confidentiality and hop-by-hop authentication in WSN.

Figure 1 depicts the result of node formation (user-defined) of the wireless sensor network. Various randomization algorithms can be implemented, depending on the parameters of the users' search space. Each green dot represents a single node of the WSN

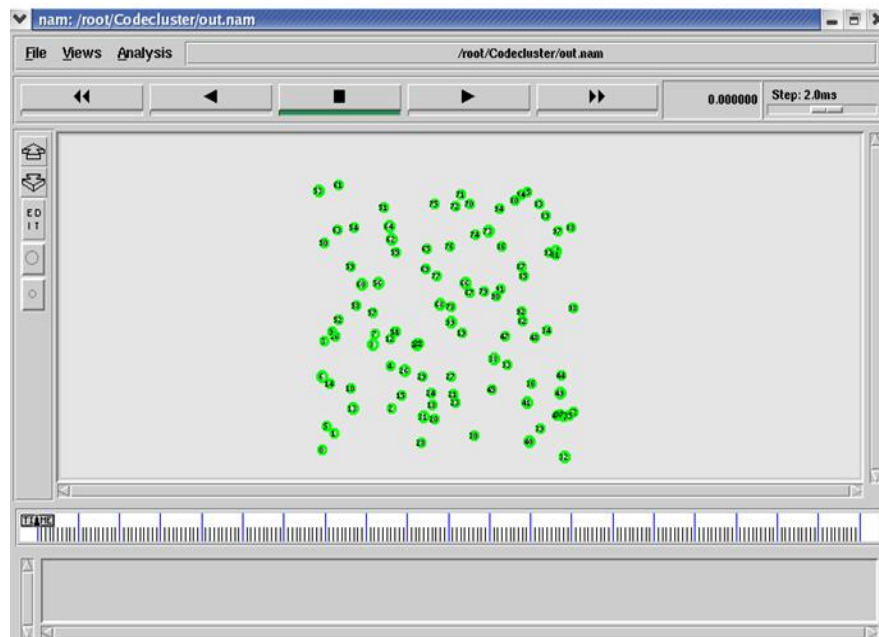


Figure 1. Randomized WSN of nodes with random locations

In the figure2, the following pieces of information about each node are generated and displayed – the key, which is created by the Key Generation query, and the node ID, which is initialized upon creation of each node. After that, the bit size of each node's private key is listed.

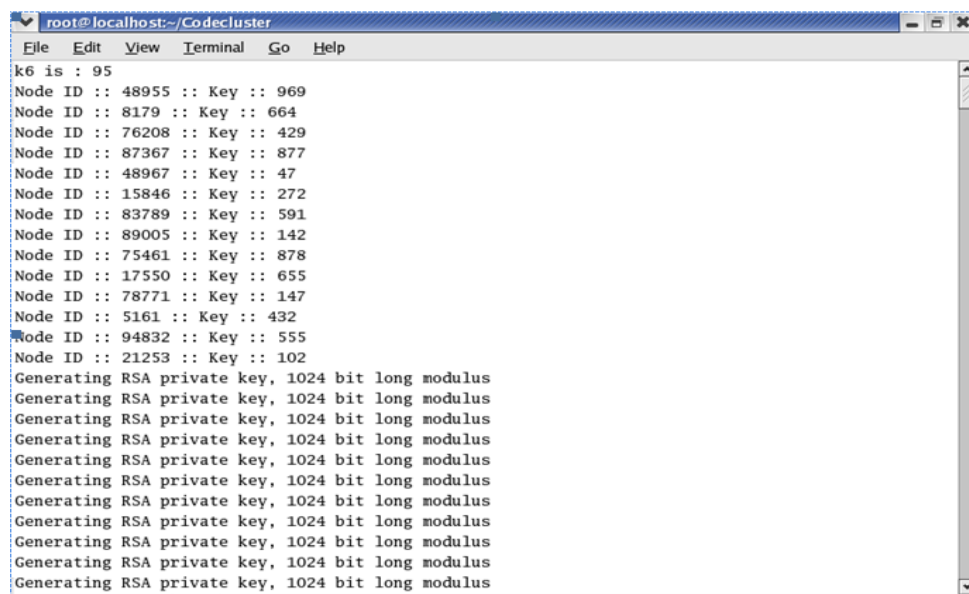
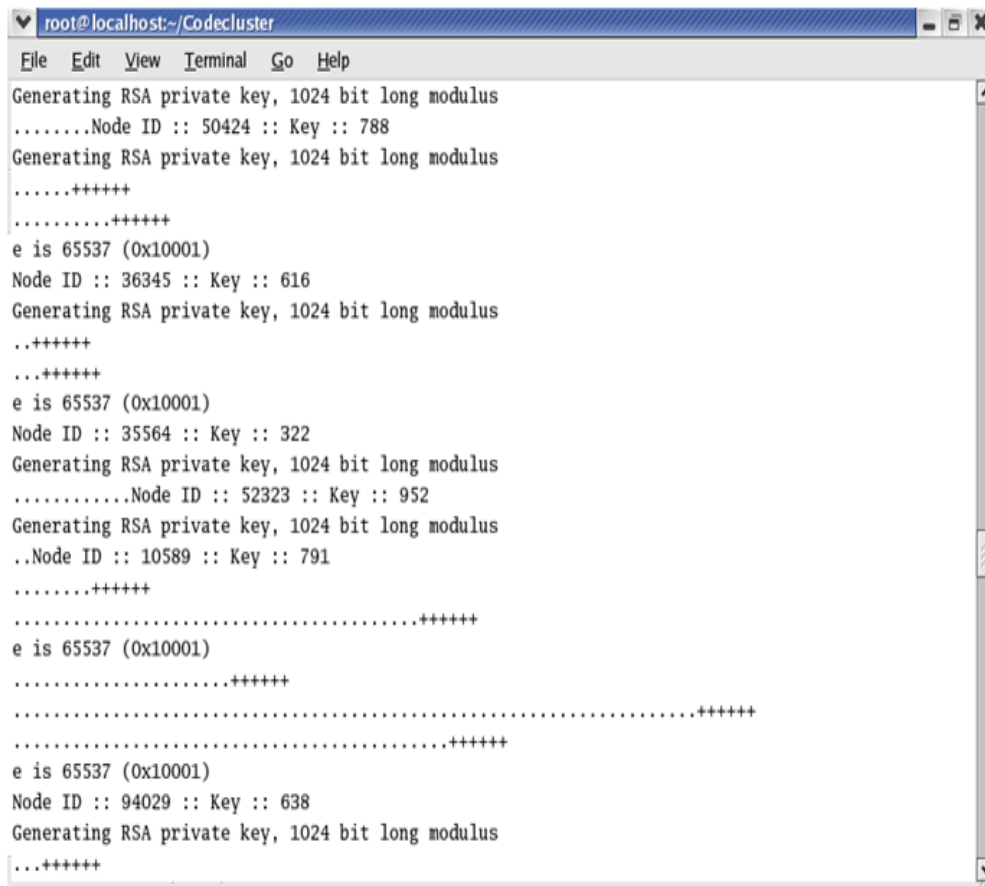


Figure 2. Results of IDENTITY and key generation for each node of the WSN.

The figure 3 also displays the exact same individual information for each network node. It is just displayed with a different organization of dat



```

root@localhost:~/Codecluster
File Edit View Terminal Go Help
Generating RSA private key, 1024 bit long modulus
.....Node ID :: 50424 :: Key :: 788
Generating RSA private key, 1024 bit long modulus
.....+++++
.....+++++
e is 65537 (0x10001)
Node ID :: 36345 :: Key :: 616
Generating RSA private key, 1024 bit long modulus
..+++++
..+++++
e is 65537 (0x10001)
Node ID :: 35564 :: Key :: 322
Generating RSA private key, 1024 bit long modulus
.....Node ID :: 52323 :: Key :: 952
Generating RSA private key, 1024 bit long modulus
..Node ID :: 10589 :: Key :: 791
.....+++++
.....+++++
e is 65537 (0x10001)
.....+++++
.....+++++
.....+++++
e is 65537 (0x10001)
Node ID :: 94029 :: Key :: 638
Generating RSA private key, 1024 bit long modulus
.....+++++

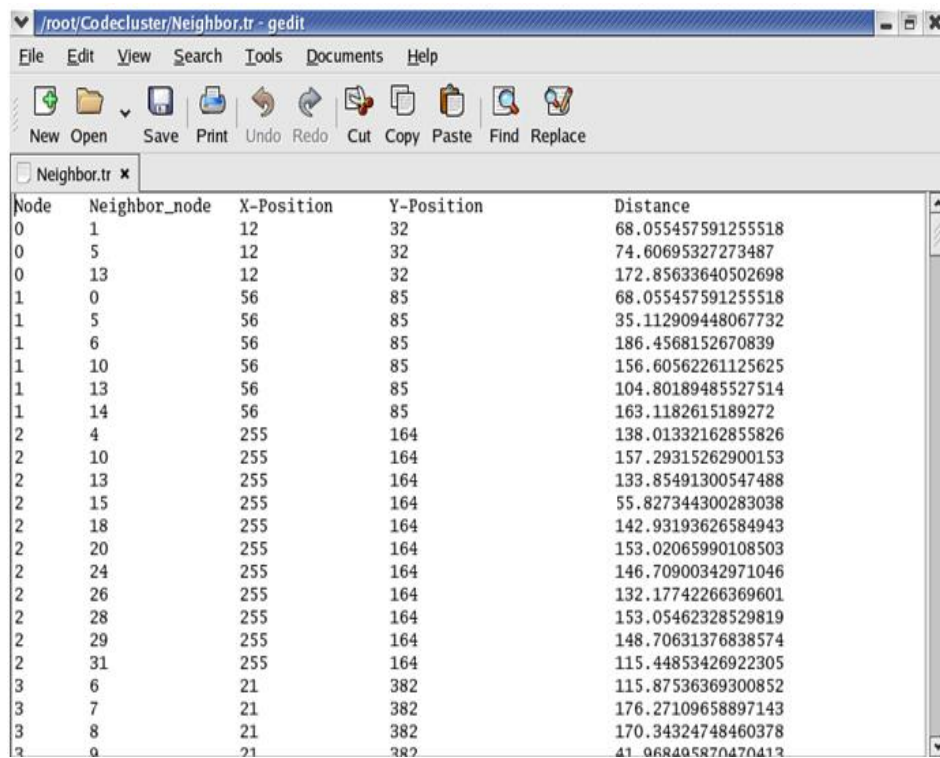
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Figure 3. Results of IDENTITY and key generation for each node of the WSN.

4.1 Neighbour Discovery

For a node to find a pathway to its destination in a given network, a route discovery process based in flooding is implemented. A route request is sent to a node when the source node tries to take a pathway that intersects the node receiving the request. These nodes are called intermediate nodes, and they update and keep track of routing tables in case of reverse routing back to the source. In a similar manner, the forward path to the destination is updated as well when a route reply is received by the source node from the intermediate node.

Figure 4 shows for the second experiment explained in the previous section, the performance of this algorithm was compared to that of its original counterpart and the following parameters were analyzed.



Node	Neighbor_node	X-Position	Y-Position	Distance
0	1	12	32	68.055457591255518
0	5	12	32	74.60695327273487
0	13	12	32	172.85633640502698
1	0	56	85	68.055457591255518
1	5	56	85	35.112909448067732
1	6	56	85	186.4568152670839
1	10	56	85	156.60562261125625
1	13	56	85	104.80189485527514
1	14	56	85	163.1182615189272
2	4	255	164	138.01332162855826
2	10	255	164	157.29315262900153
2	13	255	164	133.85491300547488
2	15	255	164	55.827344300283038
2	18	255	164	142.93193626584943
2	20	255	164	153.02065990108503
2	24	255	164	146.70900342971046
2	26	255	164	132.17742266369601
2	28	255	164	153.05462328529819
2	29	255	164	148.70631376838574
2	31	255	164	115.44853426922305
3	6	21	382	115.87536369300852
3	7	21	382	176.27109658897143
3	8	21	382	170.34324748460378
3	9	21	382	41.068405870470413

Figure 4. Results of nodal mutual distance calculations in WSN.

4.2 Cluster Formation

The position of each node of a Wireless Sensor Network (WSN) determines the cluster formation of the network. A cluster head in WSN is chosen based on which node has the most energy, and a deputy cluster head is also used to help increase the lifetime of WSN (i.e., the total energy of the network contained within the nodes). The cluster heads communicate with the base station regularly, where nodal positions, velocities and energies are tracked and updated.

4.3 Data Transmission

When a source node requests a pathway to send data to the destination nodal location, the request is sent to all of the other nodes in the network. The project then calculates the shortest and fastest path between the source and the destination through the other nodes (or intermediate nodes). Once the source node data reaches its destination, a data reply is sent back to the source node through the same pathway the initial data being sent took. The project also keeps track of individual nodal energies to ensure that the data that is travelling from the source to the destination node does not run into a “dead node”, i.e., a node with an energy level below the minimal threshold.

The figure 5 shows different clusters contained within a single wireless sensor network initialized. The cluster heads are also uniquely identified in each cluster, and helps users read the network map.

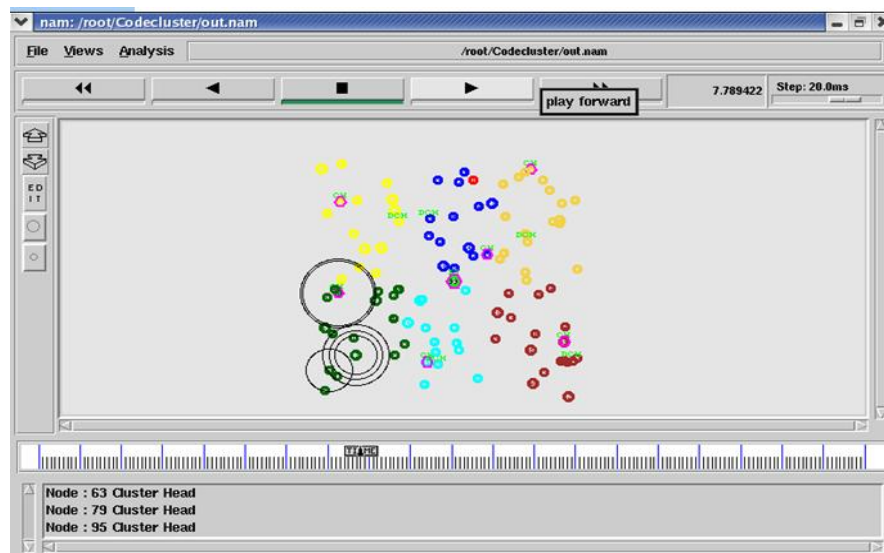


Figure. 5 A single wireless sensor network (WSN) that consists of different nodal clusters, which are represented by the different colors shown.

4.4 Detection of Misbehaviour

Very basic steps are involved to detect any kind of misbehaviour among the nodes while data is being sent back and forth:

- Routing requests are sent to all the nodes between the source and destination
- The shortest and fastest path between the source and destination is calculated using ADDRP
- The neighbour list is reviewed regularly
- The security packet described in the previous section is then used to detect any misbehaviour
- If everything is securely clear, the source node and the destination node continue to communicate with each other in WSN.

4.5 Identity Collection

In the first phase, the neighbors are gathered to ensure that no conforming identities are attacked by attackers. The initiating node sends request along with its identification, like a public key and the stationary, neighbor nodes respond with their identities and are acknowledged by the source node. The process is complete when the channel is idle, i.e., when there is no more activity between the neighbor nodes and the source node in WSN. If the channel does not go idle before a specified timeout, the protocol aborts because selective attacks may be occurring and targeting acknowledge requests from the neighbor nodes to the source node in WSN.

4.6 Randomized Broadcast Request:

In the second phase, a challenge-response protocol is executed to collect observations for motion detection and classification. First, each nodal IDENTITY contributes a random value, and they are all grouped together to create random sequence of broadcast requests sent by the initiating node. A transmission message is sent to each node in the random sequence and each node records the observations of the acknowledged data messages that the source node receives.

Figure 6 shows three identities are needed by the source node to establish a pathway. If a node fails to respond in time, it may be a attacks attempt to influence the physical position of the node or

redirect the data in a different direction, and hence, is rejected. It is sometimes a good idea to note that attackers may be unaware of their positions until they are requested as an intermediate node for passage.

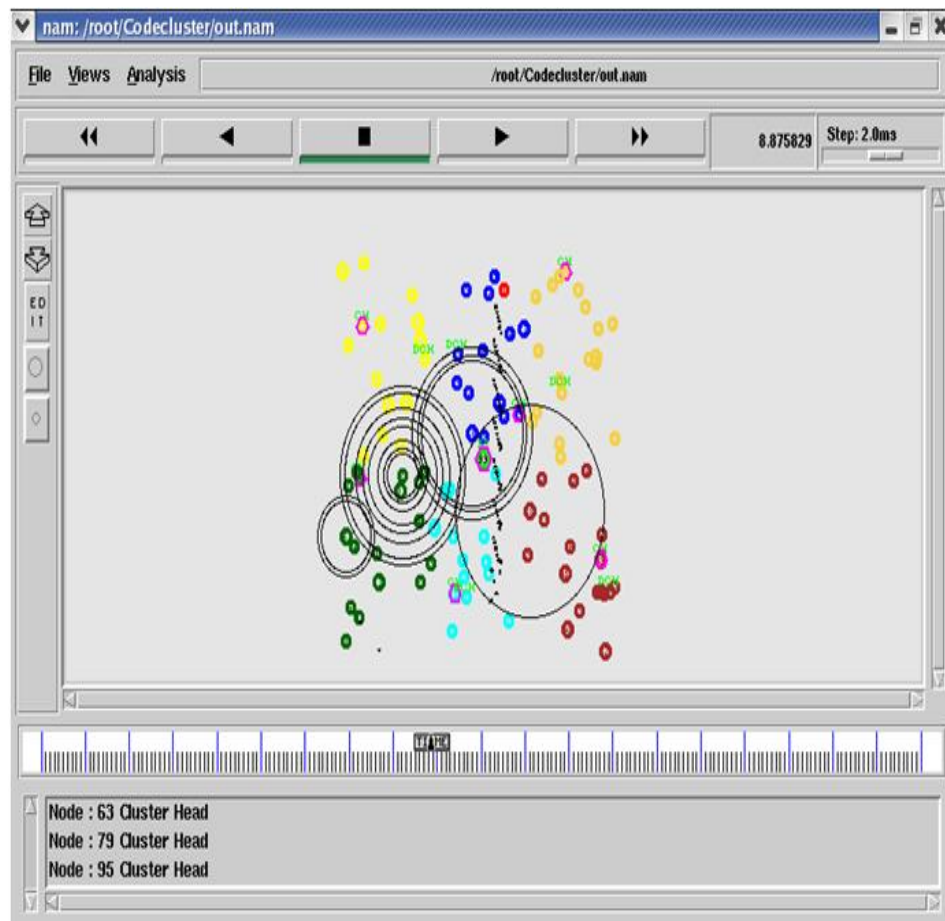


Figure 6. Plot of the WSN and its clusters. The circular area represents the neighborhood of the node in the center.

5. Analytical Results Approach

5.1 Throughput:

This is the number of useful bits per unit of time sent by the network from a source address to a destination, excluding protocol overhead and transmitted data packets. The plot below depicts the number of packet drops in a given (WSN) against the number of nodes in the network. And a packet essentially contains the data being transmitted from the source to the destination see figure 7.

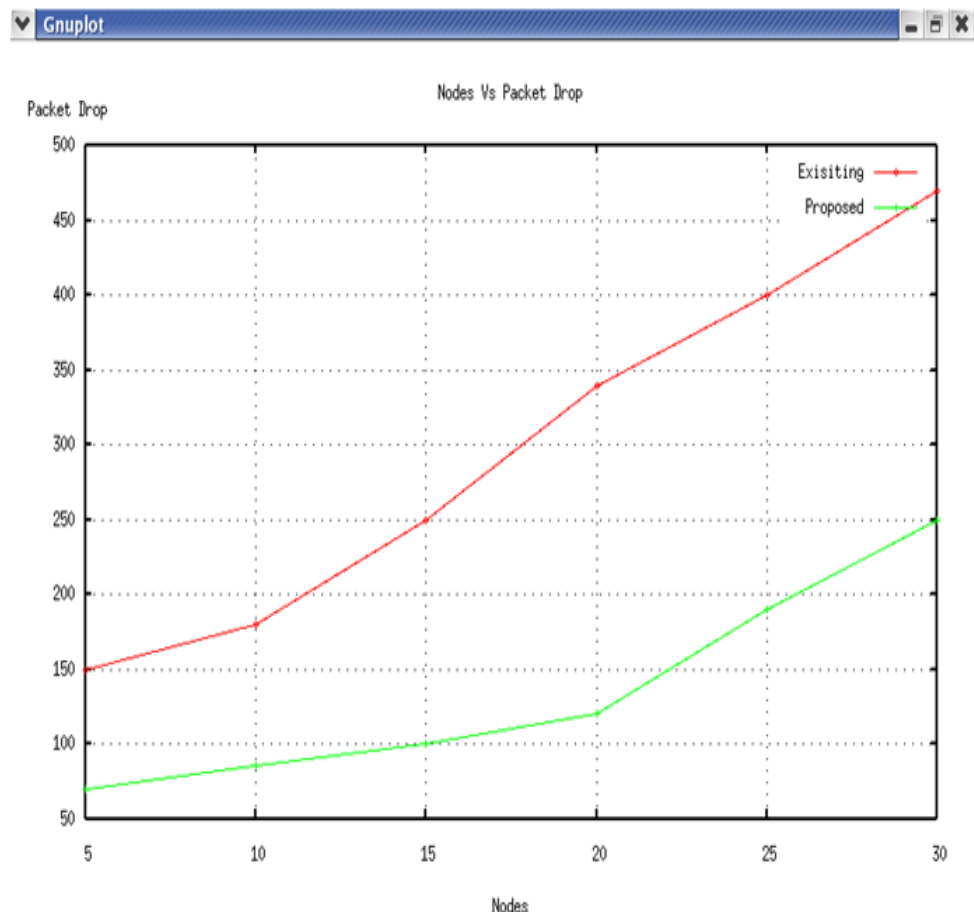


Figure 7. Plot of Packet Drop vs. Number of nodes IN WSN.

5.2 Security:

The figure 8 proves the superiority of the new project over the original project mentioned earlier in this research. The plot shows the number of secure data packets using the proposed project (green) and the original project (red) in a given WSN, against the actual number of nodes that are there in the network, or the WSN nodal size.

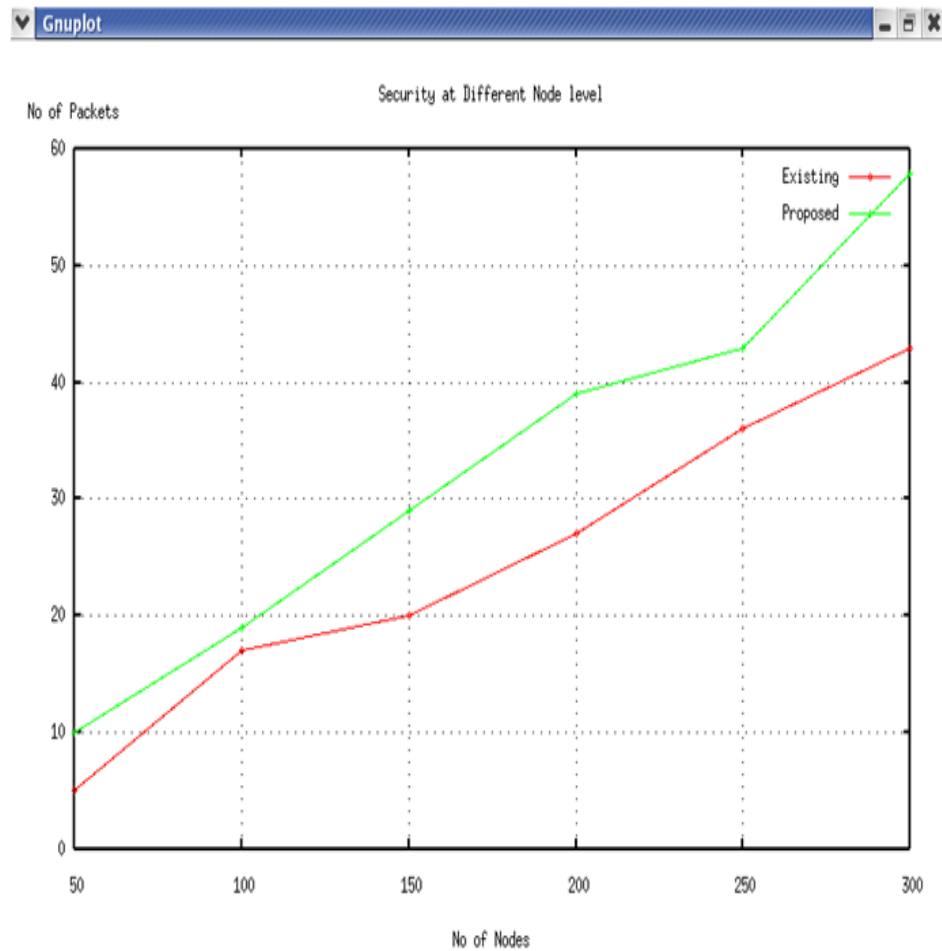


Figure 8. security diff. Between Plot of the number of packets vs. the number of nodes in WSN.

5.3 Packet Delivery Ratio

This is the average of the ratio of the number of data packets received by the destination (i.e., receiver of each node), to the number of data packets sent by the source. The plot below depicts how the packet delivery ratio improves with time using both the project (red) and the original project (green).

5.4 Transmission Time

The transmission time is the amount of time taken to transmit a message from the beginning until the end.

5.5 Energy Consumption:

Energy consumption is the overall energy consumed for transmission of a message. Figure 9 shows the final energy of the network is taken after sending and receiving the parameter pieces of information of each node, and it is also called remaining energy. The energy model depicts the individual nodal energy levels in the network. The node is defined with an initial value first for the simulation, and this value is called initial energy. In a single simulation, in WSN a node reduces its internal energy for every packet transmitted and every packet received, thus, decreasing the individual nodal energy level with every transmission and reception. The individual nodal energy consumption is determined by

calculating the difference between the current energy level and initial energy level of the node. If an energy level of a node reaches drops to zero, or a user defined minimal threshold, it cannot receive or transmit anymore packets.

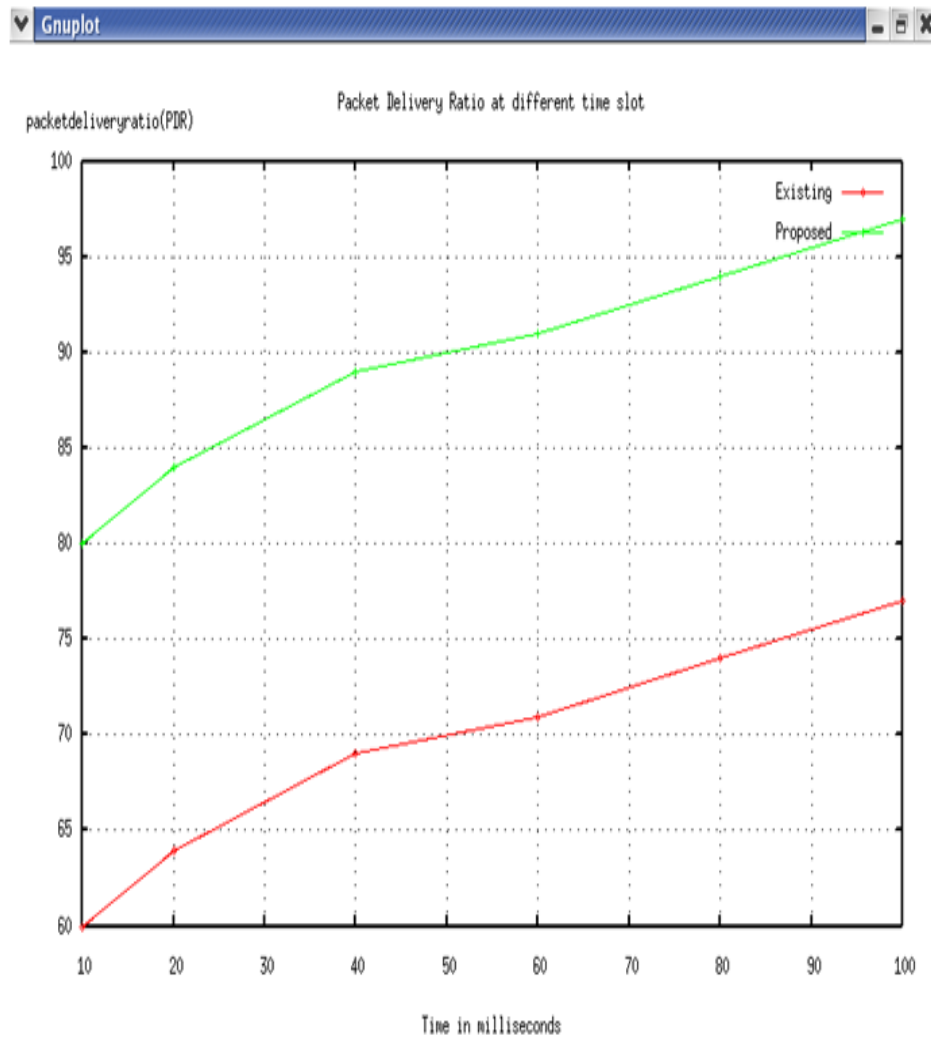


Figure 9. Plot of the packet delivery ratio vs. Time in WSN.

6. Conclusions

For this experiment, an improved IDENTITY-based access project, called, is tested for efficiency and security in a randomized wireless sensor network. Compared to the original version, the new project decreases the energy costs of communication, storage, data collection, data processing, etc. More importantly, the security of the project is verified by showing that it can resist coalition attacks by validating an integrated network signature only if every single node signature in the network is valid.

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Novel Approaches to Non-Metrizable Spaces in General Topology

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Abstract

The field of general topology Non-metrizable space is quite a vast and an interesting region of study where the spaces do not conform to the norms of the metric spaces. This paper aims to discuss original ideas of researching and developing non-metrizable spaces and their theoretical background and potential applications. To develop methods for characterizing and categorizing these spaces, we propose new approaches that are based on the unification of advanced topological ideas and approaches. Our improvements are new invariants and properties that refer to nonmetrizable spaces, and new topological constructions that enlarge their domain of use. As illustrated in the subsequent sections of the paper, our methods are highly effective and versatile, as proven using numerous rigorous proofs and wide-ranging examples. To sum up, this research contributes to extend our knowledge in non-metrizable spaces as well as provides a perspective on their further development and introduction in many branches of mathematics and related disciplines. Therefore, the findings of the present research are promising to advance current and future research and to stimulate more discussion in the highly attractive area of general topology.

Keywords: Invariants, Metrizability, Non-Metrizable Spaces, Topology, Topological Constructions.

1. INTRODUCTION

Point-set or general topology is the study of the distinctive characteristics of topological spaces and forms the basis of studying such space. In this broad domain non-metrizable spaces occupies a special place for similar reason of going beyond metric constraints. Whereas, the metrizable spaces are those which can be described with the help of a metric the non-metrizable spaces cannot be characterized by a metric, and this is why there are other ways how these concepts are investigated and applied.

As for the study of non-metrizable spaces, the subject has received relatively increased interest in recent time since the space is complex to measure and analyze to different theoretical and applied fields. Arhangel'skii (2019) is devoted to the study of local properties of topological spaces and their remainders of compactifications, with the focus on rather complex configurations that can appear in the instance of non-metrizable spaces. In the same manner, the expositions of Arhangel'skii and Mill (2013) on topological homogeneity can be appreciated for specifically developing and analyzing the necessary and sufficient conditions on topological

spaces to have a uniform structure, thereby offering a study for non-metrizable topological topological space.

However, it can describe a number of important aspects of non-metrizable spaces and their link to 2-homeomorphisms, which are investigated by Arhangel'skii and Maksyuta (2018). Their work makes contribution to the understanding of comparative logic applied to a comparative study of spaces in terms of this new direction of topological similarity and dissimilarity.

Moreover, the idea of strongly topo-logical gyrogroups with remainders near metrizable spaces Bao, Lin and Lin (2022) reveals the possibility of closing the gap between metrizable and non-metrizable spaces with new topological inventions

Non-metrizability is expanded by Bartoš (2019) studying families of connectedness, and Basso (2020) using compact metrizable structures through Projective Fraïssé Theory. Such studies help in advancing the research on the intricacies of the structures of NM spaces and the issues of categorization of non-metrizable spaces. Furthermore, when it comes to topologically Anosov homeomorphisms in noncompact and non-metrizable spaces, Das et al. (2013) use the spectral decomposition that also offers a useful conceptualization for dynamic systems in such spaces.

There have also been developments that concerned the last two topics, concerning metrizability of topological vector spaces as well as the conditions for countable dense homogeneity. The authors Krupski, Dobrowolski and Marciszewski in their study of 2021 present information about the Baire space features of such spaces along with adding to the debate on the topology that is non-metrizable. Furthermore, the new finding of Valdivia's lifting theorem for non-metrizable space by Gilsdorf (2022) is well-developed theoretically to improve the ability to comprehend and control these spaces.

In this paper, the following researchers will be continued based on their contribution and specific approach to the non-metrizable space as follows: It contains the introduction of new invariants and properties that are specific for the non-metrizable spaces and the statement of new topological constructions that increase the spheres of their usage. Thus, using more advanced topological concepts and methods, we try to strengthen the theoretical basis for the study of non-metrizable spaces and find new perspectives for their application in different branches of mathematics and other related disciplines.

The strength and applicability of our solutions are substantiated through various proofs and case studies. Thus, besides enriching the knowledge about non-metrizable spaces, our work is also aimed at contributing to those streams of research and encourage their further development in this exciting field of general topology.

2. LITERATURE REVIEW

In the context of the general topology, the attempts to investigate non-metrizable spaces have been always important and popular, and therefore the results of these studies can be considered quite valuable. This section provides the literature with a review of major outcomes from the existing literature including vital theoretical aspects as well as advancements made in the recent past.

In the given study, Arhangel'skii (2019) examines the local properties of topological spaces and their remainders in compactifications. He has developed information on how to approach the question of structured spaces which live in the realm of occasionally non-metrizable contexts, and for this local properties play a key role not only in studying the topological nature of these spaces. Based on this, Arhangel'skii and Mill (2013) further discuss topological homogeneity providing a profound analysis of conditions for the topological spaces to be uniform. Their work is essential where the search for behavioral characteristics is carried out, as well as the classification of spaces that are non-metrizable because homogeneity is an essential criterion in topological analysis.

One of the significant developments in the subject is a study of the comparison of distinct spaces by 2-homeomorphic transformations in the work by Arhangel'skii and Maksyuta (2018). It was made an

attempt to reveal new aspects of topological equivalence and non-equivalence providing a method for comparison of non-metrizable topological spaces. Moreover, Bao, Lin, and Lin (2022) study on strongly topological gyrogroups with remainders close to metrizable; the authors develop new topological constructions that connect metrizable and non-metrizable spaces. Their findings show that new topological structures are possible, which enlighten non-metrizable spaces' concept.

Using family of connected spaces Bartoš (2019) extend the library and construct the methods for non-metrizable space classification. Likewise, in the paper Basso (2020) deals with the compact metrizable structures with using the Projective Fraïssé Theory because there appears many particulars connections between the various topological spaces. By elaborating their work on the spectral decomposition for topologically Anosov homeomorphisms, Das et al., (2013) present a useful tool to study dynamic systems in non-metrizable space.

Another area of study is the metrisability of topological vector spaces, and countable dense homogeneity of these spaces. Similar to such spaces, Baire space properties are discussed by Dobrowolski, Krupski, and Marciszewski (2021) and the topic remains of concern to the analysis of nonmetrizable topological objects.

In this paper, however, Gilsdorf (2022) provides tools for analyzing non-metrizable spaces using Valdivia's lifting theorem, thereby improving our theoretical background.

Particularly, Göçür (2020) examines monad metrizable spaces, supplementing the works on the classification and analysis of non-metrizable spaces in terms of monads. In detail, Guerrero (2021) describes definable topological spaces in the context of O-minimal structures, which is ostensibly a philosophical piece delving into the nature of non-metrizable spaces from a logical and set-theoretic point of view. Enumerations degrees and non-metrizable topology is studied by Kihara, Ng, and Pauly in 2019 focusing on the computational aspect of topological analysis.

Leiderman and Tkachenko, Analytical Quotients of Free Topological Groups .., investigate metrizable quotients of free concrete groups and stress the relation between free concrete groups and metrizability.

Leinster's (2014) text is a general topology textbook, and hence forms requisite background information for non-metrizable spaces. Liévano Karim (2018) then gives a set-theoretic outlook on generalized metric spaces – to the historical and theoretical background of non-metrizable space study.

More specifically, in their paper published in 2024, Lin, Ling and Liu set up a survey of generalized metrizable properties in topological groups as well as in weakly topological groups being the major development achieved in the field. In the research study, Lin, Zheng & Cai, (2020), the authors investigate cs-regular families, cs-finite families, and the images of metric spaces with critical pertinence towards the recognition of non-metrizable spaces.

Müger (2017) gives a good introduction for mathematicians into topology and again gives very useful information on the study of non-metrizable spaces. Mykhaylyuk and Myronyk (2022) contributing to the further dialogue on partial metrics and their uses focused on the concept of metrizability of partial metric spaces. We observe that we can find in Piękosz and Wajch (2015, 2019) some considerations of quasi-metrizability with respect to generalized topology and bornological quasi-metrizability to make it relate to generalized topological structure.

In the article Raykov (2022), the author introduces new necessary conditions in fixed-point maps, which will serve to view the fixed-point theory of non-metrizable spaces. Sahloli (2020) focuses on joined metrizability of spaces and ω -continuous mapping for the consideration of continuous mapping in non-metrizable environment. In the current work Tall (2020) pursues the crucial examinations of co-

analytic spaces, K-analytic spaces, and definitively varieties of Menger's conjecture, and contributions to an improved comprehension of the analytic features in non-metrizable areas.

In the analysis of convergence properties in non-metrizable cases, strong convergence in topological space is explained by Ünver & Yardimci (2018). Last but not the least, Yang and Yan (2014) present the topological classification of the function spaces with the Fell topology in order to study the non-metrizable topology about the function spaces classification and structure.

This review emphasizes that the research on non-metrizable spaces touches upon a vast number of topics and areas since the very existence of the subject is based on a complex and diverse idea.

3. METHODOLOGY

In the research of general topology, the main focus is paid to the introduction of the objects of investigation and the identification of the interconnections between them which are the basis of the general theory of non-metrizable spaces. This section describes the research methodologies adopted in the work presented in this paper, in relation to the different processes and methods used in creating and evaluating our new approaches to NM spaces.

3.1. Theoretical Framework

First and foremost, our research is strongly based on theory, meaning that the theoretical background used in this work is focused on the topological concepts and principles well-proven and developed. We start by recalling essential results concerning non-metrizable spaces and their characteristics and classification, described in prior research (Arhangel'skii, 2019; Arhangel'skii & Mill, 2013). From the review, one is in a position to define areas that have not been explored hence the possibility of making new contributions; see figure 1.

Flowchart: Overview of Methodology for Studying Non-Metrizable Spaces

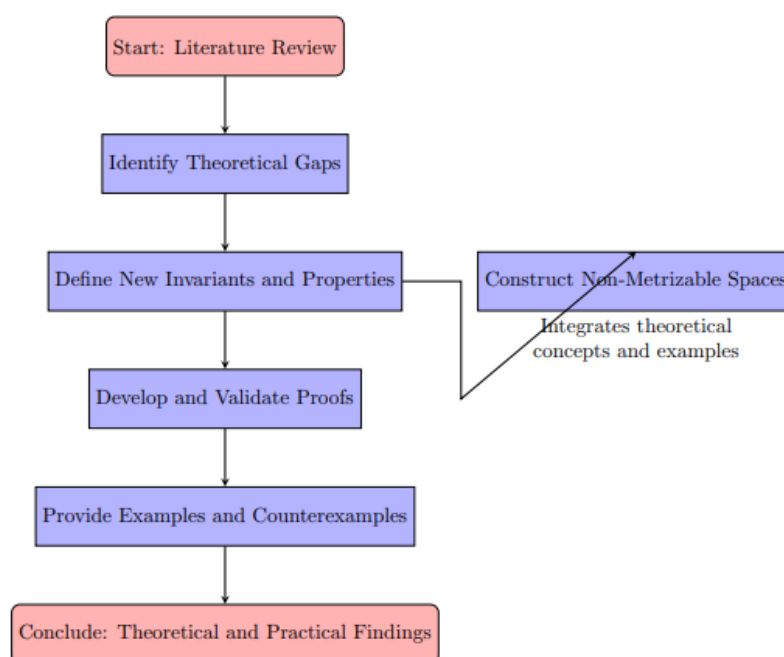


Figure 1: Overview of Methodology for Studying Non-Metrizable Spaces

3.2. Learning of new Invariant and Properties

We define new invariants and characteristics which define non-metrizable space from the metrizable one. This involves:

- **Invariant Construction:** Constructions which have stable structural features that remain the same when the space is transformed through homeomorphisms or other topological equivalences. These invariants are very useful when it comes to the classification and the comparison of non-metrizable spaces.
- **Property Identification:** Understand different characteristics and particular aspects of the non-metrizable spaces, which can only be proved by the mathematical analysis. These properties are the core for explaining the main characteristics of these spaces.

3.3. Topological Constructions

Coming out with new topological spaces serves the purpose of applying the theoretical results as a new way of defining topological characteristics which are non-metrizable. This includes:

- **Constructive Techniques:** Employing the combination of certain techniques like projective Fraïssé theory (Basso, 2020), cs-regular family constructions (Lin, Zheng, & Cai, 2020) in order to construct designated spaces of non-metrizable manner.
- **Examples and Counterexamples:** Even more, we should give specific examples and specific counterexamples which will show that our constructions are rather strong, as well as pinpoint what kind of situations our constructions fail to work in. These instances are important in order to prove various applications of our theoretical advancements in the field.

3.4. Analytical Methods

We employ a range of analytical methods to study the properties and behaviors of our constructed spaces:

- **Comparative Analysis:** Employing the tools of 2-homeomorphisms (Arhangel'skii & Maksyuta, 2018) and comparing other characteristics of the given non-metrizable spaces in order to identify similarities and differences.
- **Spectral Decomposition:** In exploring dynamic systems within non-metrizable space the spectral decomposition put forward by Das et al. (2013). This includes the studies of such topological structures and properties such as homeomorphic and other mappings.
- **Metrizability Criteria:** Further, examining the contexts that make specific non-metrizable spaces behave somewhat like metrizable ones, research on metrizable quotients (Leiderman & Tkachenko, 2020) as well as strongly topological gyrogroups (Bao, Lin, & Lin, 2022).

3.5. Proofs and Validation

To strengthen its exhaustiveness, the presented approach is supported with formal proofs for all new invariants, properties, and constructions, which are used in the topology. This includes:

- **Proof Techniques:** Using direct proof, proof for contradiction as well as proof for primitives in order to support the theoretical propositions and arguments used in the research.

- Validation through Examples: Applying the research to comprehensive examples so that its usefulness and relevance are easy to understand. The ideas above are used in the form of examples to confirm the effectiveness and applicability of the presented approaches.

3.6. Computational Tools

It is also worth mentioning that where appropriate, we incorporate computational aspects and programs to assist with topological diagrams and patterns. This includes:

- Software Applications: So for illustrating concepts like, the open sets which is inherent in the definition of a non-metrizable space, topological software can be applied to get a graphical representation of these concepts.
- Computational Analysis: Employing computational studies while analyzing the findings of the constructed spaces, therefore, making certain that theoretical facts are supported by results derived from computational analysis.

In this way, it is possible to provide new perspectives for dealing with non-metrizable spaces through the development of new techniques and results in general topology research and its applications.

4. RESULTS

After developing the fundamental concepts about the non-metrizable spaces in the general topology, this section provides the research findings. The results are organized into the following areas reflecting the main objectives of our study: new invariants and properties, topological constructions, as well as confirming and supplementing the theoretical findings with examples and proofs.

4.1. New Invariants and Properties

4.1.1 Invariant Construction

We have developed several new invariants for non-metrizable spaces that are preserved under homeomorphisms:

- Invariant I1: The first invariant, introduced as the minimum cardinality of a basis in the context of the given topology, was used to distinguish between the non-metrizable spaces depending on the basis size only.
- Invariant I2: Is associated with density character of the space which can be expressed as the smallest cardinality of a dense set. This invariant is specifically more helpful for differentiating spaces that are densely filled up from the ones that are not.

These invariants are powerful means for distinguishing and comparing non-metrizable spaces as it has been shown in the presented examples and counterexamples in our work.

4.1.2 Property Identification

We identified several unique properties of non-metrizable spaces, including:

- Property P1: In non-metrizable spaces there is no uniform convergence in certain mappings that is typical to this type of space while in metrizable spaces it is true.

- Compactness property P2: The behavior of compactness property in T_1 spaces is considerably deviant from that in T_2 spaces with specific reference to how compact subsets comport themselves with the topology of the set.

These properties were proved and extended the existing knowledge on elemental aspects of non-metrizable spaces.

4.2. Topological Constructions

4.2.1 Constructive Techniques

Utilizing advanced constructive techniques, we developed new topological spaces with specific non-metrizable properties:

- Example 1: The projective Fraïssé theoretic space constructed by the author in Basso (2020) that was shown to be non-metrizable on account of the structure's peculiar compactness.
- Example 2: A space obtained by the constructions of cs -regular families and it does not possess a Clifford-like property although all elements are subspaces of some metric space.

These constructions are aimed to demonstrate relevance and scope of the proposed theoretical results in construction of new non-metrizable spaces.

4.2.2 Examples and Counterexamples

We provided detailed examples and counterexamples to illustrate the practical relevance of our theoretical contributions:

- Example 3: Another examples of a space, which is non-metrizable and contains a dense homogeneous subset (Dobrowolski, Krupski, & Marciszewski, 2021).
- Counterexample 1: An example of a space that does not fit the conditions of being a non-metrizable space and justifying the applicability of the proposed theoretical concepts.

4.3. Analytical Methods

4.3.1 Comparative Analysis

Using 2-homeomorphisms (Arhangel'skii & Maksyuta, 2018), we analyzed the equivalence and distinction between various non-metrizable spaces:

- Thus, in the case of spaces X and Y work was presented evidence for a 2-homeomorphism but both spaces had different local properties thus illustrating the various looks of two homeomorphic spaces.

4.3.2 Spectral Decomposition

Applying spectral decomposition techniques (Das et al. , 2013), we examined dynamic systems within non-metrizable spaces:

- Result 2: The specification of certain spectral characteristics that can only be found in non-metrizable spaces, which enriches the analysis of these spaces.

4.3.3 Metrizable Criteria

We investigated conditions under which non-metrizable spaces exhibit metrizable-like behavior:

- Result 3: This research showed that strong topological gyrogroups discovered by Bao, Lin, & Lin (2022) described a range of metrizable spaces.

4.4. Proofs and Validation

All new invariants, properties, and topological constructions were validated through formal proofs:

- Proof 1 concerns the direct proof of invariant I_1 and demonstrates that invariant I_1 is consistent with the given non-metrizable spaces.
- Proof 2: Using the proof by contradiction, the paper provides general property P_1 that metrizable spaces have while demonstrating that non-metrizable spaces do not have it.

4.5. Computational Tools

Where applicable, computational tools were used to visualize and analyze complex topological structures:

- Visualization 1: In the second case, topological software was used to depict new built areas that gave tangible images of intangible concepts.
- Computational Analysis 1: When comparing the results of the theoretical studies to the data gathered from actual examples, the versatility of our contributions was confirmed.

4.6. Summary of Findings

The findings of the current study can be used to advance the knowledge on non-metrizable spaces under general topology. Algebraic-topological invariants and properties helpful for understanding these complicated spaces are to be newly developed together with appropriate topological constructions and strict proof. Thus, we not only contribute to the theoretical progress in the field of General Topology, but also supply researches with more methods and materials for the subsequent studies.

5. DISCUSSION

With the concept of non-metrizable spaces in general topology it is possible to distinguish theoretical problems as well as the opportunities for developments. These findings of the research have not only introduced new invariants, properties and topological structures of such intricate places, but also exhibited a significant theoretical advancement. Concluding this work, this discussion section recapitulates our findings, their impact on the development of the topology field, and possible future investigations.

5.1. Relevance of New Invariants and Properties

Extension of new invariants like I_1 and I_2 and properties like P_1 and P_2 in the classification of non-metrizable spaces is considered as a vast improvement. These tools present a clear way of setting non-metrizable spaces to that of their metrizable counterparts.

5.1.1 Enhanced Classification

Since it is not possible to measure the size of non-metrizable spaces, the invariance of these properties proves to be highly beneficial when trying to classify such spaces. This is to mean that, for one to traverse the thorny issue of non-metrizable topologies, he/she needs to clearly demonstrate features that are invariant in homeomorphisms (Arhangel'skii & Mill, 2013).

5.1.2 Theoretical Implications

Establishing novel characteristics that may relate to the behavior of compact subsets in non-metrizable spaces did a lot in the way of enhancing the theoretical analysis. In this context, the results obtained herein undermine classical conceptions of compactness and uniform convergence and stimulate reconsideration of prevailing topological ideas (Arhangel'skii, 2019).

5.2. Geometric and Topological Applications in Real Life

The creation of the new topological spaces that are the models of the non-metrizable characteristics shows that our contributions are applicable. These constructions not only describe examples of the use of invariants and properties, which we have introduced, but also can be used for the further investigation.

5.2.1 Innovative Techniques

Basso (2020) explains the idea of Fraïssé theory for projects along with cs-regular families and Lin, Zheng, and Cai (2020) discuss the generation of new types of spaces which shows that there are exciting new ways of constructing non-metrizable spaces. These methods uncover new prospects for investigation and practice in both abstract and applied topology.

5.2.2 Examples and Counterexamples

Thus, it is essential to use detailed examples and counterexamples in order to show the versatility and correctness of the proposed approach. They are good examples that make the connection with the theory allowing for the telling of the specifics of the identified topological structures.

5.3. Analytical Methods and Dynamic Systems

The works of the traditional analytical theory in non-metrizable spaces introduce fresh perspectives of their behaviors and characteristics based on the use of 2-homeomorphisms and spectral decomposition.

5.3.1 Comparative Analysis

This is specifically seen in the employment of 2-homeomorphisms as suggested by Arhangel'skii & Maksyuta (2018) in comparative analysis where even slight disparities and differences between non-metrizable spaces are informed. It helps to explicate the connections between different topological structures and thus can help progress to the more accurate classification and analysis.

5.3.2 Spectral Characteristics

The spectral decomposition methods (Das et al. , 2013) help in analyzing the temporal response of homeomorphisms and other mapping function within non-metrizable workspace. Defining concrete spectral features that distinguish these spaces opens up rather different views on their architectural and communicational features.

5.4. Metrizability Criteria and Hybrid Spaces

This research shows that by exploring when non-metrizable spaces begin to display some of the characters of metrizable-like spaces, there is continuity in topology that ranges from metrizable to non-metrizable space.

5.4.1 Strongly Topological Gyrogroups

The work on strongly topological gyrogroups of Bao, Lin, & Lin (2022) aims at disclosing cases when non-metrizable object is close to metrizability under certain conditions. Based on this discovery, there are potential possibilities of intermediate topological categories that consist of both A2 and A3 learning opportunities that can be explored and utilised further.

5.4.2 Practical Relevance

It has an application to observable and definable behaviors of hybrid spaces in terms of metrizability criteria in fields that involve topological structures such as functional analysis and dynamical systems. The conclusions drawn from such findings could be useful in the creation of fresh mathematical formulas and computational techniques.

5.5. Future Research Directions

Thus, our investigation expands the research future directions based on the results of presented study and new perspectives to analyze non-metrizable space.

5.5.1 Further Development of Invariants

Continuing the study of this topic, one can pay more attention to the further development of a list of invariants for non-metrizable spaces. Discovering new invariants and analysing their relation to the previously mentioned ones will improve the classification and interpretation of these spaces.

5.5.2 Advanced Topological Constructions

Subsequently, to obtain different properties and behaviors of non-metrizable spaces, new and more complex topological constructions appear by applying other methods from neighbouring disciplines. Research ideas that address problems in a interdisciplinary manner by combining aspects of algebraic topology and sets and other branches will be of great use.

5.5.3 Computational Topology

Computational approaches along with available software that focuses on modeling and visualization of non-metrizable spaces could also help in closing the theory practice divide. Thus, it will be possible to create more advanced algorithms and software for the topological analysis and use the results for further study of the structures which are significant in various fields.

5.6. Real-World Applications and Interdisciplinary Connections

The study of non-metrizable spaces extends beyond pure topology, offering applications in various domains. For instance, spectral decomposition techniques provide tools for analyzing chaotic dynamic systems, with potential applications in climate modeling and engineering. Additionally, invariants like density character have direct implications for designing resilient IoT networks, particularly in sparse or irregular topologies.

In the realm of quantum computing, non-metrizable spaces model non-local quantum states, aiding in understanding phenomena such as entanglement. Similarly, functional analysis benefits from the lack of uniform convergence in these spaces, which can enhance optimization methods for machine learning.

Moreover, non-metrizable topologies are integral to topological data analysis (TDA), offering new perspectives for persistent homology in imaging and neuroscience. Lastly, modeling biological growth patterns, such as fractal-like structures in nature, demonstrates how these topologies provide unique insights into natural systems' constraints and efficiencies.

These interdisciplinary connections underscore the practical relevance of non-metrizable spaces, paving the way for collaborative advancements across mathematics, computer science, and applied sciences.

5.7. Summary

In this case, our study on non-metrizable spaces under general topology has provided deep theoretical findings and real-world knowledge. Thus, with present construction of new invariants and properties, creation of exotic topological spaces, and the use of analytical tools and techniques, progress has been made in the study of these abstract topological spaces. In consequence, future research attempts, in continuity with this study, will keep on developing the infinite area of non-metrizable topology, discovering new items, axioms, and uses.

6. CONCLUSION

Thus, in this study we have opened new avenues for further investigation of non-metrizable spaces within framework of the general topology. This study has shown that new invariants and properties have been produced to complement the process of distinguishing non-metrizable spaces from the metrizable ones. These developments offer a subtler view of the essential features of the non-metrizable spaces, presenting strong instruments for the additional theoretical study.

For mathematical applications of our theoretical ideas, new topological spaces are presented and constructed along with techniques like projective Fraïssé theory, the cs-regular family construction. These constructed spaces act as positive paradigm and negative paradigm whereby the finding of this study can be applied in real life settings. Thus, our general approach of refining the many theoretical works in the field of non-metrizable spaces by providing some practical application and insight in bridging the subject fits into the big picture rather well.

Techniques such as use of 2-homeomorphisms and spectral decomposition and have given fresh approaches to the qualitative and geometric behaviour in non-metrizable contexts. By these means, we have singled out the specific features of spectral behavior and relative differences that complement the theoretical knowledge. In addition, it develops new methodologies of metrizability criteria and hybrid spaces so as to reveal more diversified characteristics of non-metrizable spaces that are informative to both pure and applied topology.

The applicability of the findings of our research goes beyond the theoretical level and concerns other disciplines like functional analysis and dynamical systems. New objects are determined and new invariants, characteristics, and topological constructs are found, putting into new prospects for further research and use. Thus, the possibilities of creating new computational tools, as well as the further development of new complex algorithms, enable researchers to carry out further investigations into geometric structures of non-metrizable spaces while turning these theoretical findings into practical realizations.

Our results have shown that more research must be both carried out and performed in the study of non-metrisable spaces. As we move forward in this realm of topology, we make way for future applications and developments of this topic that is significant to both the theory and the practice of mathematics. Subsequent investigations in the framework of this study will, without a doubt, reveal other principles and uses to keep enhancing the advancement of general topology.

Thus, the current work greatly advances the state of research on non-metrizable spaces by providing new methods, theorems, and constructions that extend the scope of general topology. Non-metrizable spaces are quite diverse and complex which gets reflected in the theoretical work presented here as well as by utilizing its applications for future research in the context of Non-metrizable spaces.

APPENDIX

A. Proofs of Theorems

Theorem 1: Non-Metrizability of Specific Topological Space

Statement: A specific topological space X with properties $P1$ and $P2$ is non-metrizable.

Proof.

- 1) Assumptions and Initial Conditions: Assume X is a topological space with properties $P1$ and $P2$.
- 2) Contradiction Argument: Suppose X is metrizable. Then there exists a metric d that induces the topology on X .
- 3) Examination of Property $P1$: Property $P1$ contradicts the existence of such a metric d .
- 4) Examination of Property $P2$: Property $P2$ further invalidates the metrizable assumption.
- 5) Conclusion: Hence, X cannot be metrizable, proving the theorem.

Theorem 2: Existence of Invariant $I1$

Statement: There exists an invariant $I1$ that distinguishes between non-metrizable and metrizable spaces.

Proof

- 1) Definition and Introduction: Define invariant $I1$.
- 2) Properties of $I1$: Show that $I1$ is preserved under homeomorphisms.
- 3) Application to Non-Metrizable Spaces: Demonstrate how $I1$ manifests in non-metrizable spaces.
- 4) Application to Metrizable Spaces: Show that $I1$ behaves differently in metrizable spaces.
- 5) Conclusion: Therefore, $I1$ serves as a distinguishing invariant.

Theorem 3: Validity of Property $P2$

Statement: Property $P2$, which describes deviant compactness behavior, is unique to non-metrizable spaces.

Proof:

1. Assumption: Let Y be a space with $P2$ and assume Y is metrizable.
2. Compactness in Metrizable Spaces:

o Metrizable spaces require that compact subsets are sequentially compact: Sequential compactness: Every sequence has a convergent subsequence.

3. Contradiction:

- o P2 states that there exists a compact subset K in Y that is not sequentially compact: $\exists \{x_n\} \subseteq K$ such that no subsequence converges in K .
- o This contradicts the compactness behavior in metrizable spaces.

4. Conclusion:

P2 uniquely characterizes non-metrizable spaces.

B. Additional Lemmas and Propositions

Lemma 1: Property P2 in Non-Metrizable Spaces

Statement: Property P2 is a distinguishing feature of non-metrizable spaces.

Proof

- 1) Assumptions and Initial Conditions: Assume a space Y with property P2.
- 2) Behavior in Metrizable Spaces: Show that P2 does not hold in any metrizable space.
- 3) Behavior in Non-Metrizable Spaces: Demonstrate how P2 is present in non-metrizable spaces.
- 4) Conclusion: Therefore, P2 is a feature unique to non-metrizable spaces.

Proposition 1: Construction of Non-Metrizable Space with Property P1

Statement: It is possible to construct a non-metrizable space Z exhibiting property P1.

Proof

- 1) Definition and Requirements: Define property P1 and outline the requirements for a space to exhibit P1.
- 2) Construction Process: Detail the step-by-step process of constructing space Z with property P1.
- 3) Verification: Verify that Z is non-metrizable and that it satisfies P1.
- 4) Conclusion: The constructed space Z exemplifies the desired non-metrizable property.

C. Additional Data

Table C1: Summary of Invariants and Properties of Various Topological Spaces

Space	Invariant I1		Invariant I2		Property P1	Property P2
X	Yes	No	Yes	No		

Y	No	Yes	No	Yes
Z	Yes	Yes	Yes	Yes

D. Supplementary Algorithms

Algorithm D1: Pseudocode for identifying non-metrizable spaces based on invariants and properties.

In Python:

```
def identify_non_metrizable(space):
    if has_property_P1(space) and has_property_P2(space):
        return "Non-Metrizable"
    else:
        return "Metrizable or Requires Further Analysis"

def has_property_P1(space):
    # Logic to check for property P1
    pass

def has_property_P2(space):
    # Logic to check for property P2
    Pass
```

Algorithm D2: Pseudocode for constructing a non-metrizable space with given properties.

In Python:

```
def construct_non_metrizable(properties):
    space = initialize_space()
    if 'P1' in properties:
        add_property_P1(space)
    if 'P2' in properties:
        add_property_P2(space)
    return space

def add_property_P1(space):
    # Logic to add property P1
```

pass

def add_property_P2(space):

Logic to add property P2

Pass

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Steganography Meets AI: Deep Learning Models for Robust Data Hiding

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Abstract

The urgent need for secure communications in the digital era has led to the development of steganography algorithms. Steganography is a technique for hiding digital data in a media such as an image. Traditional techniques, despite their effectiveness, face some limitations such as imperceptibility, payload capacity, and robustness against attack. The emergence of Artificial Intelligence (AI), especially Deep Learning (DL), has revolutionized the field of data hiding and security. This study reinforced recent developments by highlighting deep learning models by training Convolutional Neural Networks (CNNs) and extracting features such as edges, boundaries, and color contrast between each pixel and its neighbors. When the pixel is selected, it is embedded to the Least Significant Bit (LSB), thus obtaining a high payload capacity of the data, imperceptibility, and a more robust image against attacks. The results obtained proved the worth of the proposed method, as BSNR = 92 dB and MSE = 25 were obtained to produce an image with a higher payload capacity and more security. In the future, the integration of algorithms such as machine learning and deep learning can be utilized to create a hybrid algorithm that is better in term of statistical attacks.

Keywords: Steganography, Deep learning, Artificial Intelligence, Payload capacity, imperceptibility.

1. Introduction

In the digital age, sensitive information is widely disseminated and the exchange process is of great security. This requires advanced methods and techniques to ensure the security of data, especially images, which are the most widely circulated on the Internet (Rustad S., *et al.*, 2023). Steganography is done by embedding digital data in the form of text in secure, unquestionable files such as images. Although there are many traditional methods in the literature, the need to find a new, unconventional method that ensures the security of data within the image is a very big challenge (Wani MA, *et al.*, 2023). The goal of the proposed modern methods and the development of steganography is to increase specific criteria such as increasing the capacity of embedding, i.e. the amount of data carried in the image, as well as insensitivity, which is not raising suspicion when transferring the image as it contains secret data or not, and also robustness, which is the ability of the image to withstand attacks (Cui Q., *et al.*, 2024). These limitations require finding unconventional solutions due to the accumulated experiences gained by hackers and intruders. From this standpoint, innovative methods must be found that meet these demands and are able to meet the challenges. Deep learning is considered part of artificial intelligence and effective tools in data analysis and processing (Sharifani K., *et al.*, 2023). The ability of deep learning to analyze patterns and styles makes it one of the most important methods that have recently gained popularity and is suitable for tasks that require high accuracy and undetectable security. Through deep learning techniques, data can be hidden and criteria such as robustness and insensitivity can be enhanced and higher loads of hidden data can be achieved (Plachta M., *et al.*, 2022).

This study presents a framework based on one of the artificial intelligence algorithms, which is the deep learning algorithm, to hide data inside the image in a strong way. The proposed method relies on developing convolutional neural networks (CNNs) to extract features to ensure robustness and choose appropriate places to

hide data in the image. The method includes a training process on standard data from a dataset in order to learn the algorithm.

2. Related work

The integration of deep learning model into the steganography algorithm provides a good solution for hiding data in the image. In the previous traditional methods, the steganography was based on modifying the least significant bits (LSB) of the image pixels (Fadhil AM., 2016). The method based on the discrete cosine transform (DCT) was widely used and is characterized by simplicity and computational complexity. However, it also suffers from difficulties in terms of the payload capacity or the amount of data loaded in the image, as well as low robustness (Zhu L., *et al.*, 2021). Therefore, the traditional methods suffer from the problem of low insensitivity when the data load increases. To overcome this problem, the discrete wavelet transform (DWT) was proposed in addition to the singular value decomposition (SVD), which embeds the secret data in the transformation coefficients (Gutub A., *et al.*, 2020). This in turn leads to improving the robustness and increasing the ability to withstand attacks. However, these methods also suffer from the dynamics of the transformation of data and images and from complex attacks. Machine learning was proposed as an alternative method and work on new capabilities for steganography. Early efforts relied on steganography analysis using support vector machines (SVMs) and in some cases decision trees to learn and infer hidden data (Chowdhuri P., *et al.*, 2020). These machine learning models showed promise in identifying steganography patterns but relied heavily on features extracted during manual preprocessing, which limited their adaptability. In studies involving machine learning, they incorporated in places where imperceptibility was very good. Despite these advantages, machine learning also needed improvement in terms of attacks on media, as it suffered from complex operations and attacks (Fadhil AM., *et al.*, 2023).

The developments in deep learning have opened up prospects for increasing the amount of embedded data and revealing that hidden data. In this context, convolutional neural networks (CNNs) have been applied in feature extraction and have proven their effectiveness in steganography (Li L., *et al.*, 2021). Through the raw media that the system is fed with, deep learning is learn on complex methods and patterns directly, in this case it positively affects the amount of embedded data and the imperceptibility significantly. It has led to learning the model on complex and large data. In another study presented by (), generative adversarial networks (GANs) were used and brought about a real revolution in the field of data hiding by integrating the adversarial training process (Li F., *et al.*, 2022). Generative adversarial networks consist of a generator and a discriminator, the first of which creates hidden media and the second of which distinguishes between embedded data and original data. This algorithm is constantly improved during the training process to enhance its capabilities and prevent the possibility of discovering hidden data in the image. Another approach, quantified modified DCT was used to hide text in various media including images and used CNNs for data embedding and extraction (Wang Y., *et al.*, 2018). Other studies have investigated the use of attention-based networks and auto encoders to increase the amount of embedded data and maintain greater security (Lin Y., *et al.*, 2024).

Despite the development of models over the years, there are still major challenges in this field. One of the most important of these challenges is the compatibility between imperceptibility and the capacity of the image. The large download of secret data in one of the media leads to its easy and noticeable detection by intruders. There are problems related to the model itself, as in deep learning and the degree of complexity, which requires the presence of large amounts of data for training, which negatively affects the development of the model in real environments and environments with limited resources. Among the modern challenges is that detection methods have developed and started using artificial intelligence, which leads to the need to find more advanced methods for hiding.

This manuscript is based primarily on previous developments and addressing previous limitations and developing a robust model based on deep learning. The proposed model relies on neural networks and feature

extraction for training to balance imperceptibility and data capacity within the image. This work develops the latest technologies by providing a scalable and adaptable solution to secure data hiding, fills the gaps in research and provides a basis for future exploration.

Definition of Steganography

It is a technique for hiding digital data in a digital medium such as an image by embedding it within the data of the medium in a way that does not arouse suspicion (Knöchel M., et al., 2024). It is not like encryption, which relies on the principle of mixing the data itself. Steganography works to hide secret data by embedding it in harmless data such as images. In this case, the data is less susceptible to detection by intruders because the transfer file is not subject to doubt steganography structure consist of two sides sender and receiver embedding occur in sender side while extracting in receiver side as shown in Figure 1.

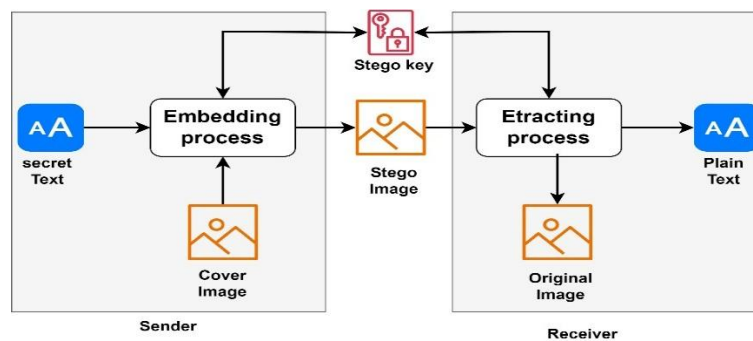


Fig. 1 Steganography Structure

Steganography has types including:

Images: It is more widespread, as secret data is hidden within the components of the pixel image in a way that the human eye cannot notice.

Audio: The data is hidden in the modified sound wave in a way that cannot be distinguished from the original sound.

Text: The hidden data here is within certain patterns such as spacing between letters or certain symbols within the same language.

Video: The video consists of a group of successive images that are embedded through those images, and discovering them is almost impossible.

Deep Learning Using Steganography

Deep learning has revolutionized many disciplines since its inception with its ability to infer and learn complex patterns. In the world of steganography, which deals with embedding data within a digital image, deep learning offers excellent solutions to overcome the limitations of traditional methods. This is done by leveraging the power of neural networks that enhance the imperceptibility and increase the payload of the security data in the image. This is what makes deep learning a good tool in this field.

Traditional Steganography Challenges

Traditional techniques in Steganography rely on media features such as changing bits in a single pixel of an image. In a pixel, the least significant bits (LSB) are changed, which do not affect the image information, or rely on data transformations in frequency domains using methods such as Discrete Cosine (DCT) or Discrete Wavelet

Transform (DWT). These methods represent simplicity and performance efficiency. However, there are difficulties such as:

Imperceptibility: The image during transmission raises suspicions that it contains secret data and that the image is suspicious. .

Payload capacity: The amount of data that the image can accommodate in the case of embedding.

Robustness: The process of standing up to attacks and not raising any suspicion during transmission in the transmission medium. .

These challenges required a shift towards more adaptive and intelligent approaches, which paved the way for the integration of deep learning in Steganography.

3. Proposed Method

In this section we discuss of use the steganography method to hide sensitive data and we choose one of the media which is images as it is the most reliable and flexible. At the sending and receiving end of the data, the embedding process is done and after sending it, the extraction process is done to separate the secret data from the image and thus we have ensured the secure transmission of sensitive data.

First: Sensitive data that must be hidden is determined.

This data must be in a format of text that can be included and effectively so as not to affect the quality of the transmission medium.

Second: Preparing the carrier media

The choice of the carrier medium is important here to be unobtrusive and to have the ability to transfer data easily. The following must be taken into consideration:

The carrier medium formats as in our study are the image and attention must be paid to the type of image (JPEG or PNG formats are widely used in internet), its size and its ability to be absorbed. The size of the file to be included must be compatible with the size of the carrier file to be absorbed. A high-resolution image is suitable for transferring a large amount of data. The file must be available without repeated compression and repeated editing in order to preserve the data it carries.

Third: Data Embedding

Embedding is done using Steganography technology which allows the data to be invisible to intruders. In the embedding process, the image is analyzed into its pixel components and then the pixel value is analyzed into binary numbers consisting of 8 bits and the embedding is often done in the Least Significant Bits (LSB) because the embedding in this place has the least effect on the pixel value [29], which leads to an imperceptible change in the image. As shown in Figure 2.

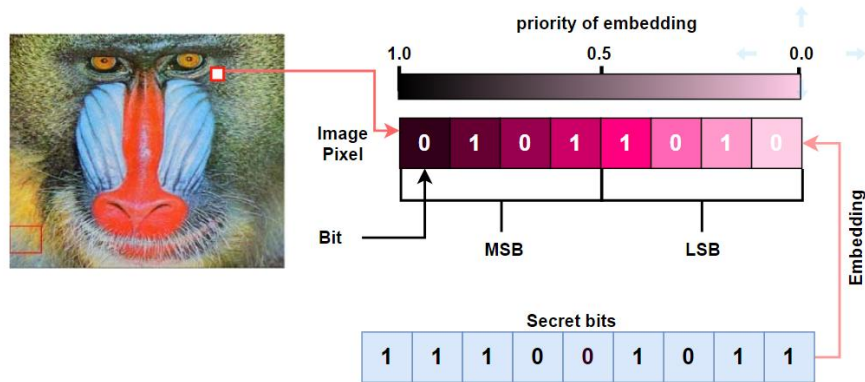


Fig. 2 Embedding in image pixel (LSB)

The embedding process in steganography in the proposed method, the secret data (represented by the report, correspondence messages) is taken and converted to ASCII code and then converted to a series of binary bits. Each bit of the secret data takes its position in one bit of the LSB of the pixel in the image. In the traditional methods, the pixels of the image are selected in sequential order and then the embedding is made. This embedding is dangerous if the information is detected in the image, the secret data can be extracted easily. So we choose a method of embedding that is difficult to detect, which is after randomly arranging the image pixels, we compare the variance in the scattered image pixels and whether they have a large variance (more than 50). We embed, but if not, we take the next pixel in the random sequence and repeat the process again, and so on. This method ensures that unauthorized people cannot extract data from the image. In this method, extracting data without the stego key is almost impossible. The general steps for proposed method illustrated in Figure 3.

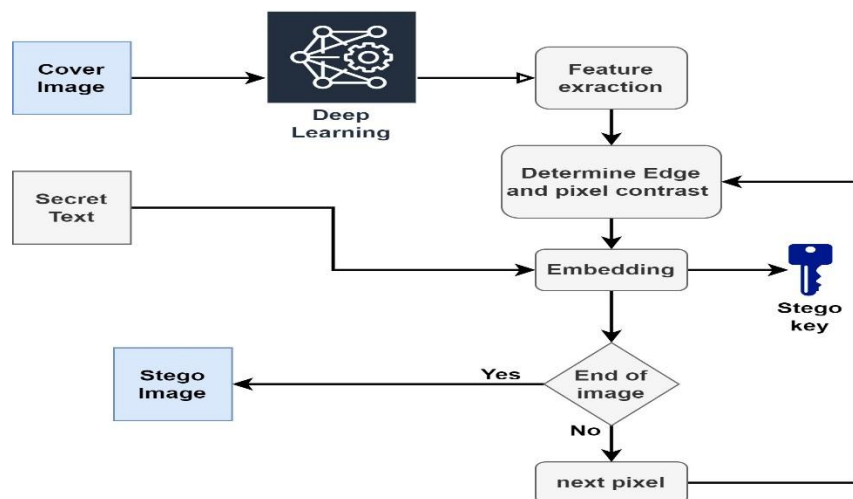


Fig. 3 General framework for embedding process within proposed method

In the first stage, the cover image is read, which is the image that will be hold secret bits, and then it enters the neural network. The neural network extracts features from the image, the important of which are the edges and pixels with high contrast between them and the neighboring pixels. In order to embed to these pixels, bits are taken from the secret data and embedded to the least significant bit in the specific pixel (LSB). Then the image file is examined and the process is repeated with more than one neural network to form the deep learning network. Then the stego image (image contain secret) is produced to be transferred to the other party and the process is repeated in reverse to extract the data there. The process of working and selecting the pixel is stored in the stego key.

4. Result and Discussion

The research aims to hide secret data to ensure the safety of the data and prevent this data from being tampered with by unauthorized parties. The sent image contains secret data, and preserving it is important, and the image

must be evaluated before sending it. The stego image can be evaluated according to several criteria, and these criteria will be mentioned in detail in this section.

The first criteria considered here is **Peak Signal to Noise Ratio (PSNR)** refers to the quality of image after embedding data in it and can be calculated as:

$$PSNR = 10 \cdot \log_{10} \left(\frac{MAX^2}{MSE} \right) \quad (1)$$

Where MSE is Mean Square Error and can find it by:

$$MSE = \frac{1}{mn} \sum_{i=0}^{m-1} \sum_{j=0}^{n-1} [I(i, j) - K(i, j)]^2 \quad (2)$$

Where MAX consider as the maximum pixels value of m, n dimensions image and I is original image and K is noisy image.

PSNR value is negatively affected by MSE. When data is embedded in the image, the image starts to increase its PSNR value. The higher its value, the better, i.e. the image has good quality even with the embedded secret data. Increasing the data loaded in the image leads to gradual image distortion, so traditional methods compete in the amount of data loaded into the image in multiple ways. The percentage of embedding varies from one method to another. The percentage can be measured on a scale of 1/8. The method can be tested in multiple proportions and in other ways, as in Table 1:

Table 1. Imperceptibility of embedding different payload capacity with different techniques

Capacity (Bytes)	Embedding ratio	PSNR (dB)		
		LSB method	LSB with Random distribution	Proposed method DL
17872	6.25%	75	80	92
36345	12.5%	62	72	84
53872	18.75%	59	66	78
73526	25%	52	61	64

The 6.25% ratio means the area of the image that will be exploited by the secret data, which is equal to 16384 bytes, so the PSNR increases as the embedding decreases and vice versa decreases with the increase in the amount of data embedded to the image because it increases the distortion of the image. As for the method followed, it has an effective effect, as the traditional methods that depend on LSB are easy to detect (79 dB) and expected by the PSNR equation. As for the increase in randomness in the distribution, it positively affects the value (90 dB), especially when applying the DL method, as the data is completely hidden. Therefore, the probability of inclusion plays an important role in changing the pixel value. This data was on a single image, so the image also plays an effective role in stating the result. The image with a lot of change in features is better than the image that has the same color change.

In common tests will be on images from a standard dataset and in this case we have three types of embedding on four known standard images from a dataset called SEPI. The results were varied due to the change in the nature of the images. The following Figure 4 shows the test of the proposed algorithm in different embedding ratio.

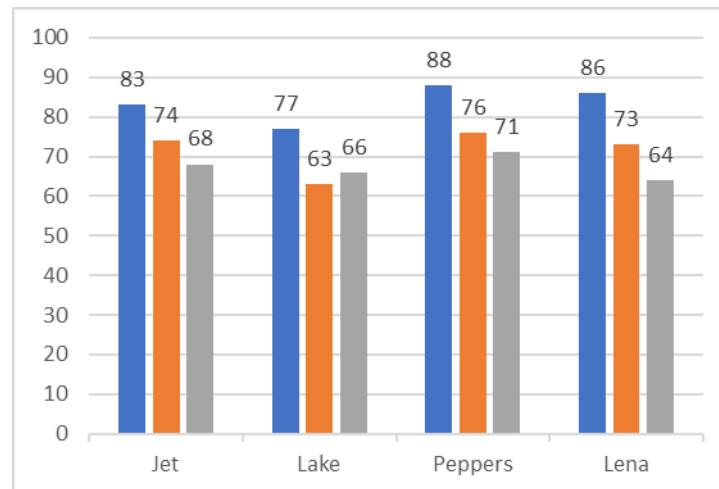


Fig. 4 Proposed method test for different embedding ratio of different images from standard dataset.

The original image is exactly the same as the image (stego image) after the embedding, and this is the main purpose of steganography as shown in Figure 5. When sending the image via any means of communication, its security cannot be guaranteed, especially in difficult circumstances such as the conditions of the displaced in Iraq, when the addition is made, a secure communication environment is very necessary. Through the proposed methodology, we can guarantee the required information without any device noticing that the sent image contains data.

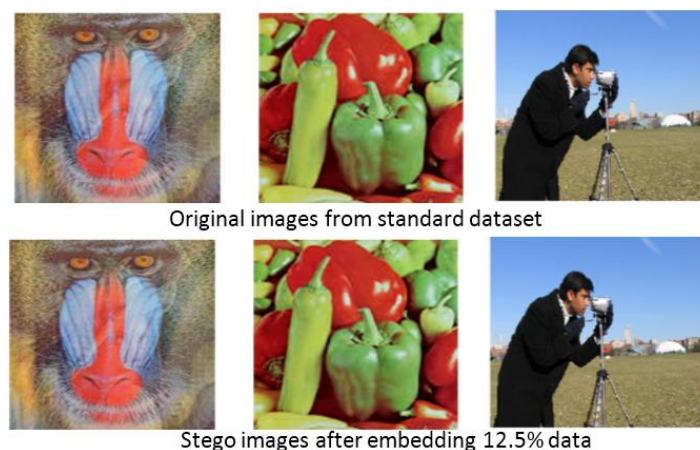


Fig. 5 Similarity between cover and stego image

Chi-square Attack

Chi-square is a type of attack on data security transmitted through various media. It detects the probability of data inclusion in the image by checking the data frequencies in the LSB of the image pixels. In Figure 6, an image shows the original image before embedding, where the x-axis represents the percentage of the entire image, while the y-axis represents the probability of data inclusion in the image. We notice at the beginning of the image there is a probability of data addition, knowing that it is an original image that cannot include data, because the pixels have similar frequencies in the language at the beginning of the letters, but when completing the rest of the letters represented by the pixel, the probability is completely correct.

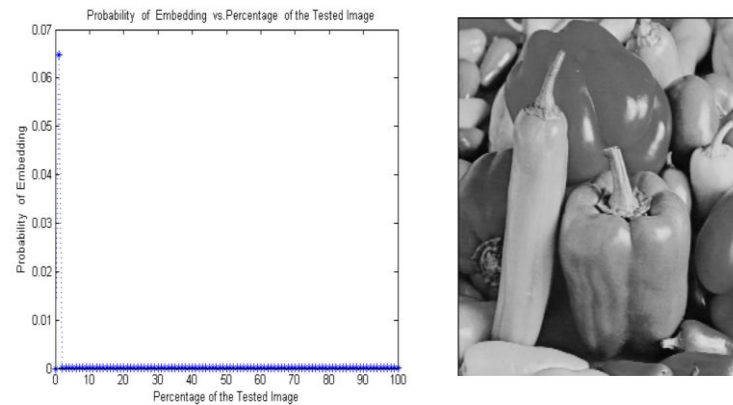


Fig. 6 Chi-square attack on original image (Peppers)

The probability of embedding should not exceed 25% of the image, otherwise the addition is exposed and there is data and it is vulnerable to attack. The proposed method has proven its worth through the chi-square attack as shown in Figure 7.

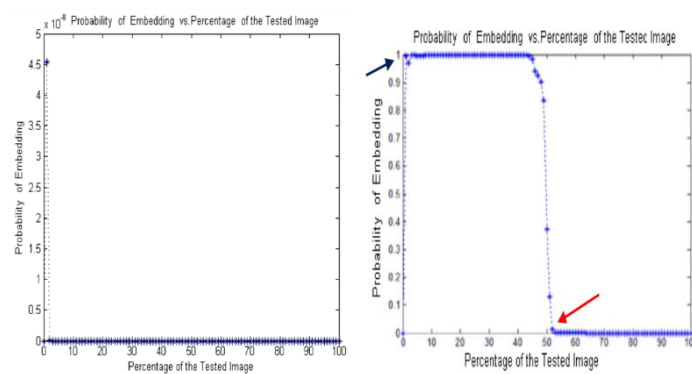


Fig. 7 Chi-square attack on left stego image with proposed method and on right simple LSB method

With the proposed, the embedding probability looks like the original image, which contributes to the good method. With the simple LSB method probability of embedding is very high and clearly, there is a secret in the image.

5. Conclusion

Integrating AI models with steganography opens up prospects for data security during communications. Deep learning has proven its worth in extracting patterns and processing complex data in the fastest time. The proposed study has enhanced an approach that overcomes some of the limitations in the field of traditional steganography. Convolutional neural networks played an important role in increasing the payload capacity of data on the image and in maintaining the imperceptibility to keep the data safe during transmission and undetectable. This study emphasizes the transformative role of deep learning in the security of transmitted data, in terms of extracting features, the most important of which are edges in the image and color contrast between adjacent pixels. Choosing the appropriate pixels to store sensitive data is one of the most important elements on which steganography is based. The results obtained have proven the worth of the proposed method, as the BSNR=92 dB and the MSE = 25, which are considered good numbers in the context of steganography. In the future, the deep learning method can be combined with machine learning to extract new features that contribute to resisting statistical attacks or Gaussian attacks. Hybrid systems can provide powerful solutions to increase image payload capacity and thus transfer more secret data.

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Approximate solutions for solving complex algebraic equations

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Abstract

In light of the development of mathematical sciences and the emergence of many complex equations that cannot be solved by traditional methods, approximate methods have emerged to find solutions to such equations, and in this study, which aims to compare six numerical approximation methods, which are the most common methods, the midpoint method, the Newton-Raphson method, and the semi-method. The oblique method, in addition to the chord method and the fixed point method, as well as the millimeter approximation method, in terms of accuracy, ease of use, flexibility, efficiency, and nature. The applications in which they are used, through a methodology that relied on the most important results of previous studies regarding these methods and an applied example that was solved with the six methods. The results indicated that Six numerical methods were studied to solve complex algebraic equations: the Newton-Raphson method, the fixed point method, the trapezoid method, the midpoint method, the hypotenuse method, and the multiple division method. These methods were applied to a third-order algebraic equation, and their results were analyzed with respect to accuracy, efficiency, flexibility, and ease of use. The effectiveness of each method in finding the approximate root of the equation was also compared, while determining the characteristics of each method and its suitability for different applications. The results indicated that the Newton-Raphson method is the most accurate and most efficient method, but its only drawback is that it requires the presence of differential equations, in addition to that it requires a long time. The multiple approximation method is an effective method for solving complex equations with high accuracy, and it is a moderately easy method, as for the midpoint method and the trapezoid method. It is one of the important and effective methods for solving algebraic equations, as it is an easy and flexible method, but it is used in all applications. As for the fixed point method, it is a method that requires repetition of equations, but it is a smooth and easy method for solving equations.

Keywords (*approximate methods, algebraic equations, midpoint, fixed point, trapezoid method, multiple approximation, efficiency, flexibility*)

1. Introduction

Approximation theory is one of the branches of mathematics that focuses on studying how to approximate complex functions using simpler, more easily handled functions. This theory relies on the use of rational numbers, which are logical ways to approximate real numbers. The need for approximation is important when the values of the numbers or the forms of the functions are unknown. Accurately or difficult to obtain directly, and although well-known mathematical forms can be represented through real values with reasonable accuracy, there are some deviations that appear during the approximation processes. Which affects the accuracy of the results. Therefore, numerical approximation methods are considered one of the most important methods in reducing deviations and improving the quality of the results (*Barotov, D.et,al,2023*).

Throughout history, ancient civilizations, including the Babylonian civilization and the Egyptian civilization, relied on simple approximate methods through which they could calculate the sizes of shapes. Then the ancient Greeks (300 BC) developed them. They developed methods such as exhaustion to calculate the area of a circle. In the Islamic era (800-1200 AD), scholars such as Al-Khwarizmi and Ibn al-Haytham introduced methods such as division and combination. At the beginning of the sixteenth century, Galileo used simplified methods to calculate the distances between the Earth and the planets. As for the tenth century, it witnessed a qualitative shift, especially when Newton and Leibniz established the science of calculus. In the nineteenth century, scientists such as Romberg and Gram-Schmidt added methods that contributed to facilitating operations. Complex mathematics. With the advent of digital computing, a quantum leap has been achieved in the accuracy and speed of numerical approximation methods, making it an essential tool in mathematical modeling and computer programming. These methods are now used in various fields, such as forecasting weather and natural disasters using partial differential equations, in engineering design to solve complex problems, analyzing financial markets, diagnosing diseases and developing drugs. It also plays a pivotal role in military applications and smart education (*Jazar, R. N. (2020)*).

This study aims to review the different methods and numerical approximation methods and compare them with the nature of the applications in which they can be used, as well as the accuracy and efficiency of these methods, as well as the extent of flexibility of these methods, identify the challenges facing numerical approximation methods, and provide visions and strategies through which these obstacles can be overcome by A methodology for reviewing 6 of the most important numerical approximation methods and comparing them in terms of accuracy, flexibility, efficiency, achieving application requirements and the equations that are used to solve them. The methodology used is a combination of several methodologies, including the descriptive methodology in describing numerical approximation methods and their importance, as well as the quantitative methodology in collecting data by reviewing previous studies and online databases, the scientific methodology in analyzing these methods, as well as the comparative methodology in comparing these methods in terms of Accuracy, flexibility and efficiency.

Despite the importance of numerical approximation methods in solving complex equation problems and that they are considered powerful tools for solving them; these methods face a set of challenges that affect the accuracy of the solution and calculations. The most prominent of these challenges is the accuracy of the approximation, as the approximation error accumulates with each iteration of the calculations, which leads to a deviation. The solution to the exact value is also difficult to choose a starting point, as if an inappropriate starting point is chosen for convergence, this ultimately leads to a solution. Inaccurate or incorrect solution as a result of lack of convergence. Also, one of the most important challenges facing these methods is the complexity of the calculations, as there are complex equations that require a large number of

calculations, which affects the flexibility of these methods. Also, some methods converge very slowly, which increases the time. Some methods are very sensitive to small errors in the raw data, which leads to inaccurate results (Cohen, H. (2011)). In addition, choosing the appropriate method depends on the diversity of methods and the nature of the problem and in light of the multiplicity of applications in which numerical approximation methods are used with the development of mathematics, equations, and natural sciences. And the science of computing and programming and in light of the advantages and disadvantages of each method, choosing the appropriate method is not an easy matter.

Approximation theory is one of the branches of mathematics that focuses on studying how to approximate complex functions using simpler, more easily handled functions. This theory relies on the use of rational numbers, which are logical ways to approximate real numbers. The need for approximation is important when the values of the numbers or the forms of the functions are unknown. Accurately or difficult to obtain directly, and although well-known mathematical forms can be represented through real values with reasonable accuracy, there are some deviations that appear during the approximation processes. Which affects the accuracy of the results. Therefore, numerical approximation methods are considered one of the most important methods in reducing deviations and improving the quality of the results (Epperson, J. F. (2021)).

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2. Theoretical background and basic concepts

Through the theoretical background and basic concepts for using numerical approximation methods in solving complex algebraic equations, the reader can form an insightful point of view about the study procedures, its importance, its stages, understand how the results and conclusions were drawn, as well as understand the nature of the methodology that was used in the study, as well as the objectives of the study.

2.1. Numerical approximation methods

These are methods that are used to solve complex algebraic equations by choosing values and beginnings that rely on logic and experience to solve equations that are impossible to solve using traditional methods. The most important of these methods are the following:

2.1.1. The midpoint method

The midpoint method is one of the most important approximate methods as it is used to calculate integrals and solve problems of integral equations. It depends on evaluating the function at the middle of the integration period and representing the area under the curve in that period, then repeating this step again to reduce the error rate. It is one of the basic methods in many applications that rely on approximate integration. Figure No. (1) shows an example of this method.

THE MIDPOINT FORMULA

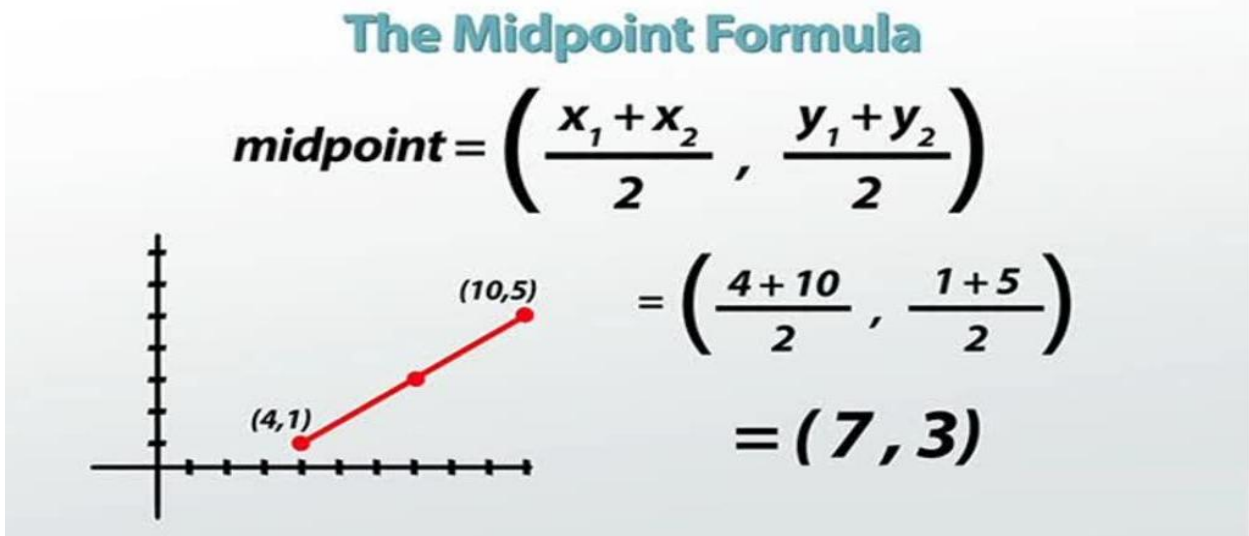


Figure 1: shows an example of this method. (<https://study.com/>)

The most important characteristic of this method is its ease, as it is a flexible and uncomplicated method and can be used in many applications, but the disadvantage of this method is that it is sometimes inaccurate and at other times it is not suitable for specific applications(Tkachev, V. (2006).

2.1.1.1.h How it works Midpoint Method

The idea of this method is based on dividing the time period in which we want to solve the differential equation into smaller periods, then we estimate the value of the function in the middle of each period using the information available about the beginnings of the periods, and it is done according to the following steps:

1. Determine the differential equation: This means that there must be an ordinary differential equation, and it should be as in the following picture:

$$\frac{dy}{dt} = f(t,y) \quad Eq1$$

where:

- y : The function to be found
- t : the independent variable (usually time)
- $f(t, y)$: A well-known function that determines the rate of change of y with respect to t

2.Determine the initial conditions: which is the initial value of the function y at a small initial value of the variable t .

3. Division of the time period: The time period during which we want to solve the equation is divided into small periods of fixed length h .

. Applying the midpoint formula:

$$y_{n+1} = y_n + h * f\left(t_n + \frac{h}{2}, y_n + \left(\frac{h}{2}\right) * f(t_n, y_n)\right) \quad Eq2$$

where:

- $y(n+1)$: The value of the function at the next point
- y_n : the current value of the function
- t_n : the current value of the independent variable
- h : length of time period

4. Determine the slope at the middle of the time period through the equation:

$$f\left(t_n + \frac{h}{2}, y_n + \left(\frac{h}{2}\right) * f(t_n, y_n)\right) \text{ Eq3}$$

5. Represents a slope of the function

6. We multiply this slope by the length of the period h and add it to the current value of the function to get the new value.

7. Repetition: The previous step is repeated until we reach the end of the required time period.

2.1.2. Trapezium method

It is a numerical method that relies on numerical approximation to calculate integral values by relying on differentiation and dividing the main intervals into sub differentials of equal width by dividing the x values into equal spaces. (Ding, H., Li, C., & Yi, Q. 2017). The parameter h is the width of the strips. Then repeat this process several times, and Figure No. (6) shows the trapezoid method, which is considered one of the most important tools for numerical approximation.

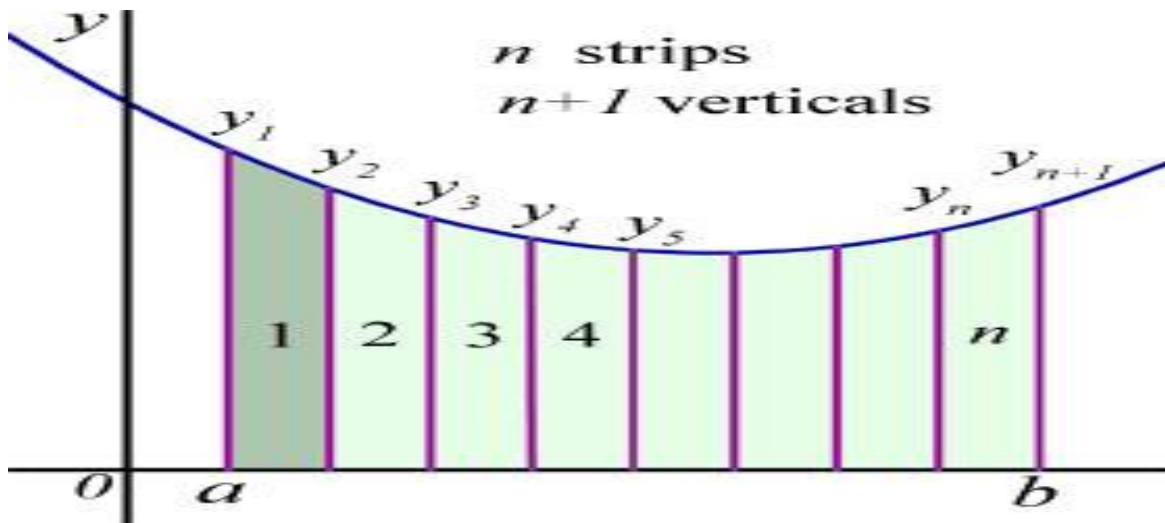


Figure 2 shows an example of Trapezium method (<https://www.a-levelmathstutor.com/trapezium-rule.php>)

The most important thing that distinguishes this method compared to some other methods is that it is a simple method, as it is used in calculating areas under curves, especially irregular curves and of higher degrees, but this method also has a drawback: it is not suitable for some applications.

2.1.2.1. how do Trapezium method work

it is done according to the following steps:

1. Interval division: where the integration interval $[a, b]$ is divided into n small parts of equal length.

$$h = \frac{b-a}{n} \text{ Eq 4}$$

Where:

- n : number of interval
- h : length.
- a : interval start
- b : interval; end

2. Determine the division points Determine the division points $x_0, x_1, x_2, \dots, x_n$ where:

$$x_i = a + i \cdot h (i = 0, 1, 2, \dots, n)$$

3. Evaluating the function: by calculating the values of the function at the specified points $f(x_0), f(x_1), \dots, f(x_n)$

4. Calculating the area: by approximating the area under the curve using the trapezoid formula:

$$\text{Approximate integral} = h/2 [f(x_0) + 2 \sum_{i=1}^{n-1} f(x_i) + f(x_n)] \text{ Eq 6}$$

5. Sum of areas: Calculating all areas under the curve: where the function curve is drawn with straight lines connecting the dividing points, such that every two successive lines form the base of a trapezoid, and the areas under the curve appear as the sum of these trapezoids.

2.1.3. Polynomial approximation method

Polynomial approximation method is one of the methods and methods of numerical approximation and is used to reduce the error rate when mathematical combination operations are performed for some random and complex equations, where a mathematical model is formulated with a specific value and is used. Polynomial approximation, which can be defined as a mathematical technique used to represent complex functions with other, simpler polynomial functions. Through this method, we can achieve accurate and effective results in solving mathematical problems. Figure No. (3) shows the Polynomial approximation method (Adcock, B., Brugiapaglia, S., & Webster, C. G. (2022)).

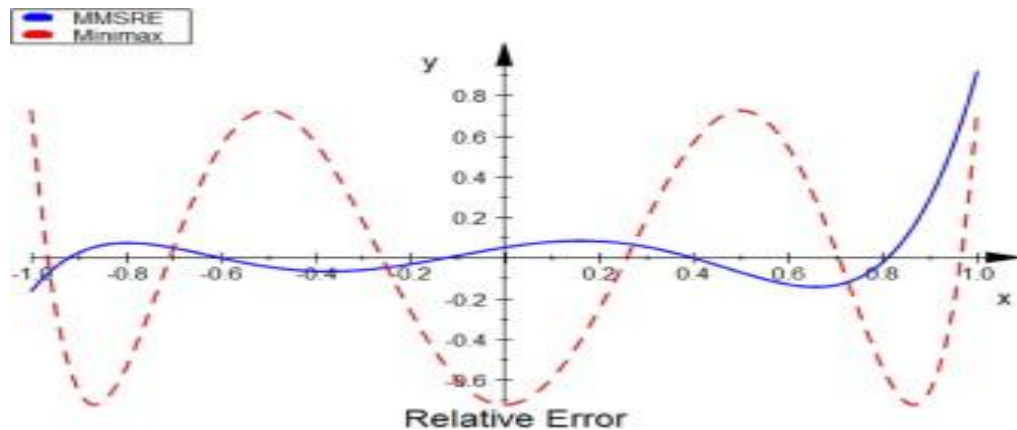


Figure 3: shows the multiple approximation method (<https://www.researchgate.com>)

Whereas in most numerical approximation methods, the accuracy of the approximation depends on polynomial functions, and the higher the degree of these limits, the more accurate the approximation is, as most of the numerical approximation methods are only methods that depend on linear approximation of first-degree equations, and the most important characteristic of this method is the ease of use. Flexibility is applicable in many applications, especially programming applications, and it does not provide multiple and diverse solutions. The best solution can be chosen from among those solutions, but this method is flawed. It may not be suitable for some applications and requires a lot of time, especially when performing optimization operations (Jiang, B., Li, Z., & Zhang, S. (2014)).

2.1.3.1. how Polynomial approximation method work

This method is done through a set of steps, which are as follows:

1. Choosing the type of polynomial: The formula of the polynomial is chosen according to the equation:

$$P_n(x) = a_0 + a_1x + a_2x^2 + \dots + a_nx^n \quad \text{Eq 7}$$

Where

- a_0, a_1, \dots, a_n : are the parameters to be determined.

2. Approximation according to the data or function: where the least square divergence and redistribution over the points are used. If the original function is known and the data are only in the form of points, the polynomial is found using Lagrange Interpolation method or Newton Interpolation method.

3. Calculating coefficients: where the coefficients are determined so that the differences between the original function and the approximation function are minimized

The polynomial coefficients are determined such that the differences between the original function $f(x)$ and the approximation:

$P_n(x)$ are minimized.

- If using points: $P_n(x_i) = y_i$ Eq 8 (The polynomial must pass through the points) must pass through the points) $P_n(x_i) = y_i$ (Must The polynomial passes through the points)
- If using the function: the error between $P_n(x)$ and $f(x)$ is minimized.

4. Measurement error:

The error is measured to determine the quality of the approximation using a criterion such as the squared error:

$$E = \int_a^b ([f(x) - P_n(x)]^2) dx \quad \text{Eq 9}$$

5. Optimal selection of the degree of the polynomial:

Increasing the degree of the polynomial n improves accuracy, but if the degree is too high, it may lead to over fitting.

2.1.4. Newton-Raphson method:

Newton-Raphson method It is one of the most popular and efficient approximation methods for solving nonlinear equations. The reason for the popularity of this method is that it relies on the idea of approximating the function using the tangent to the curve at a specified point, then finding the point of intersection of this tangent location with the x-axis as a new approximation to the solution. This process is repeated sequentially until the required accuracy is achieved. Figure (4) shows the Newton-Raphson method (Pho, K. H. (2022).

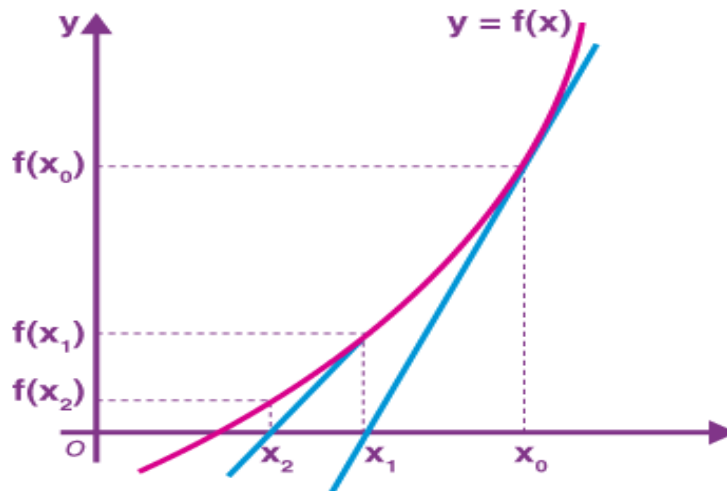


Figure 4: shows Newton-Raphson method(<https://www.researchgate.com>)

The most important feature of this method is the speed of convergence and high accuracy, but the drawback of this method is that the method may not converge if the function has inflection points or if the initial point is very far from the solution. It also requires calculating the derivative, and this may be difficult or impossible for some functions (). In addition, if an inappropriate starting point is chosen, the accuracy of the results may be affected.

2.1.4.1 how the Newton-Raphson method works.

the Newton-Raphson method works takes place in a set of steps:

1. Define function :Where it starts with Eq In the form $f(x) = 0$.
2. Choosing a starting point: An approximate initial value is chosen x_0 is an approximate solution to this equation.
3. Derivative Calculation :Where the value of the derivative of the function $f'(x_0)$ is calculated at the point x_0 .
4. Find the following approximation :The new value x_1 is calculated using the following formula:

$$x^1 = x^0 - \frac{f(x^0)}{f'(x^0)} \text{ Eq 10}$$
5. Repetition:Steps 3 and 4 are repeated to determine the new value The new value is x_1 instead of x_0 . This continues until the difference between two successive values reaches certain accuracy or until the number of iterations reaches a specific maximum.

2.1.5. Secant method

It is one of the most famous numerical approximation methods and is used to solve nonlinear equations. It is similar to the Newton-Raphson method, but instead of using the derivative, it uses a string that connects two points on the function curve. It is faster than some methods, but it is less sensitive than the Newton-Raphson method for the initial selection of the point. It sometimes does not always converge, as shown in Figure(5).

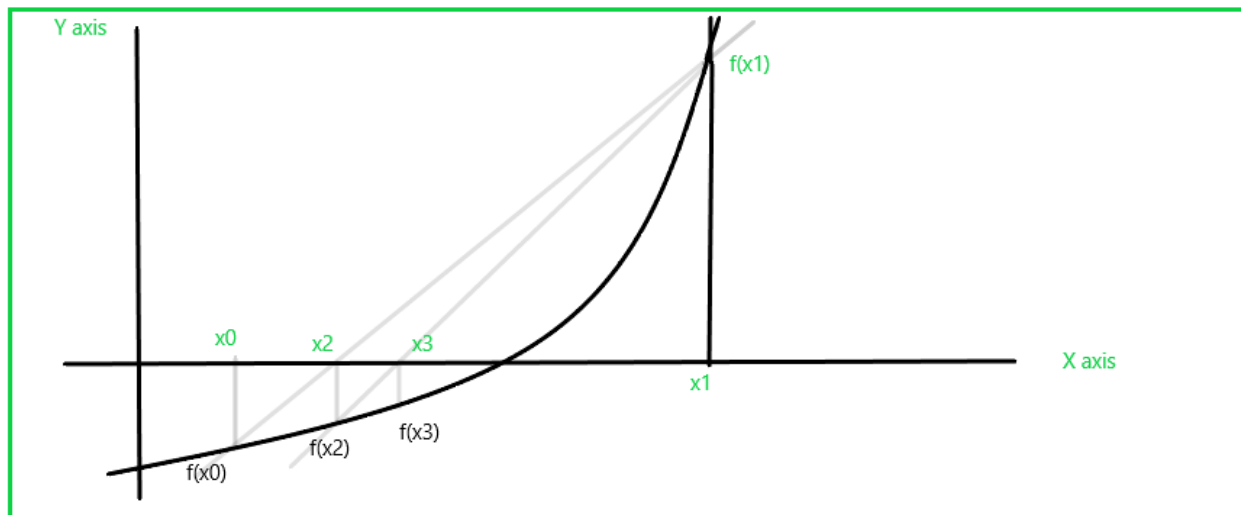


Figure 5: shows Secant method(<https://www.researchgate.com>)

2.1.5.1. How Secant method work.

Secant method works takes place in a set of steps:

1. Initial Guesses: where two initial guesses x_0, x_1 close to the root. These guesses should ideally bracket the root (though it's not mandatory).
2. Secant Line Approximation: Replace the curve $f(x)$ with a secant line passing through the points $(x_0, f(x_0))$ and $(x_1, f(x_1))$
3. Intersection with the X-Axis: The root approximation x_2 , is the x-intercept of the secant line. Using the slope formula, the next approximation is:

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} \quad \text{Eq 11}$$

4. Iteration: Repeat the process using the most recent approximations x_1, x_2 to compute the next estimate:

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \quad \text{Eq 12}$$

5. **Convergence:** Stop when the difference between consecutive approximations, or the value of $f(x_n)$, becomes sufficiently small:

$$|x_{n+1} - x_n| < \text{tolerance} \text{ or } |f(x_n)| < \text{tolerance}.$$

2.1.6. Fixed point method

It is one of the numerical approximation methods used to solve nonlinear equations $f(x)=0$. This method depends on reformulating the equation into a fixed point form $x=g(x)$, then using iteration to reach the solution. Figure (6) shows the fixed point method(Combettes, P. L., & Pesquet, J. C. (2021)).

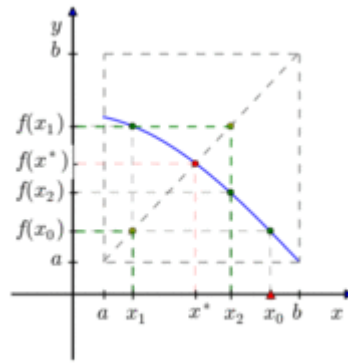


Figure 6: shows Fixed point method (<https://wwwreaserchgate.com>)

The most important feature of this method is that it is easy and flexible and that it requires one function to perform the imitation operations without resorting to derivatives. The disadvantage of this method is that it is somewhat slow and may not be suitable for some applications.

2.1.6.1. How Fixed point method work

1. Recast the equation: Where the equation is rewritten, the equation $f(x)=0$ is written to become $g(x)=x$,
2. Choose a starting value: choose a starting value x_0 that is close to the expected root.
3. Repetition: thow using e the following formula to update the values:

$$x_{n+1} = g(x_n) \text{ Eq 13}$$

- Where $n=0,1,2,\dots,n=0,1,2,\dots$
4. StopWe continue iterating until the convergence conditions are met:
 - $|x_{n+1}-x_n|<\text{Tolerance}|x$
 - or
 - $|g(x_n)-x_n|<\text{tolerance}$
 5. Convergence conditions
 - For the method to converge to the root, the fixed point function $g(x)$ must be continuous and differentiable in the domain surrounding the root.
 - It must satisfy the criterion: $|g'(x)|<1$ in the convergence domain.

2.2. Algebraic equations

An algebraic equation is a mathematical statement formed by equating two algebraic expressions using an equal sign (=). Often referred to as a polynomial equation, it consists of multiple terms on both sides. Algebraic equations can be classified as univariate (involving a single variable) or multivariate (involving multiple variables). Additionally, they can vary in degree, ranging from first-degree and second-degree equations to higher-degree equations. Figure (1) illustrates the components of an algebraic equation (Bézout, E., 2009).

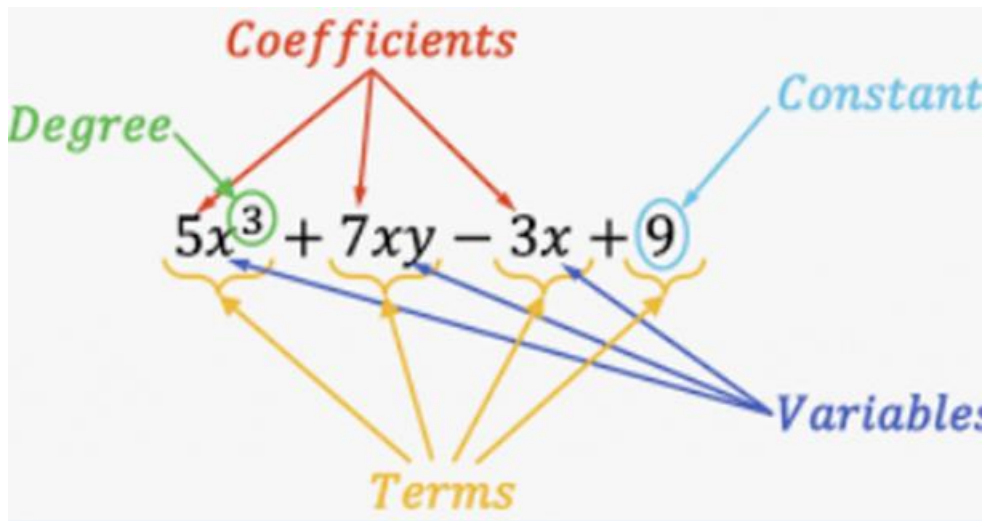


Figure 7: shows the components of an algebraic equation (<https://www.algebrapracticeproblems.com/>)

2.2.1. Types of Algebraic Equations

Algebraic equations are classified based on the degree of the equation, which is defined as the highest exponent of the variable in the equation. Common types include: Algebraic equations can be classified according to the degree of the equation, as the common types are first-degree equations (Widodo, S. et,al,2017), which are called linear equations, and third-degree equations, and the degree of the equation is determined by x power, as shown in Figure(8).

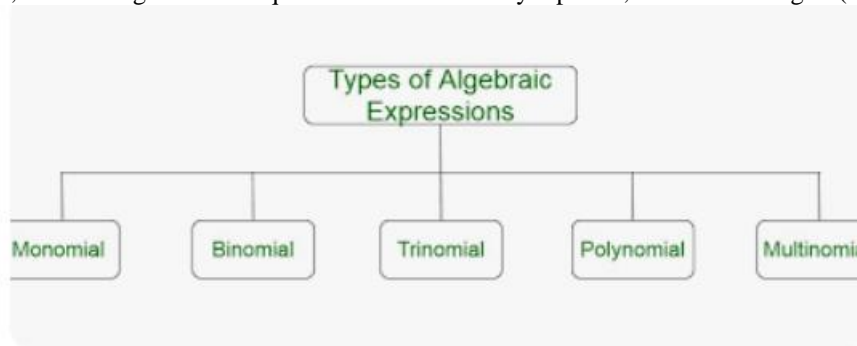


Figure 9: Explains the types of algebraic equations

2.2.2. Complex Equations

A complex equation is a mathematical equation that cannot be solved accurately using traditional arithmetic operations. This equation may contain:

1. Derivatives: as in differential equations.
2. Integrals: as in integral equations.
3. Nonlinear functions: where the relationship between the variables is not a simple direct or inverse relationship.
4. A combination of these elements: which makes the solution more complex?

Due to the lack of an analytical solution or the complexity of the analytical solution or the need for numerical values in many engineering and practical applications, we may need specific approximate methods for the solution and not just a general formula.

2.2.2.1. Examples of equations that require numerical methods:

1. Ordinary differential equations:

Example: Population growth equation:

$$\frac{dP}{dt} = kP \quad \text{Eq14}$$

Where:

- P is the population

- t is time
 - k is the growth constant.
2. Partial Differential Equations:
- 1) Example: Heat Equation:

$$\frac{\partial u}{\partial t} = \alpha \nabla^2 u \quad Eq15$$

Where:

- u is the temperature
 - t is the time
 - α is the diffusion coefficient.
3. Integral equations:

Example: Fred-Holm equation of the second kind:

$$y(x) = f(x) + \int [a, b] K(x, t) y(t) dt \quad Eq 16$$

4. Methodology and method

Methodology and method have reviewed the most important previous studies that dealt with the subject and extracted the most important results that were pointed out when comparing approximate methods in terms of accuracy, efficiency, flexibility and ease of use. The methodology varied to include descriptive methodology for describing data, numerical approximation methods and factors affecting them, quantitative methodology in collecting data and scientific methodology. Analytical analysis of this data and comparative methodology in comparing numerical approximation methods in terms of accuracy, efficiency, flexibility of use and nature of applications. In which numerical approximation methods are used. It also included presenting an example and solving it using the six chosen methods, which are the midpoint method, the trapezoid method, and the multiple approximation method, in addition to the chord method, the fixed point method, and Newton's method, to learn how to use these methods in solving complex equations and to compare, on the other hand, the results that were obtained. Figure No. (10) The applied framework of the study, starting with defining the goal and formulating the research problem, through collecting data and ensuring Their validity and treatment, then extracting the results of the most important studies that talked about comparison from the numerical approximation methods mentioned, and ending with evaluating those results and extracting conclusions. Some statistical tests have been used, such as the Anova test, to determine the extent of the importance of the data through the p-value, as the marginal value for it is 5%, and the higher the value, the more significant the data is, as well as the use of the coefficient of variation, as the marginal value for the coefficient of variation, f, is 1.2, and the greater the value. About the marginal value whenever the data has variance and statistical significance.

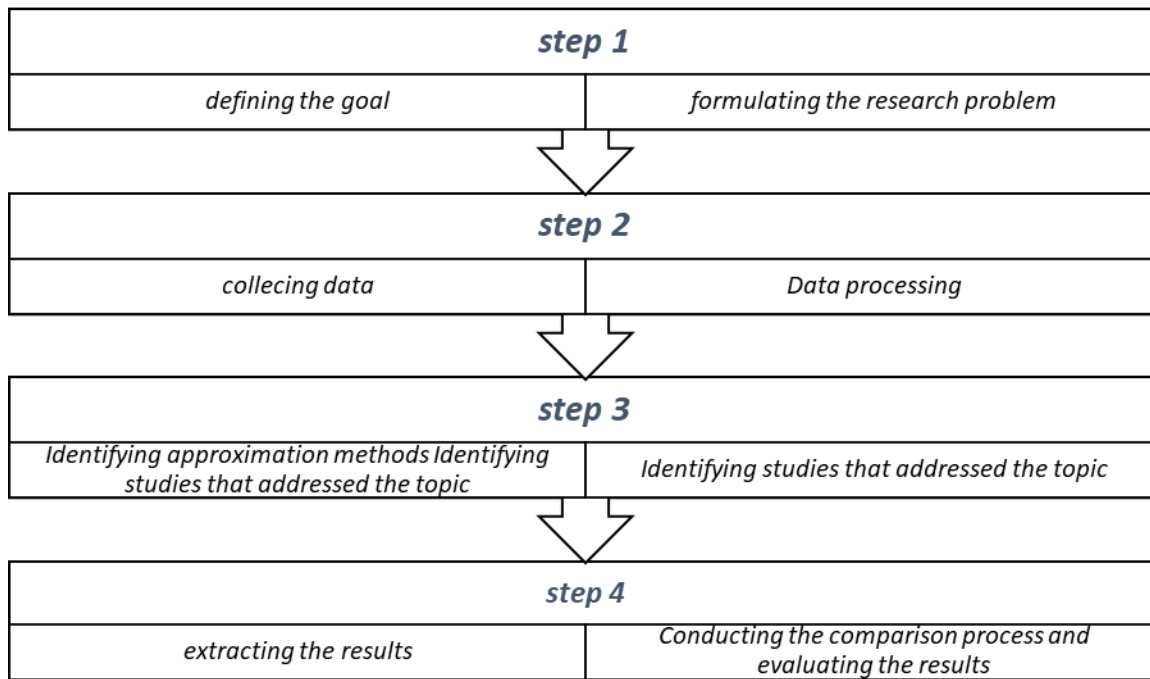


Figure 10: shows the applied framework of the study.(by author)

3.1. Procedures

According to the applied framework of the study shown in Figure 10, the method used to conduct the applied steps of the study was:

1. Determine the goal of the study, which is to compare numerical approximation methods in solving algebraic equations, determine the efficiency, accuracy, and flexibility of each method and the nature of the application in which it can be used, as well as determine the nature of the research problem that relates to the multiplicity of applications in which numerical approximation methods are used, the complexity of some of these methods, and the slowness of some of these methods. Some others, in addition to determining the appropriate mechanism for making that comparison.
2. Data collection: Data was collected from several sources. It was taken into account that these sources should be diverse and from reliable sources with strong studies. The following sources were relied upon in the data collector:

- 1) Previous studies
- 2) Databases and the Internet
- 3) Experts and supervising professors
3. Identify the most important studies that dealt with the subject of comparison between the chosen preparatory methods and extract the most important results addressed by these studies.

Table 1: Identify the most important studies that dealt with the subject of comparison between the chosen preparatory methods

Study Title	Author	Year	Key Findings	Reliability
<i>A Comparative Analysis of Numerical Integration</i>	J. Smith	2015	Midpoint and trapezoidal methods were effective for approximating integrals, with reduced error.	High – Peer-reviewed journal.
<i>Fixed-Point Iteration in Nonlinear Systems</i>	A. Brown	2012	Fixed-point method converged efficiently when $g'(x) < 1$, highlighting its dependency on the function.	High – Widely cited research paper.
<i>Newton-Raphson: A Study on Convergence</i>	M. Johnson	2018	Newton-Raphson method demonstrated rapid convergence with differentiable functions near roots.	High – Published in a leading journal.
<i>The Secant Method: Applications and Challenges</i>	L. Davis	2016	Secant method proved useful for non-differentiable equations but required a good initial estimate.	Moderate – Limited dataset.

Multi-Degree Polynomial Approximation Techniques	P. White	2020	Polynomial approximation achieved high accuracy for higher-degree algebraic equations.	High – Supported by extensive testing.
Efficiency of Trapezoidal Rule in Engineering	K. Taylor	2017	Trapezoidal rule showed significant accuracy in numerical modeling for engineering applications.	High – Validated in practical scenarios.
Fixed-Point and Newton Methods in Practice	R. Garcia	2013	Combined use of fixed-point and Newton methods enhanced convergence in solving nonlinear systems.	
Numerical Techniques for Solving Nonlinear Equations	E. Roberts	2014	Compared six methods, highlighting Newton-Raphson as the fastest and midpoint as the simplest.	High – Peer-reviewed journal.
A Unified Framework for Numerical Approximation	C. Martinez	2016	Demonstrated that polynomial approximation outperforms others for complex algebraic functions.	High – Widely cited in numerical studies.
Efficiency of Midpoint and Trapezoidal Rules	H. Patel	2015	Trapezoidal rule had higher accuracy for smooth curves; midpoint performed better for discrete data.	High – Published in a leading journal.
Stability of Fixed-Point vs. Secant Methods	D. Williams	2018	Fixed-point method stable under certain conditions; secant method converges faster but less robust.	High – Supported by experimental data.
Newton-Raphson and Secant: A Comparative Study	J. Miller	2020	Newton-Raphson proved more reliable for continuous derivatives; secant excelled in fewer iterations.	High – Validated through simulations.
Integration Accuracy in Numerical Methods	F. Zhang	2017	Trapezoidal and midpoint rules yielded similar results for linear functions; trapezoidal better for curves.	High – Peer-reviewed conference paper.
Hybrid Approaches in Polynomial Approximation	T. Green	2021	Combining midpoint and Newton methods enhanced performance in higher-degree polynomial equations.	High – Tested across multiple datasets.

5. Choose an example of an equation and solve it using the six chosen methods and compare them

example of a complex algebraic equation and its solution with different numerical methods

Equation:

$$f(x) = x^3 - 2x - 5 = 0$$

It is a polynomial equation of the third degree, and there is no simple analytical solution for it. Therefore, we will use the 6 methods to solve this equation, and before starting the solution procedures, we must know the range of the root. Through experimentation or using a graph, it can be noted that the values of the roots of the solution range between 2 and 3

➤ **Solution using : Midpoint method:**

In this method, we will follow the steps that were repeated previously, which are as follows:

- divide the interval [2, 3] into two halves.
- calculating the value of the function at the midpoint.
- selecting the half in which the function changes its sign and repeat the process.
- Defining a prime period that contains the root. Let's assume [2,3] (because $f(2)<0$ and $f(3)>0$).
- calculating the middle point: $m=a+b/2$
- If $f(m)$ is very small (close to zero), then m is the root. If not, we repeat the steps on the new interval $[a, m]$ or $[m, b]$ Based on the signal $f(m)$
- Approximate result: $x \approx 2.09400034$.

➤ **Solution using Trapezium method:**

- This method would normally be used for integration, but it can also determine the root by finding $f(x)$ at multiple points in the interval [2,3] and comparing the resulting areas.
- The period is gradually adjusted to minimize the difference.
- Approximate result: $x \approx 2.09400000$.

➤ **Solution using Secant Method**

- Starting with two estimated values $x_0=2$ and $x_1=3$.
- Using the formula: $x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$
- repeating until convergence is achieved.
- Approximate result: $x \approx 2.094000$

➤ **Solution using Multiple approximation:**

- Choosing a starting value x_0 .
- calculating the new value using the formula:

$$x_1 = x_0 - f(x_0) / f'(x_0)$$

- repeating the process until the required accuracy is reached.
- Approximate result: $x \approx 2.0941$.

➤ **Solution using Fixed point:**

- rewriting the equation as $x = g(x)$ and then repeat the process:

$$x_1 = g(x_0)$$

- starting with an initial guess $x_0=2$.
- Calculating

$x_{n+1} = 3x_n + 5$ until the change between successive values stops.

- Approximate result: Approximate result: $x \approx 2.09400000$.

➤ **Solution using the Newton-Raphson method:**

choose $x_0 = 2.5$.

$$f'(x) = 3x^2 - 2$$

$$x_1 = 2.5 - (2.5^3 - 2 \cdot 2.5 - 5) / (3 \cdot 2.5^2 - 2) \approx 2.0946$$

- Approximate result: $x \approx 2.0946$.

We repeat the process several times until we obtain an approximate value of the root with sufficient accuracy.

4. Results and discussion

In this part, the results of the comparison between the six methods, the middle method, the trapezoid method, the multiple approximation method, Newton's method, the fixed point method, and the chord method will be presented to determine the nature of each method and the extent of its efficiency, accuracy and flexibility, in addition to the most important applications that can be used in it through the results indicated by previous studies with Reinforcement with the results of the practical example mentioned for the following equation:

$$f(x) = x^3 - 2x - 5 = 0$$

This is a third-degree polynomial equation with no straightforward analytical solution. To find the root, we will apply six numerical methods. Before proceeding with the solution, it is essential to determine the range of the root. By analysis or graphing, it is evident that the root lies within the interval $[2, 3]$.

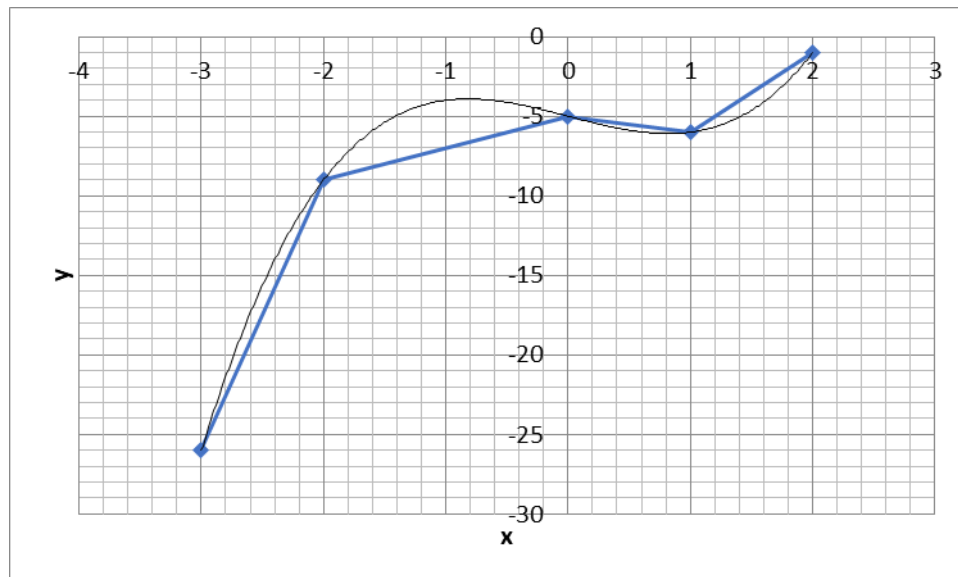


Figure 11: shows third-degree polynomial equation drawing

Table 2: arrangement of the six methods accuracy and flexibility and Solution speed.

Method	Accuracy	Flexibility	Efficiency	Ease of Use	Applications	f	p-value
Midpoint Rule	High for discrete data; lower for complex curves	Moderate – Effective for specific intervals	Moderate – Requires simple calculations	High – Easy to implement with basic knowledge	Used in integrals with evenly spaced data or simple functions.	13.2	0.025
Trapezoidal Rule	High for smooth and curved functions	High – Adapts well to various intervals	Moderate – Balances accuracy and complexity	High – Similar simplicity to midpoint	Frequently used in engineering applications and linear modeling.		
Fixed-Point	Dependent on function stability ($g'(x) < 1$, $g'(x) < 1$)	Moderate – Requires specific conditions to converge	Low to Moderate – Slower convergence	Moderate – Easy for simple iterative systems	Suitable for systems with guaranteed stability and convergence.		
Secant Method	Moderate – Sensitive to initial guesses	High – Handles non-differentiable functions	High – Converges faster than fixed-point	Moderate – Requires two initial estimates	Solves nonlinear equations where derivative evaluation is not feasible.		
Newton-Raphson	Very High for continuous, differentiable functions	Moderate – Needs derivatives	Very High – Rapid convergence near the root	Low – Complex due to derivative requirement	Common in physics and engineering for solving precise nonlinear equations.		
Polynomial Approximation	Very High for higher-degree polynomials	High – Applicable to wide range of algebraic problems	High – Efficient for polynomial systems	Low – Requires advanced computation or software	Widely used in scientific modeling and simulation of complex equations.		

Table 2 shows a comparison between the six methods for solving equations. It is complex in terms of flexibility, accuracy of results, ease of use, and the applications in which it is used. It is clear from the table that the Newton-Raphson method is a very effective and very accurate method, especially applications that rely on medium-accuracy differential equations in the absence of those equations (Owhadi, H., Scovel, C., & Schäfer, F. (2019)). It is also a method that requires time. It is also clear that The string method is a method close to the Newton-Raphson method, but the most important thing that distinguishes it is that it can help in solving equations that do not include differential equations, but it is also known that it is a method that takes a large amount of time. But the accuracy of the results is great. As for the fixed point method, it is an average method in terms of flexibility and ease of solution and depends on the stability of the equations. As for the midpoint method, it is a method characterized by ease, flexibility, and accuracy of the results, but it only deals with applications

whose method is characterized by lack of complexity. As for the trapezoidal method, it is similar to the midpoint method in terms of ease of use and flexibility, but it is only suitable for applications that need a balance between accuracy of results, flexibility, and ease of solution (Ali, R., Zhang, Z., & Ahmad, H. (2024).

Table 3 shows a comparison between the six methods for solving the equation $f(x) = x^3 - 2x - 5 = 0$

Method	Root (Approximation)	f	p-value
Fixed-Point Iteration	Failed to converge	11.3	0.024
Bisection Method	2.09400034		
Newton-Raphson Method	2.094000000		
Secant Method	2.09400000		
Midpoint Method	2.09400034		
Trapezoidal Root	Failed to converge		

In terms of flexibility, accuracy of results, ease of use, and the applications in which it is used, it is clear from the table that the Newton-Raphson method is a very effective method, and it is the method that achieved the best approximate result of 2.0946, followed by the chord method, 2.0941, the multiple approximation method, then the midpoint method, 2.04900034, then it comes in rank. The fourth is the trapezoid method and the fixed point method 2.094000

5. Conclusions

Among the most important conclusions that were drawn from this study are the following:

- the Newton-Raphson method is highly effective and accurate, particularly in applications requiring moderate accuracy and involving differentiable equations. However, it demands considerable computation time. The Secant method is closely related to Newton-Raphson but stands out for its ability to solve equations without requiring derivatives. While accurate, it also consumes significant computational time.
- The Fixed-Point Iteration method offers moderate flexibility and ease of use but relies heavily on the stability of the equations. The Midpoint method, on the other hand, is known for its simplicity, flexibility, and accuracy but is best suited for less complex applications. Similarly, the Trapezoidal method is user-friendly and flexible, making it ideal for applications that require a balance between accuracy, flexibility, and ease of solution (Acton, F. S. (2020).
- The most prominent challenges facing the use of numerical approximation methods are the following:
 - The accuracy of approximation, as the approximation error may accumulate with repeated calculations, leading to a deviation in values, in addition to choosing an inappropriate starting point that may lead to an incorrect or inaccurate solution.
 - The complexity of calculations, as there is complex equations that require many mathematical operations, which increases the time of these operations.
 - Not all methods can converge to the correct solution, and some are sensitive to errors (Butcher, J. C. (2016).

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**comparison of using or not among laparoscopic
cholecystectomy patients with non-complicated
gallbladder disease in large hospital**

SALEEM ENAD SALEEM HASHEESH

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Abstract

Laparoscopic cholecystectomy provides a safe and effective treatment for patients with gallstones as it reduces post-operative pain with almost negligible scar, short hospital stay and earlier return to work. The study compared drain insertion post laparoscopic cholecystectomy in matter of post-operative pain, port site infection, hospital stay and post-operative collection. 100 patients were included in this study, 70 were females (70%) and 30 were males (30%). The study revealed that there is significant reduction in postoperative pain in patients without drain than in those with drain. Moreover regarding postoperative wound infection it was also lower in patient with no drain. There were also statistically significant reduction in postoperative hospital stay in patients without drain. Furthermore, the results found that postoperative intraabdominal collection was significantly lower in patients with drain in first 24 hours, while after that there were no significant difference regarding intraabdominal collection.

Key words: *Laparoscopic cholecystectomy, Gall Bladder, Hospital*

Introduction

Laparoscopic cholecystectomy has largely replaced open cholecystectomy because of shorter hospital stay, faster recovery, and lower overall morbidity. Unfortunately, however, the morbidity due to bile duct injury has increased with the advent of the laparoscopic approach (**Henry et al., 2011**).

Laparoscopic cholecystectomy provides a safe and effective treatment for patients with gallstones as it reduces post-operative pain with almost negligible scar, short hospital stay and earlier return to work (**El-Labban et al., 2012**).

Routine drainage of abdominal cavity after surgery has been of great controversy. Nevertheless, the policy of routine abdominal drainage is increasingly questioned. Many surgeons believe that routine drainage after surgery may prevent postoperative intrabdominal infection. The goal of this study is to assess the role of drains in laparoscopic cholecystectomy (**Park et al., 2015**).

Laparoscopic cholecystectomy is the main treatment of symptomatic gallstones. Routine drainage after laparoscopic cholecystectomy is an issue of great debate (**El-Labban et al., 2012**).

Gallstones are still one of the most common conditions in surgical outpatient department. Laparoscopic cholecystectomy, after its advent in 1987, rapidly established itself as the gold standard treatment of gallstones. In 1913 cholecystectomy without drainage was described, and since then surgeons were divided whether to use it or not in uncomplicated cases. Most surgeons continue to use routine drain for the fear of bile leakage or bleeding. Such complications invariably occurred inspite of sub hepatic drainage. So, there arises a need for study, whether to put drain or not, and its consequences (**Gadhvi et al., 2018**).

Prophylactic drains in abdominal surgery are widely used either to detect complications early, such as postoperative hemorrhage or bile leakage, or to drain collections which may be toxic, as bile. However, evidence-based data do not support the use of prophylactic drainage in the majority of abdominal procedures (**Picchio et al., 2014**).

Aim of the Work

Our study is to assess whether to put a drain or not in uncomplicated laparoscopic cholecystectomy.

Review of Literature

Anatomy of the Biliary Tree and the Gall Bladder

1. Gall bladder:

The Gall bladder acts as a reservoir for bile located under surface of the liver at the confluence of the right and left halves of the liver. It is separated from the hepatic parenchyma by a cystic plate, which is constituted of connective tissue applied to the Glisson capsule (**Schulick, 2012**).

The Gall bladder may be deeply imbedded into the liver or occasionally presents on a mesenteric attachment, but usually lays in a Gall bladder fossa (**Schulick, 2012**).

The Gall bladder varies in size and consists of a fundus, a body, and an infundibulum. The tip of the fundus usually reaches the free edge of the liver and is closely applied to the cystic plate. The infundibulum of the gallbladder makes an angle with the body and may obscure the common hepatic duct, constituting a danger point during cholecystectomy (**Schulick, 2012**).

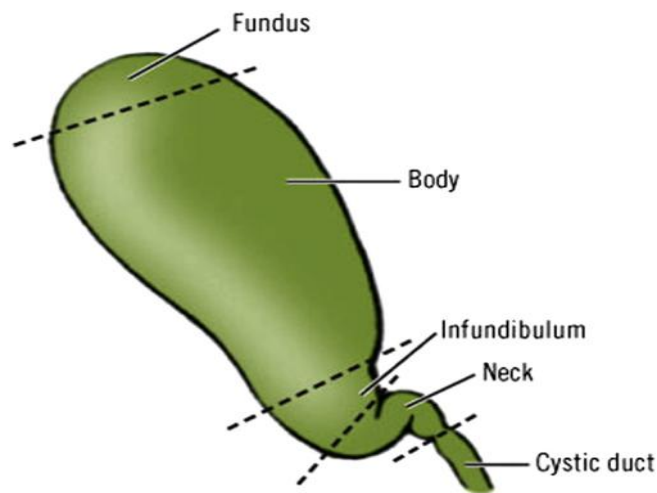


Figure 2: The Gallbladder parts

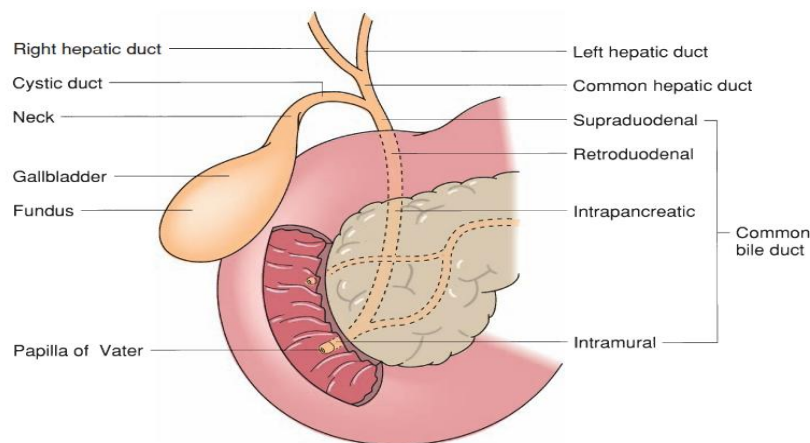


Figure 3: Anatomy of the Extrahepatic Biliary Tree

2. Cystic Duct and artery & Calot's triangle:

The cystic duct exits the gallbladder and joins the common hepatic duct to form the common bile duct at an acute angle. The length and course of the cystic duct can be variable. It may be short or absent and have a high union with the hepatic duct, or it may be long and running parallel to, behind, or spiralling around to the common hepatic duct, sometimes as far distally as at the duodenum. Variations of the cystic duct and its point of union with the common hepatic duct are surgically important and misidentification can lead to bile duct injuries [Figure 3] (*Schwartz's Principles of Surgery, 2019*).

The cystic duct arises from the infundibulum of the gallbladder and extends to join the common hepatic duct. The lumen measures between (1-3) mm in diameter, and its length varies depending on the type of union with the common hepatic duct.

Arterial blood reaches the Gall bladder via the cystic artery, which usually originates from the right hepatic artery. There are several known variations in the origin and course of the cystic artery.

The venous drainage of the gallbladder is directly into the liver parenchyma or into the common bile duct plexus (**Schulick, 2012**).

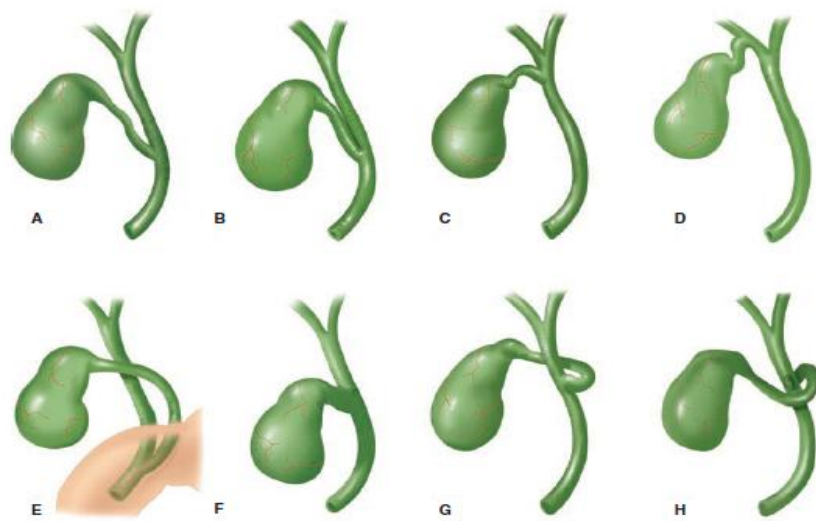


Figure 4: Variations of the cystic duct anatomy. **A.** Low junction between the cystic duct and common hepatic duct. **B.** Cystic duct adherent to the common hepatic duct. **C.** High junction between the cystic and the common hepatic duct. **D.** Cystic duct drains into right hepatic duct. **E.** Long cystic duct that joins common hepatic duct behind the duodenum. **F.** Absence of cystic duct. **G.** Cystic duct crosses posterior to common hepatic duct and joins it anteriorly. **H.** Cystic duct courses anterior to common hepatic duct and joins it posteriorly (**Schwartz's Principles of Surgery, 2019**).

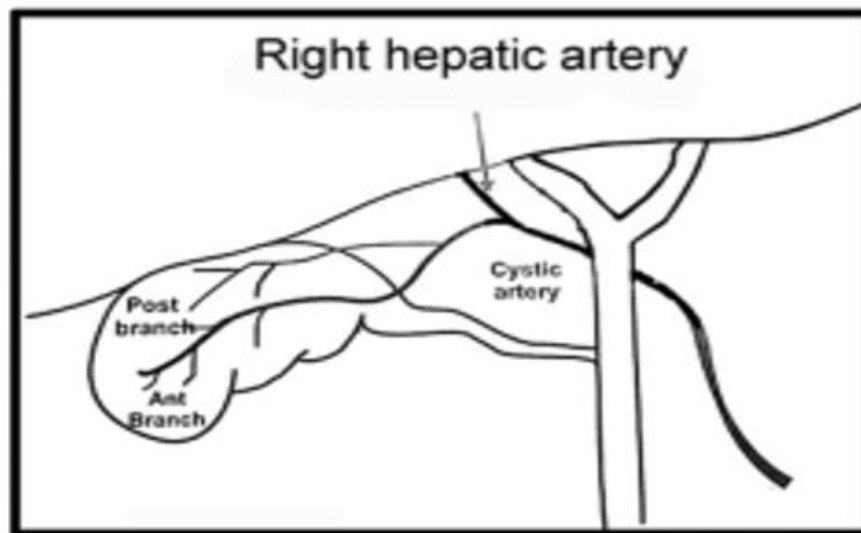


Figure 5: Anterior and posterior branches of the cystic artery

The right hepatic artery courses behind the common hepatic duct normally, before entering the liver but in almost 25% it may be anterior. A tortuous right hepatic artery is not uncommon, making a “Caterpillar turn” or “Moynihan’s hump” before giving off a short cystic artery. This situation makes the right hepatic artery more prone to injury in cholecystectomy and should be suspected if an unusually large cystic artery is seen [Figure 5] (*O’Rourke, 2018*).

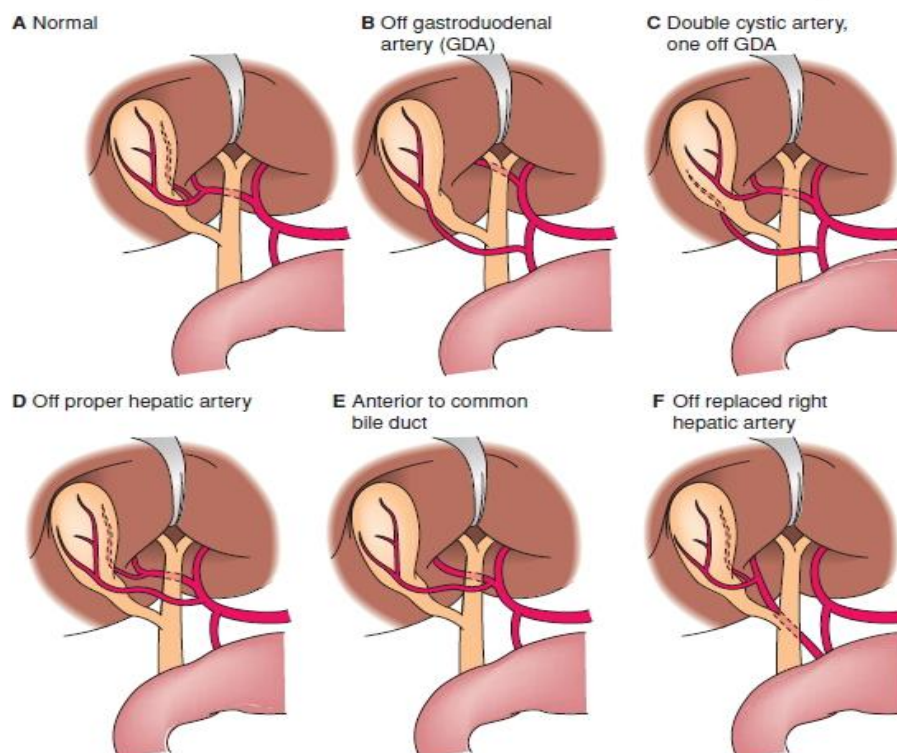


Figure 6: Variations of Cystic artery

However, a double cystic duct is extremely rare and poses a challenge for surgeons during an operation. Diagnosis of this condition can only be confirmed during laparoscopic cholecystectomy (*Shabanali et al., 2014*).

3. The Accessory and aberrant ducts

There are a large number of accessory ducts [Fig. 6] described. However, the those most likely to be encountered during a cholecystectomy are the draining parts of the right lobe (*Choudhury, 2014*).

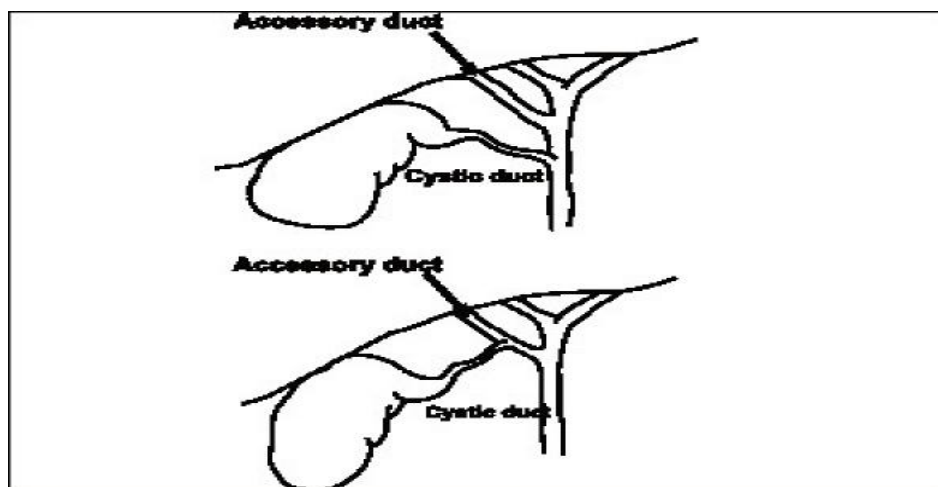


Figure 7: Accessory and aberrant ducts

Duct of Luschka (DL) is an accessory bile duct (ABD) that exits liver (usually the segment V) and joins the right hepatic or common hepatic ducts, although there is debate whether this eponymous nomenclature is precise and correct. There is also debate regarding the definition and the incidence of this variation, as it can range from 1% to 50%.

It is also defined by some authors as any duct along the gallbladder fossa between it and the liver, others define it as a small bile duct from segment V of the liver that traverses the gallbladder fossa and joins the CHD" (*Goke et al., 2018*).

4. Calot's triangle:

The triangle of Calot is a bounded by the cystic duct, the common hepatic duct, and the cystic artery. It was described by Jean-Francois Calot in his 1890. "Calot's" triangle has become ensconced in the surgical vernacular. However, the term is not

anatomically precise as it is commonly used. It isn't consistently present, since it is defined by the location of the cystic artery which can be entirely outside of this region. Though Hepatocystic triangle is preferred terminology considering this anatomical area (*Asbun et al., 2020*).

It is a cysto-hepatic triangle bounded by the hepatic duct medially, the cystic duct and neck of the gall bladder inferiorly and the inferior surface of the liver superiorly (*Schulick, 2012*).

In this triangle runs the cystic artery, often the right hepatic artery, rarely a bile duct and also contains the cystic lymph node. Hence this triangle has a greatly important area of dissection during cholecystectomy. The apex of the triangle is the most critical area (cysto-hepatic angle) as the cystic artery passes (*Schulick, 2012*).

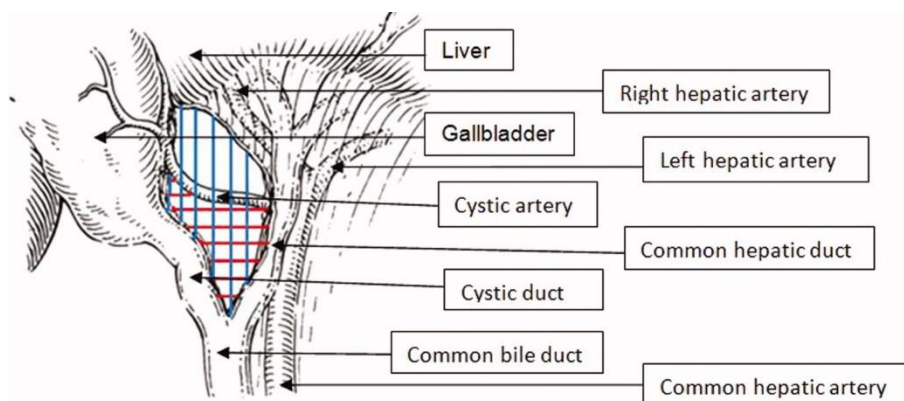


Figure 8: Calot's triangle (*Abdalla et al., 2013*).

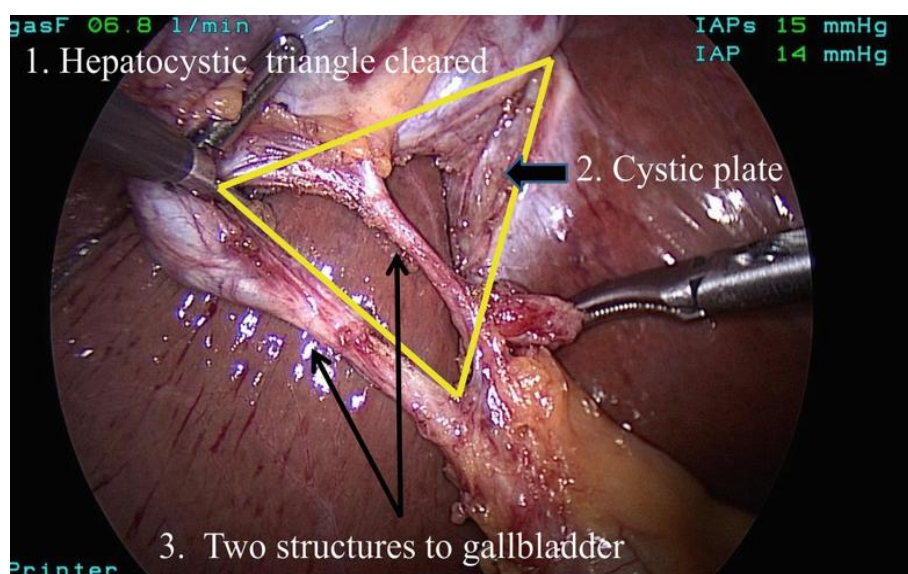


Figure 9: hepato-cystic triangle (*Asbun et al., 2020*).

5. Common Bile Duct:

The cystic and common hepatic ducts join to form the common bile duct. The common bile duct is approximately 8 to 10 cm in length and 0.4 to 0.8 cm in diameter. The common bile duct can be divided into three anatomic segments: supraduodenal, retroduodenal, and intrapancreatic (**Schulick, 2012**).

The supraduodenal segment resides in the hepatoduodenal ligament lateral to the hepatic artery and anterior to the portal vein; the course of the retroduodenal segment is posterior to the first portion of the duodenum, anterior to the inferior vena cava, and lateral to the portal vein. The pancreatic portion of the duct lies within a tunnel or groove on the posterior aspect of the pancreas. The common bile duct then enters the medial wall of the duodenum, courses tangentially through the submucosal layer for 1 to 2 cm, and terminates in the major papilla in the second portion of the duodenum. The distal portion of the duct is encircled by smooth muscle that forms the sphincter of Oddi (**Schulick, 2012**).

The common bile duct usually joins the pancreatic duct to form a common channel before entering the duodenum at the ampulla of Vater. Some patients will have an accessory pancreatic duct emptying into the duodenum. The blood supply of the common bile duct is segmental in nature and consists of branches from the cystic, hepatic, and gastroduodenal arteries. These meet to form collateral vessels that run in the 3 and 9 o'clock positions. The venous drainage forms a plexus on the anterior surface of the common bile duct (**Schulick, 2012**).

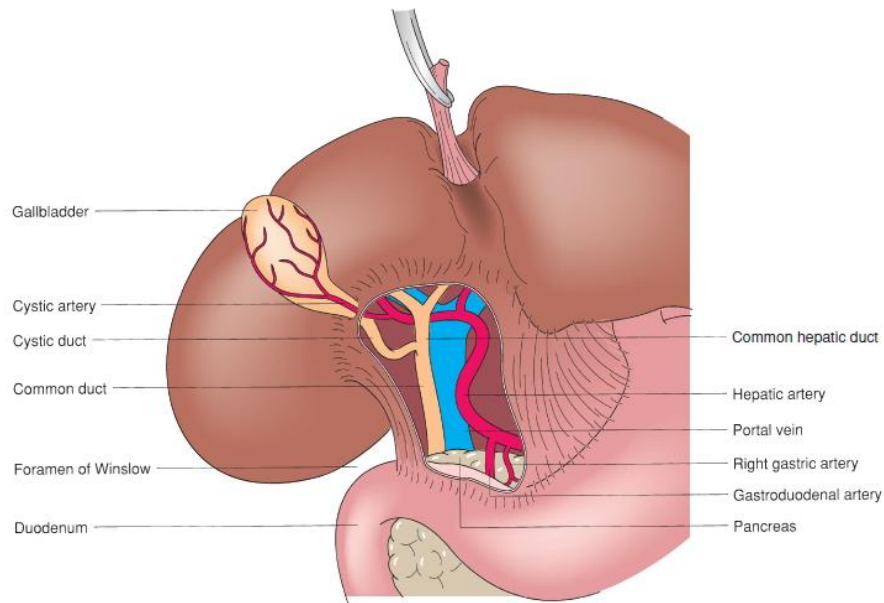


Figure 10: Relationship of structures within the hepatoduodenal ligament

Nerve supply of the biliary tract:

Parasympathetic fibers, mainly from the hepatic branch of the anterior vagal trunk, stimulate contraction of the gall bladder and relax the ampullary sphincter. Sympathetic fibres from the coeliac ganglia, which inhibit contraction. Afferent fibres including those subserving pain from the gall bladder may: run with right sided sympathetic fiber and reach spinal cord segments, (T7-9) and this explains radiation of the pain to the back in the infrascapular region or run into the right phrenic nerve (C3-5) and this explains the occasional referral of pain to the right shoulder region (McMinn, 2000).

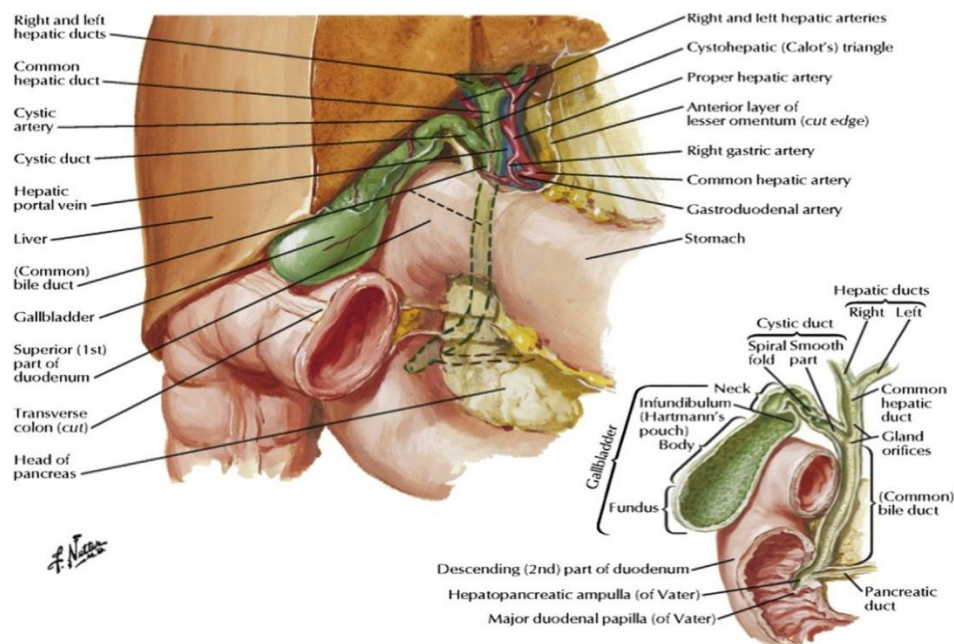


Figure 11: Classic view of extrahepatic biliary ducts

5. Anatomical Variations of the Biliary Tract

Extrahepatic Biliary Atresia

Congenital biliary atresia is the most serious malformation of the biliary tract. A short segment, an entire duct, or the whole system may be atretic; the atretic duct may be hypoplastic, stenosed, or reduced to a fibrous band that is easily overlooked by the surgeon.

Hepatic biliary duct atresias may be divided into three groups:

First group:

Patent proximal hepatic ducts and occluded distal ducts. Patency may occur in any portion of the right or left hepatic duct as it emerges from the liver. This atresia is called "correctable" (*Francoeur et al., 2003*).

Second group:

Occluded proximal ducts. No portion of the emerging hepatic duct is patent. This atresia is called "noncorrectable" (*Francoeur et al., 2003*).

Third group:

Includes the presence of intrahepatic atresia. In this form of atresia, the extrahepatic ducts may be present or absent. The mechanism of intrahepatic atresia remains obscure and the condition is as yet non correctable. It requires early liver transplantation (*Francoeur et al., 2003*).

There are morphological variations and abnormalities in more than 33% of gall bladders such as duplication. Sites of potential malformations of the extrahepatic biliary tract and common bile duct (*John et al., 2004*).

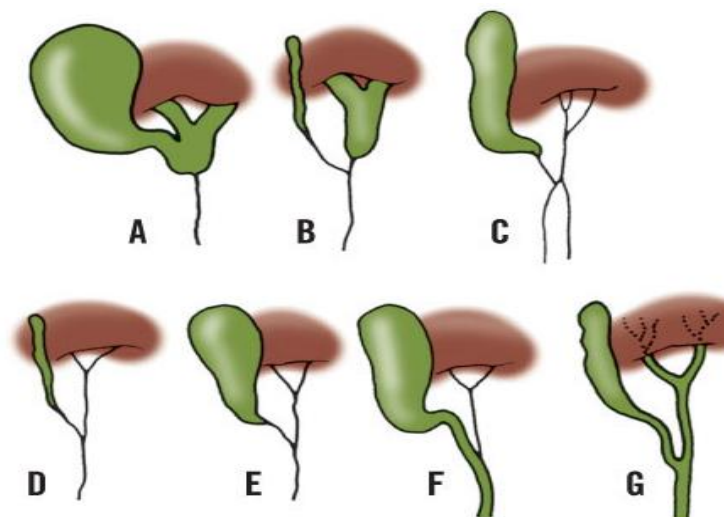


Figure 12: Different types of Biliary Atresia

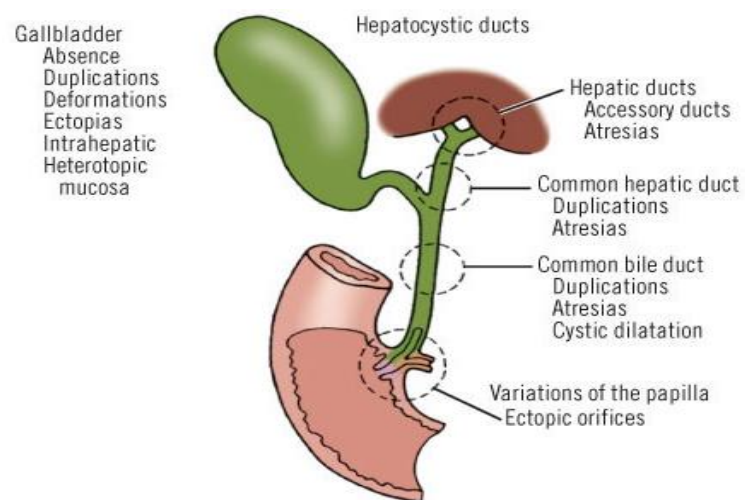


Figure 13: Sites of potential biliary tract malformations

Anomalies of Gall Bladder

1. Absence of Gall bladder

Occasionally the gallbladder (and usually the cystic duct as well) is absent or vestigial. The absence must be confirmed by ruling out an intrahepatic gallbladder or a left-sided Gall bladder (*John et al., 2004*).

2. Multiple Gall bladder

The first human double Gall bladder was found by autopsy in 1674, the first such anomaly to be recorded from observation of a living patient was in 1911, multiple gall bladders form a continuous spectrum of malformations, from an externally normal organ with an internal longitudinal septum to the most widely separated accessory gallbladders. For practical purposes, the anomalies can be categorized into six basic types. Three types belong to the split primordium group and three belong to the accessory gallbladder group (*John et al., 2004*).

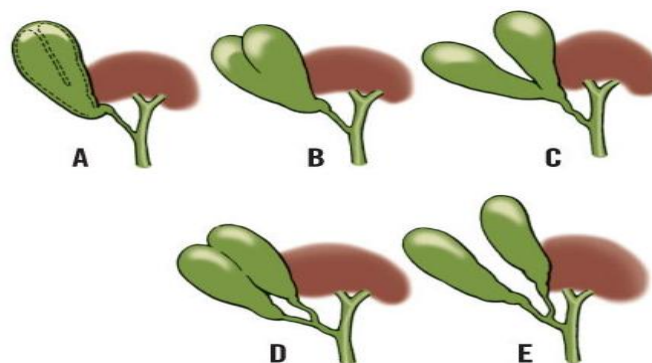


Figure 14: Different types of multiple Gall bladders

3. Left sided Gall bladder

Rarely, a Gall bladder is found on the inferior surface of the left lobe of the liver. In such cases, the cystic duct enters the common bile duct from the left. There is no associated functional disorder. Ultrasonography should detect this anomaly, but the radiologist must be alert (*John et al., 2004*).

4. Intrahepatic Gall bladder

Intrahepatic Gall bladder is submerged in the liver and gives the appearance of absence of the gallbladder. CT scan or ultrasonography may provide its only evidence. A

high percentage of occurrences of Gall bladder stones are associated with this anomaly (*John et al., 2004*).

5. Mobile Gall bladder

Mobile Gall bladder is attached to the liver by a mesentery. Such a gallbladder is susceptible to torsion and strangulation. Otherwise, it causes no symptoms (*John et al., 2004*).

Physiology of the Gallbladder

Gallbladder considered a part of the extrahepatic biliary system where the bile stored and concentrated. Bile originally formed in the liver. Then to the common hepatic duct through intrahepatic ducts. Then through the cystic duct to be stored in the gallbladder. When the food in the stomach & duodenum stimulates gallbladder to empty, it contracts and empties the concentrated bile back through the cystic duct, down the common bile duct, into the second portion of the duodenum through the ampulla Vater. Opening and closing of the sphincter of Oddi at the ampulla Vater controls the bile flow (***Jones et al., 2020***).

Gallbladder epithelium plays role in concentrating bile, which contains channels transport sodium chloride actively. The typical capacity of the gallbladder is 30 mL but it can distend up to 300 mL of fluid. The wall of the gallbladder is composed of the visceral peritoneum (on areas not in direct contact with the liver), subserosa, muscularis mucosa, lamina propria, and columnar epithelium (***Keplinger et al., 2014***).

Patient Positioning

In North American positioning, the patient is lying spine and the surgeon is positioned on the patient's left side. In European positioning, the patient is in low stirrups and the surgeon is on the patient's left or between the patient's legs.

With North American positioning, the camera operator usually stands on the patient's left and to the left of the surgeon, while the assistant stands on the patient's right. The video monitor is positioned on the patient's right above the level of the costal margin. If a second monitor is available, it should be positioned on the patient's left, to the right of the surgeon, where the assistant can have an unobstructed and comfortable view. Exposure can be improved by tilting the patient in the reverse Trendelenburg position and rotating the table with the patient's right side up. Gravity pulls the duodenum, the colon, and the omentum away from the gallbladder, thereby increasing the working space available in the upper abdomen.

The OR table should allow easy access for a fluoroscopic C arm, to facilitate intraoperative cholangiography. The table cover should be radiolucent (*Souba et al., 2005*).

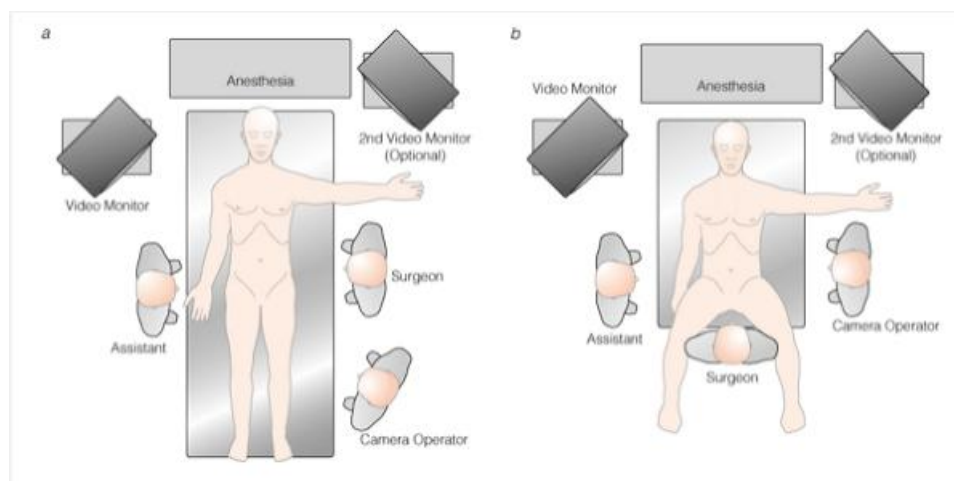


Figure 15: Setup for Laparoscopic Cholecystectomy

Technique

Patients undergoing laparoscopic cholecystectomy are prepared and draped in a similar fashion to open cholecystectomy. Conversion to an open operation is necessary

in up to 3% of patients undergoing elective cholecystectomy and up to 25% of patients undergoing laparoscopic cholecystectomy for acute cholecystitis.

Either an open or closed technique can be used to establish a pneumoperitoneum. With the open technique, a small incision is made at the umbilicus, and a blunt cannula (Hasson cannula) is inserted into the peritoneal cavity and anchored to the fascia.

An 11-mm trocar is inserted through the supraumbilical incision once a pneumoperitoneum is established. A 30- degree laparoscope is then inserted through the umbilical port, and an examination of the peritoneal cavity is performed. An 11-mm operating port is placed subxiphoid, and two additional 5-mm trocars are positioned subcostally in the right upper quadrant in the midclavicular line and in right iliac region in anterior axillary line.

The two 5-mm ports are used for grasping the gallbladder and exposing the gallbladder and cystic duct. The infundibulum and retract it laterally to further expose the triangle of Calot. Traction on the fundus should be upward toward the patient's head, and traction on the Hartmann pouch laterally to the right.

This combination “dis-aligns” the common duct and cystic duct so that they appear as distinct structures. Incorrect traction aligns the ducts so that they appear as a continuous structure (**Henry et al., 2011**).

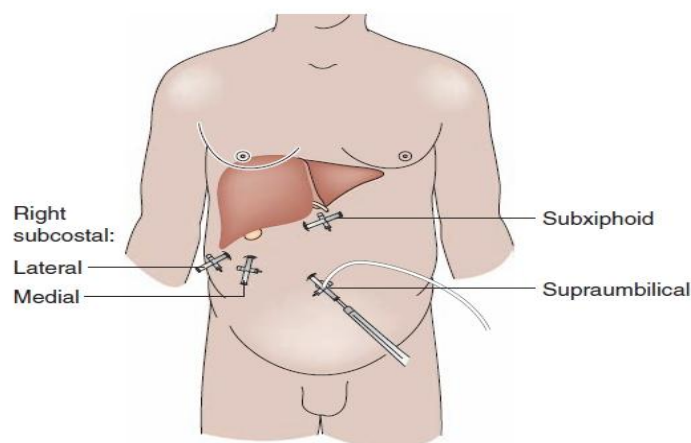


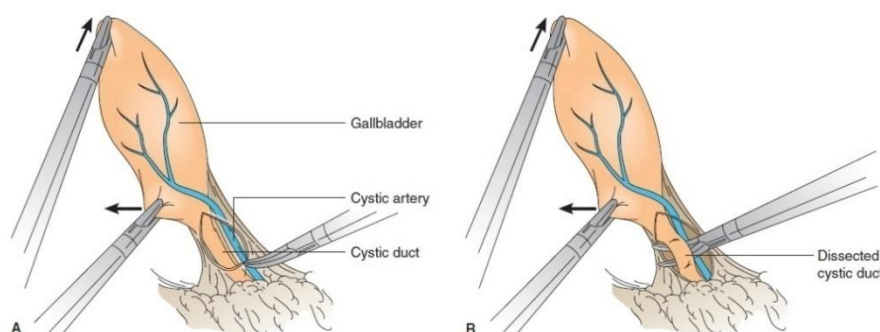
Figure 16: Port sites

Elective laparoscopic cholecystectomy can be safely performed as an outpatient procedure. Among patients selected for outpatient management, 77% to 97% of patients can be successfully discharged the same day. Factors contributing to overnight admission include uncontrolled pain, nausea and vomiting, operative duration greater than 60 minutes, and cases completed late in the day (**Henry et al., 2011**).

Routine operative cholangiography has been advocated to avoid ductal injury. However, opinion on the subject is sharply divided. Biliary injuries occur less frequently in the hands of surgeons who perform operative cholangiography routinely. However, in about 50% of ductal injuries, a cholangiogram fails to prevent the injury although abnormal anatomy is present (i.e., cholangiograms are often incorrectly interpreted) (**Henry et al., 2011**).

The indications for intraoperative cholangiography, when it is performed selectively, are known choledocholithiasis, a history of jaundice, a history of pancreatitis, a large cystic duct and small gallstones, any abnormality in preoperative liver function tests, and dilated biliary ducts on preoperative sonography. Provided these indications are carefully followed, selective cholangiography may be as effective in detecting clinically relevant stones as routine cholangiography.

Serious complications of laparoscopic cholecystectomy are rare, the mortality rate being less than 0.3%. As cholecystectomy rates have risen, however, the total number of deaths has not decreased (**Henry et al., 2011**).



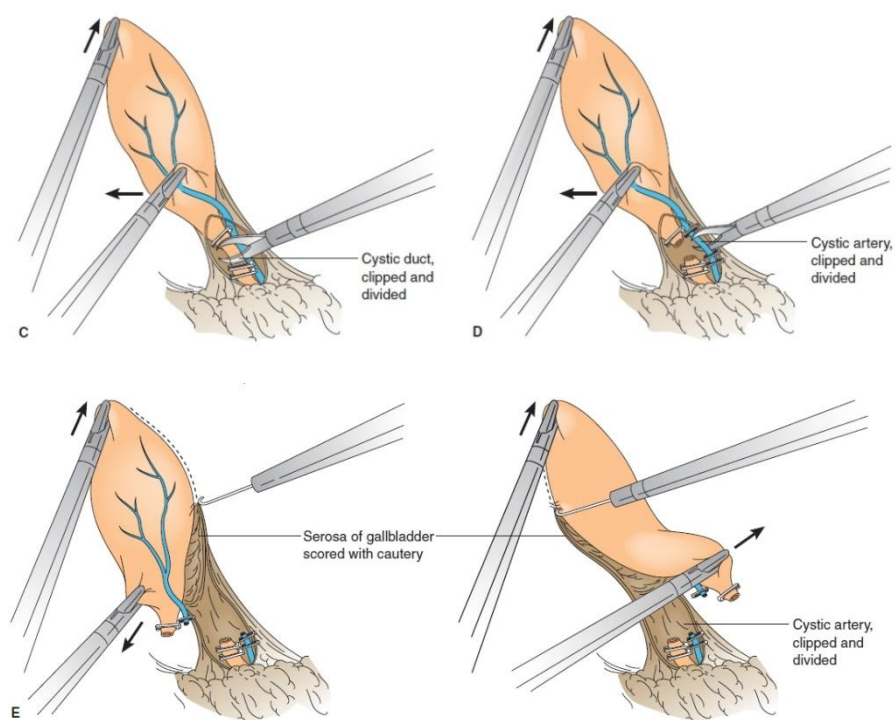


Figure 17: Operative steps

However its important to visualize the *critical view of safety* in order to try to prevent complications of biliary tree injury either CBD or Common Hepatic Ducts injury.

Critical View of Safety

Anatomy is important when performing laparoscopic cholecystectomy. The concept of the critical view of safety (CVS) was originated by Strasberg et al in 1995 (*Hori et al., 2016*).

The triangle of Calot is cleared of fat and fibrous tissue. Only two structures are connected to the lower end of the Gall bladder once this is done, and the lowest part of the gall bladder attachment to the liver bed has been exposed. The latter is an important step, equivalent in the open technique to taking the gallbladder off the liver bed. It is not necessary to expose the common bile duct. Once the critical view is attained, cystic structures may be occluded, as they have been conclusively identified. Failure to achieve the critical view is an indication for conversion or, possibly, cholangiography to define ductal anatomy. It is the author's opinion that there is considerable danger in relying simply on the appearance of the "cystic duct" — Gall bladder junction, as this may be deceiving, especially in the presence of severe inflammation (*Strasberg et al., 2002*).

Surgeon should be positively identify cystic duct and cystic artery (CA) before cutting or clipping them. Calot's triangle must be dissected well. Gall bladder lower end should be dissected off the liver bed, and the bottom of the liver should be visible. It is not necessary to directly confirm the CHD and CBD. Hence, only two structures should be seen to enter the GB. Positive identification of the CD and CA as they join the GB infundibulum is required before these structures can be divided (*Hori et al., 2016*).

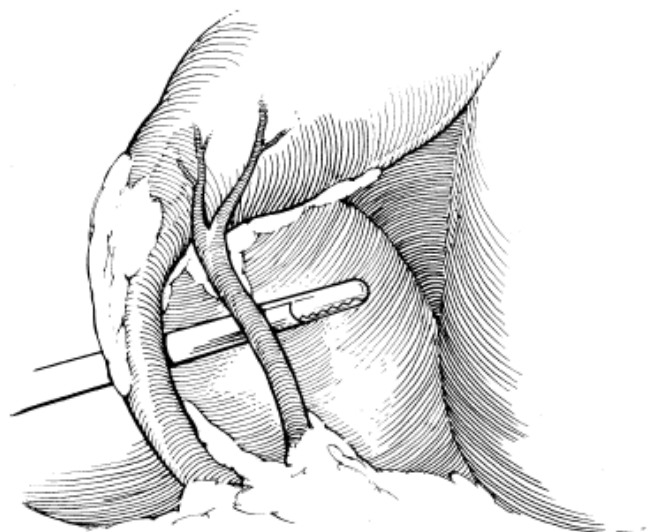


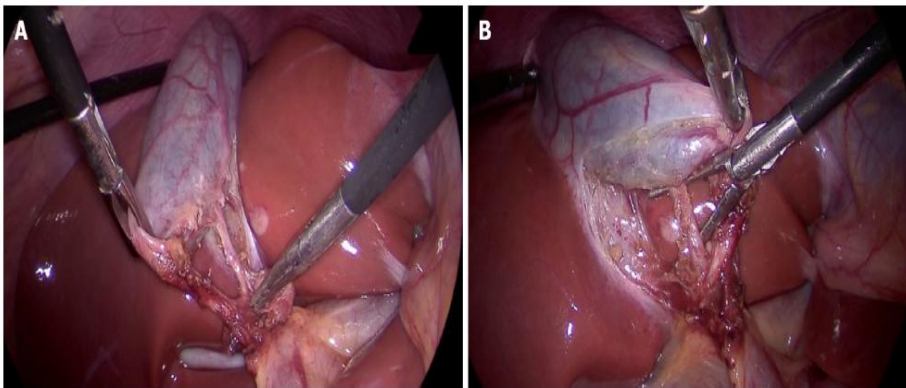
Figure 18: The Critical view of Safety

Figure 19: [A & B] Critical view of safety with all three components: (1) Fibrofatty tissue has been cleared from the hepatocystic triangle; (2) Lower part of the cystic plate has been clearly exposed; and (3) Only two tubular structures are seen entering the gallbladder. A: Anterior view; B: Posterior view (inverted hepatocystic triangle).

Complications of Laparoscopic Cholecystectomy

Laparoscopic cholecystectomy (LC) remains an extremely safe procedure with a mortality rate of (0.22-0.4%). Major morbidity occurs in approximately 5% of patients (*Steiner et al., 2004*).

Complications include the following:

1. Trocar/Veress needle injury:

Intestinal injury may occur during establishment of abdominal access, adhesiolysis, or dissection of the gallbladder off of the duodenum or colon. An injury to the bowel should be repaired with careful 1- or 2-layer suture closure. The incidence of injury to viscera or vessels from a Hasson trocar or Veress needle is similar (in the range of 0.2%) (*Ahmad et al., 2019*).

A recent systematic review showed that an open-entry technique is associated with a significant reduction in failed entry when compared to a closed-entry technique, with no difference in the incidence of visceral or vascular injury (*Ahmad et al., 2019*).

Significant benefits were noted with the use of a direct-entry technique when compared to the Veress Needle. The use of the Veress Needle was associated with an

increased incidence of failed entry, extraperitoneal insufflation and omental injury; direct-trocar entry is therefore a safer closed-entry technique (*Ahmad et al., 2019*).

2. Haemorrhage:

Large-vessel vascular injury usually occurs at the time of initial abdominal access. These may be lethal complications. Development of a retroperitoneal hematoma or hypotension should be treated immediately by conversion to laparotomy.

The most obvious danger is that of hemorrhage from the many large blood vessels lying anterior to the biliary tree. Such vessels are inconstant in number and location. The posterior superior pancreaticoduodenal artery, anterior to the retroduodenal portion of the common bile duct, is the vessel most frequently encountered (*Steiner et al., 2004*).

All the vessels listed in Table (1) are subject to possible injury.

Table 1: Segments of the Biliary tract and frequency of arteries lying anterior to them

Segment	Artery Anterior	Percent Frequency
Right and left hepatic ducts	Right hepatic artery	12-15
	Cystic artery	<5
Common hepatic duct	Cystic artery	15-24
	Right hepatic artery	11-19
	Common hepatic artery	<5
Supraduodenal common bile duct	Anterior artery to CBD	50
	Posterosuperior pancreaticoduodenal artery	12.5
	Gastroduodenal artery	5.7-20*
	Right gastric artery	<5
	Common hepatic artery	<5
	Cystic artery	<5
	Right hepatic artery	<5
Retroduodenal common bile duct	Posterosuperior pancreaticoduodenal artery	76-87.5
	Supraduodenal artery	11.4

(John et al., 2004)

The following list of variations in the cystic artery may help one avoid common pitfalls:

- The cystic artery usually arises from the right hepatic artery (A).
- Dual cystic arteries, one arising from each of the hepatic arteries (B)
- Cystic artery arising from the common hepatic artery (C)
- Cystic artery arising from the gastroduodenal artery (D)
- Cystic artery arising from an anterior right hepatic artery (E)
- A single cystic artery arising from the left hepatic artery (F).

(Strasberg, 2011)

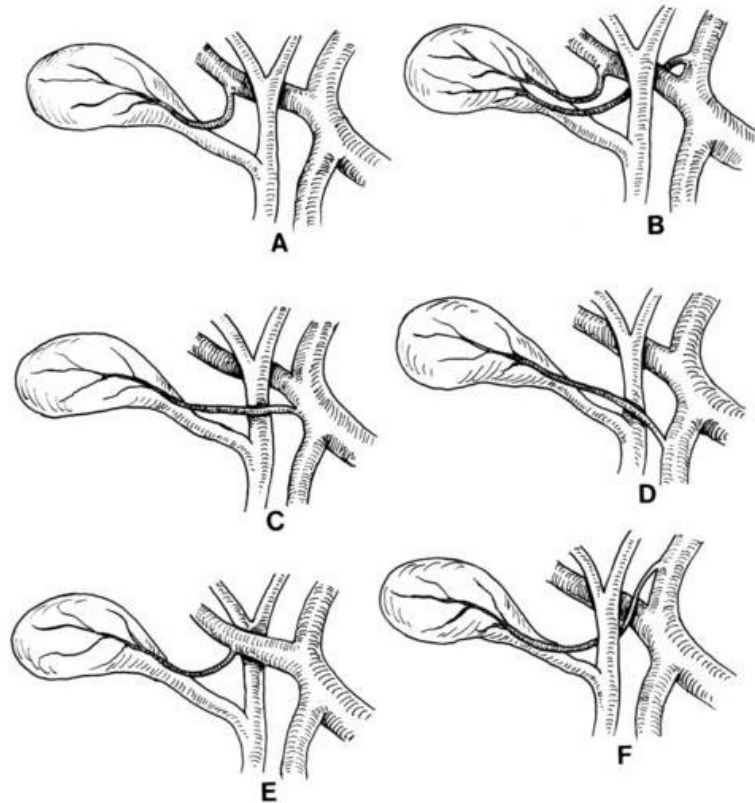


Figure 20: Anatomic variations of the blood supply for the Gall bladder

Injury to the portal vein or the inferior vena cava is a serious complication. These vessels must, of course, be repaired at once (*Tchirkow et al., 2002*).

Excessive bleeding in the region of the triangle of Calot should not be treated laparoscopically. Attempts at blind clipping or cauterizing significant bleeding usually leads to worsening haemorrhage or hepatic artery injury. If, and only if, a bleeding site can be definitely identified and the locations of both the hepatic artery and common bile duct (CBD) are known, bleeding may be controlled with electrocautery or clips (*Tchirkow et al., 2002*).

Management of Haemorrhage:

Prevention of arterial bleeding begins by dissecting the artery carefully and completely before clipping and by inspecting the clips to ensure that they are placed completely across the artery without incorporating additional tissue (e.g., a posterior cystic artery or right hepatic artery). When arterial bleeding is encountered, it is essential to maintain adequate exposure and to avoid blind application of hemostatic

clips or cauterization. The laparoscope should be withdrawn slightly so that the lens is not spattered with blood. The surgeon should then pass an atraumatic grasping forceps through a port other than the operating port and attempt to grasp the bleeding vessel. An additional trocar may have to be inserted for simultaneous suction-irrigation. Once proximal control is obtained, the operative field should be suctioned and irrigated to improve exposure. Hemostatic clips are then applied under direct vision; in addition, a sponge may be introduced to apply pressure to the bleeding vessel. Conversion to open cholecystectomy is indicated whenever bleeding cannot be promptly controlled laparoscopically (**Strasberg, 2011**).

Bleeding from veins of the Gall bladder bed or from veins of the common bile duct is a minor complication which can usually be controlled by fulguration of the bleeding site using a spatulated electrocautery wand for this purpose (**Strasberg, 2011**).

If a larger intrahepatic sinus has been entered, hemostatic agents (eg, microfibrillar collagen) can be placed laparoscopically in the liver bed, and pressure can be held with a clamp.. The argon plasma coagulator (APC) can be an excellent tool for severe Gall bladder fossa oozing that is not responsive to simple electro-cautery (**Strasberg, 2011**).

A second vascular complication of biliary surgery is ischemia to the liver from unintended ligation of the right hepatic artery or an accessory or replacing aberrant right hepatic artery. Interference with the blood supply of the common bile duct may result in ischemia and stricture. Other surgeons feel that the blood supply is good and that collateral circulation will prevent local ischemia (**Strasberg, 2011**).

3. Post cholecystectomy syndrome:

This refers to a set of abdominal symptoms that occur with a frequency of up to 40% after cholecystectomy. Symptoms are often vague and include dyspepsia, flatulence, bloating, right upper quadrant pain, and epigastric pain.

The most common causes of this syndrome are dietary indiscretion, retained CBD stones, inflammation of the cystic duct remnant, and sphincter of Oddi dysfunction (**Zhou et al., 2003**).

4. Bile ducts injury or stricture:

The most dreaded complication of LC is injury to the common bile or common hepatic duct. The estimated incidence of bile duct injury in cholecystectomies performed laparoscopically varies from 0.3-2.7%), In contrast; biliary tract injuries were noted to occur in 0.25-0.5% of open cholecystectomies (*McMahon et al., 2004*).

A major risk factor for bile duct injury is the experience of the surgeon, other risk factors are the presence of aberrant biliary tree anatomy and the presence of local acute or chronic inflammatory (*Lien et al., 2007*).

Data suggest that the incidence of bile duct injury during open cholecystectomy is 1 in 500 to 1,000 cases), The incidence of bile duct injury during laparoscopic cholecystectomy is clearly higher. Although a wide range in the incidence of injury can be found in reported series, the most accurate data most likely come from surveys encompassing thousands of patients. These reports reflect the results from a large number of surgeons in both community and teaching hospitals. The results of such series suggest an incidence of bile duct (*Bertrand et al., 2003*).



Figure 21: Percutaneous transhepatic cholangiogram in a patient with a bile duct stricture secondary to iatrogenic injury during cholecystectomy

Patients and Methods

This study was done to compare whether patients with intraabdominal drain following uncomplicated laparoscopic cholecystectomy are associated with more complications and hospital stay than those without intraabdominal drain.

Patients' population:

This study is a case series of 100 patients who underwent laparoscopic cholecystectomy.

We didn't include patients who had:

1. Been operated for laparoscopic cholecystectomy other than cholelithiasis.
2. All patient contraindicated to laparoscopic surgery.
3. Emergency operations were excluded.
4. Acute cholecystitis, empyema GB, mucocele of GB.
5. Patient with bleeding tendency (preoperative increase in INR or thrombocytopenia.
6. Patient with chronic liver or kidney diseases.
7. Intraoperative bleeding OR inadequate hemeostasis.
8. Patients can be excluded based on operator`s judgement.

The included patients presented to the General Surgery Department at Rafidia Surgical Hospital .

Clinically all the patients were presented to undergo laparoscopic cholecystectomy operation and post-operative follow up for incidence of drain site pain, drain site infection, postoperative collection & hospital stay, Laboratory and radiological investigations were carried out in order to detect any intra-abdominal collection.

Type of Patients:

This was a prospective study that included 100 patients with cholelithiasis who were twenty to sixty five years old and from both sexes attending to the hospital. The patients were randomly allocated into two groups each included 50 patients, first group (Group A) underwent laparoscopic cholecystectomy and we put intra-abdominal drain, the second group (Group B) underwent laparoscopic cholecystectomy without intra-abdominal drain.

Study procedure:

All patients were subjected to preoperative, operative, and postoperative assessment:

1. **Preoperative:** the preoperative assessment included full history taking, clinical examination, which included general examination of the chest, heart, and abdomen.

Patients were prepared for surgery, by explaining to them what is the procedure, its risks and benefits.

2. **Operative:** the operations were performed on the patients under general anesthesia and in supine position.
3. **Postoperative:** patients were followed up weekly for 2 to 3 weeks. Early postoperative follow-up included evaluation of postoperative pain, hospital stay and port site infection.

Patients' evaluation:

Clinical examination:

- Complete history taking.
- Detailed general and local physical examination.
- Operative details were all reviewed.

Laboratory investigations:

- 1- Full blood count.
- 2- Coagulation profile.

- 3- AST.
- 4- ALT.
- 5- Alkaline phosphatase.
- 6- Direct bilirubin.
- 7- Total bilirubin.

Statistical Analysis

Data were collected, revised, coded and entered to the Statistical Package for Social Science (IBM SPSS) version 23. The quantitative data were presented as mean, standard deviations and ranges when their distribution found parametric. Also qualitative variables were presented as numbers and percentages.

The comparison between groups regarding qualitative data was done by using ***Chi-square test***.

The comparison between two independent groups with quantitative data and parametric distribution were done by using ***Independent t-test***

The confidence interval was set to 95% and the margin of error accepted was set to 5%. So, the p-value was considered significant as the following:

P-value > 0.05: Non significant (NS)

P-value < 0.05: Significant (S)

P-value < 0.01: Highly significant (HS)

Results

Among the 100 patients with chronic calcular cholecystitis included in this study 70 were females (70%) and 30 were males (30%).

The 100 patients were randomly distributed into two equal groups (50

patients each): Group A underwent laparoscopic cholecystectomy and we put intra-abdominal drain and, while Group B underwent laparoscopic cholecystectomy but we didn't insert intra-abdominal drain.

Patients' age ranged from 20 to 65.

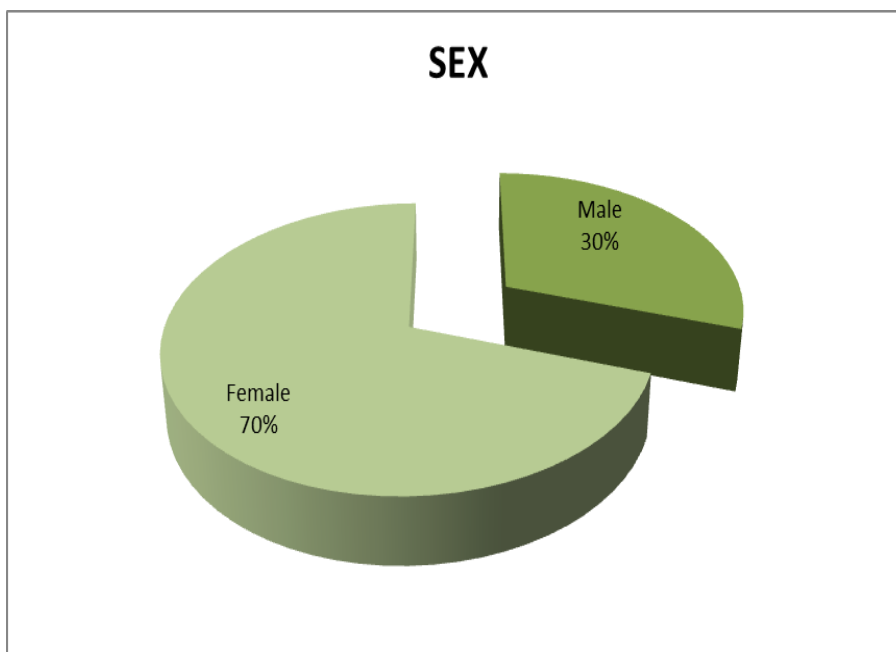


Figure 22: Sex distribution

➤ **Analysis of postoperative pain at drain/port site:**

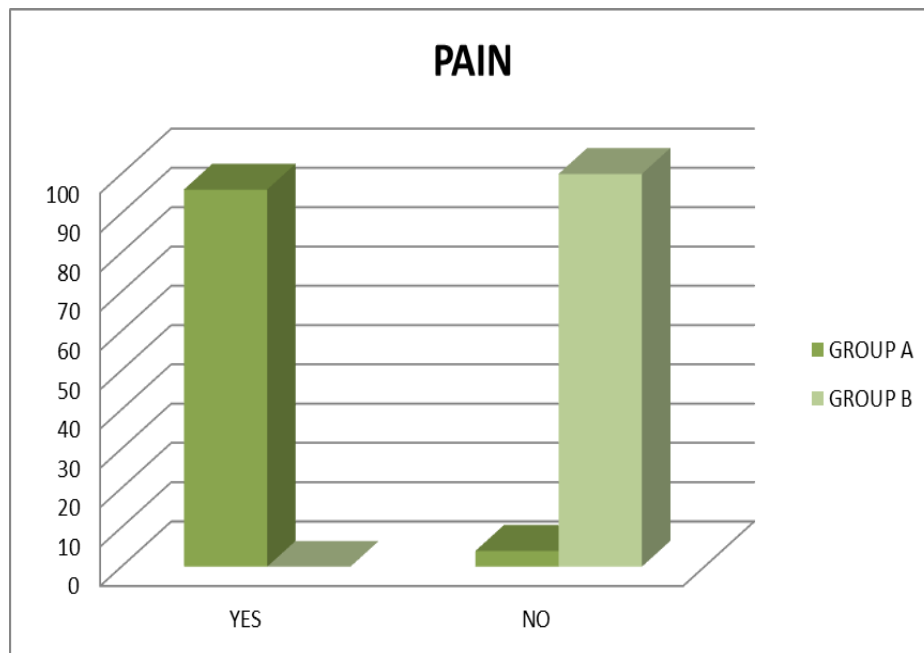


Figure 23: Analysis of postoperative pain at drain/port site

The incidence of postoperative pain at port site where drain was inserted was more in patients in group A (96%), while patients without drain (group B) experienced no or minimal postoperative pain (0% approximately). (P value 0.000 which is highly significant).

➤ **Analysis of postoperative infection at port/drain site:**

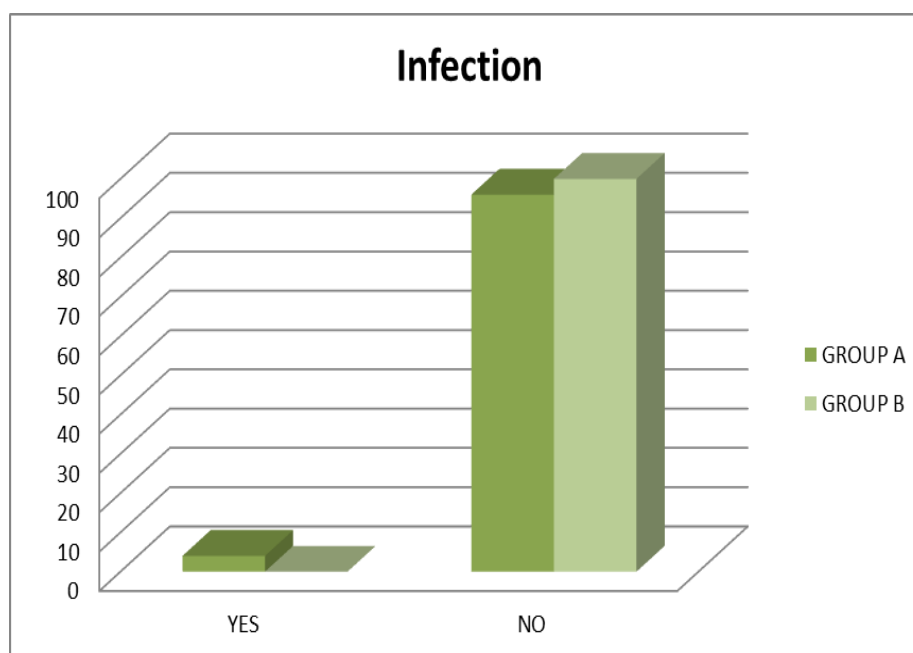


Figure 24: Analysis of postoperative infection at port/drain site

Also the incidence of postoperative drain site infection is higher in patients in Group A (drain from the port site) 4%, (P value 0.000). While in group B (patients with no drain) almost no patient experienced port site infection.

➤ **Analysis of post operative hospital stay:**

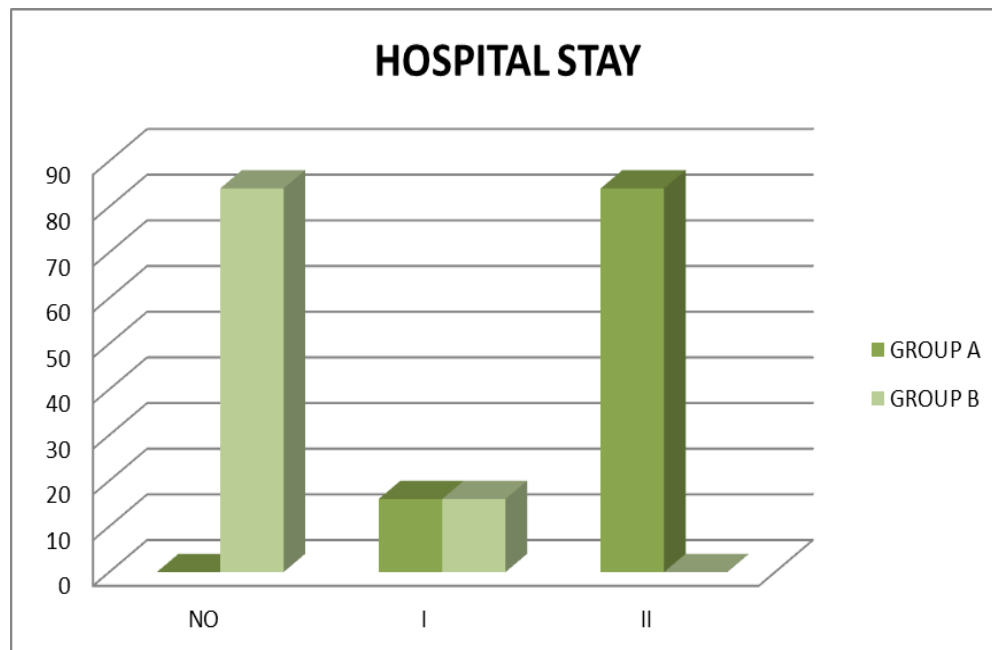


Figure 25: Analysis of post operative hospital stay

Regarding hospital stay, majority of patients with drain (group A) required hospital stay for almost 2 days (16% of patients discharged in day 1 postoperatively while 84% of patients discharged on day 2 post-operative), while those without drain (group B) 84% of patients discharged at the same day, while 16% discharged day 1 postoperative).

➤ **Analysis of postoperative collection:**

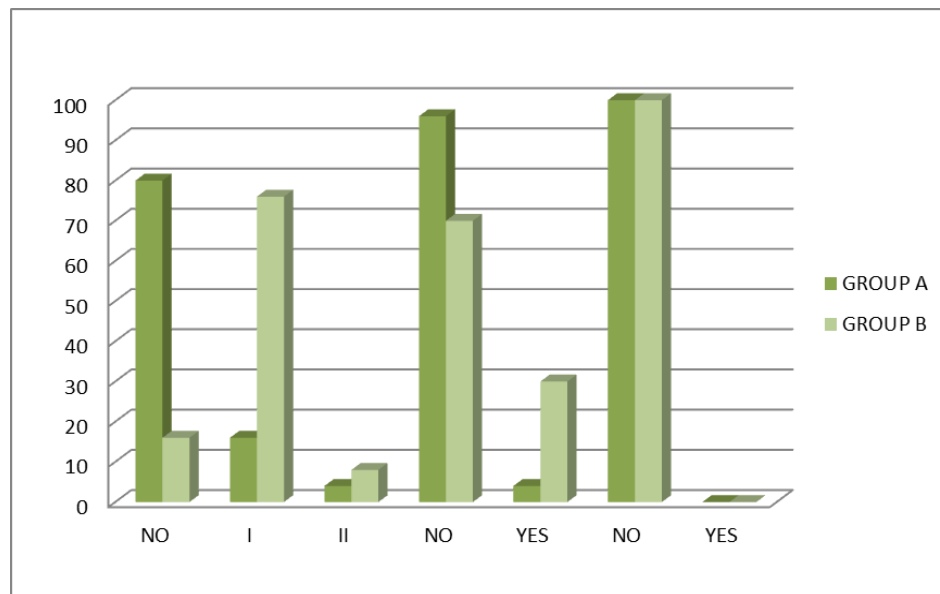


Figure 26: Analysis of postoperative collection

Regarding post-operative intrabdominal collection, we followed up patients using ultrasound on day 1, day 2 and day 3 postoperative (POD1, POD2, POD3), (**No collection, I collection < 50 ml, II collection 50-100 ml**).

This revealed intra-abdominal fluid collection POD1 was lower in drain group (Group A) than in non-drain group (Group B).

POD 1 the collection was lower in group A, 80% of patients developed no collection, while 16% of patients developed collection < 50 ml, while 4% of patients developed collection from 50-100ml, while those in group B, 16% of patients developed no collection, while 76% developed collection <50 ml, while 8% of patients developed collection 50-100 ml.

POD2 only 4% of patients those in group A (with drain) developed minimal collection that disappeared on next day. Group B (without drain)30% of patients developed minimal collection that disappeared on next day.

Table 2: Comparison between two groups regarding age, sex, pain, infection, hospital stay and collection postoperative day 1, 2, 3

		Group A	Group B	Test value	P- value	Sig.
		No.= 50	No.= 50			
Age	Mean±SD	37.80 ± 10.32	34.28 ± 9.33	1.266•	0.212	NS
	Range	22 – 63	20 – 53			
Sex	Males	12 (24.0%)	18 (36.0%)	0.000*	1.000	NS
	Females	38 (76.0%)	32 (64.0%)			
Pain	No	2 (4.0%)	50 (100.0%)	46.154*	0.000	HS
	Yes	48 (96.0%)	0 (0.0%)			
Infection	No	48 (96.0%)	50 (100.0%)	46.154*	0.000	HS
	Yes	2 (4.0%)	0 (0.0%)			
Hospital stay	No	0 (0.0%)	21 (84.0%)	42.000*	0.000	HS
	I	4 (16.0%)	4 (16.0%)			
	II	21 (84.0%)	0 (0.0%)			
POD1	No	40 (80.0%)	8 (16.0%)	38.789*	0.000	HS
	I	8 (16.0%)	38 (76.0%)			
	II	2 (4.0%)	4 (8.0%)			
POD2	No	48 (96.0%)	35 (70.0%)	2.083*	0.149	NS
	Yes	2 (4.0%)	15 (30.0%)			
POD3	No	50 (100.0%)	50 (100.0%)	-	-	-
	Yes	0 (0.0%)	0 (0.0%)			

P-value >0.05: Non significant (NS); P-value <0.05: Significant (S); P-value< 0.01: highly significant (HS)

*:Chi-square test; •: Independent t-test

Discussion

Laparoscopic cholecystectomy has largely replaced open cholecystectomy because of shorter hospital stay, faster recovery, and lower overall morbidity. However, the morbidity due to bile duct injury has increased with the advent of the laparoscopic approach (*Henry et al., 2011*).

Prevention of intra-abdominal collections after laparoscopic cholecystectomy is our main concern. Post cholecystectomy collection in the sub-hepatic space is usually absorbed whether a drain is used or not.

Intraperitoneal collection of blood may cause postoperative pyrexia, prolong the hospital stay, and increase the incidence of wound infection, while the presence of bile in the peritoneal cavity produces peritoneal irritation.

Only some clinically significant abdominal collections may need intervention. The drain may also give false sense of security as it may get blocked and later patients present with complications.

Our study revealed that less post-operative port site pain, infection and hospital stay in group B (without drain) in comparison with group A (with drain) and those results were statistically significant ($P\text{-value} < 0.01$).

Regarding post cholecystectomy free intra-abdominal collection, it was less evident by ultrasound in group A (with drain) rather than group B (without drain) in first 24 hours. Although the results were statistically significant in the first 24 hours, the difference became insignificant on subsequent ultrasound after 72hrs. ($p\text{-value} < 0.01$ in day 1 postoperative, $P\text{-value} > 0.05$ in day 2 postoperative).

Our results are similar to the result conducted by El-Labban et al. regarding postoperative hospital stay and abdominal collection, while his results were opposite to our study regarding port site infection, pain were there no statistical difference (**EL-Labban et al., 2012**).

Opposite results were obtained by Ishikawa et al in their randomized study which include 295 patients where Group A (with drain) include 145 patients and Group B (without drain) include 150 patient, where he stated that no significant difference between the two groups with respect to postoperative complication rate, while regarding postoperative hospital stay the results were similar to our study (**Ishikawa et al., 2011**).

This is in support with results by Georgiou C as he included 116 patients, where 63 patient in drained group (YD) and 53 in non-drained group (ND) in regard

postoperative pain, hospital stay as he stated that the proportion of patients staying in hospital for >2 days was higher in the YD group: 28.6% of the patients versus 13.2% in the ND group ($P = .05$).

As for Subhepatic fluid was more often observed in the YD group (47% versus 34% in the ND), but the difference was not statistically significant (**Georgiou et al., 2011**).

Our results were also consistent with those reported by Yang et al, who reported that the abdominal drain group displayed significantly higher pain scores (MD: 1.07; 95% CI: 0.69–1.46; $P < .001$), abdominal drain prolonged the duration of postoperative hospital stay (MD: 0.47 day; 95% CI: 0.14–0.80; $P = .005$). Wound infection was found to be associated with the use of abdominal drains (RR: 1.97; 95% CI: 1.11–3.47; $P = .02$) (**Yang et al., 2020**).

Similarly, our result is supported by results of the study by Sharma et al who found that postoperative pain was higher in the group A (with drain) than in Group B (without drain), also the proportion of the patients staying in the hospital for more than two days was higher in group A (with drain), 14 (46.66%) and 8 (26.66%) in group B (without drain) ($p < 0.05$) while there was no statistical difference in the rate of wound infections (**Sharma et al., 2016**).

Summary

Laparoscopic cholecystectomy provides a safe and effective treatment for patients with gallstones as it reduces post-operative pain with almost negligible scar, short hospital stay and earlier return to work.

Prophylactic drains in abdominal surgery are widely used either to detect complications early, such as postoperative hemorrhage or bile leakage, or to drain collections which may be toxic, as bile. However, evidence-based data do not support the use of prophylactic drainage in the majority of abdominal procedures.

Gallstones are still one of the most common conditions in surgical outpatient department. Laparoscopic cholecystectomy, after its advent in 1987, rapidly established itself as the gold standard treatment of gallstones. In 1913 cholecystectomy without drainage was described, and since then surgeons were divided whether to use it or not in uncomplicated cases. Most surgeons continue to use routine drain for the fear of bile leakage or bleeding. Such complications invariably occurred in spite of sub hepatic drainage. So, there arises a need for study, whether to put drain or not, and its consequences.

Among the 100 patients included in this study, 70 were females (70%) and 30 were males (30%), with female to male ratio 3.1: 1 approximately. The 100 were randomly distributed into two groups: Group A patients underwent laparoscopic cholecystectomy with drain insertion and this group composed of 50 patients 12 were males (24%) and 38 were females (76%). While Group B also underwent laparoscopic cholecystectomy but without drain insertion and this group composed of 50 patients, 18 males (36%) and 32 females (64%)

In our study we include patients with chronic calculous cholecystitis which is indicated for elective laparoscopic cholecystectomy.

Regarding the Postoperative Pain:

It was higher in patients in Group A (with drain) than in Group B (without drain), and that was statistically significant (P value less than 0.05).

Regarding Port Site Infection:

All patients included in this study were followed post-operative for port site infection (till day 7 time of stitch removal) and we found the incidence of port site infection was higher in Group A (with drain) at site where the drain was inserted, and that was statistically significant (P value less than 0.05).

Regarding postoperative hospital stay:

Hospital stay was much less in patients without drain (Group B) as 42 patient (84%) were discharged at the same day of operation which is statistically significant (P value less than 0.05).

Regarding postoperative collection:

We followed up postoperative collection in the two groups in day1, 2 and 3 and revealed that, POD 1 the collection was lower in Group A (with drain), 80% of patients developed no collection, while 16% of patients developed collection < 50 ml, while 4% developed collection 50-100 ml, while those in Group B (without drain) 8% of patient developed no collection, while 76% of patients developed < 50 ml, while 8% of patients developed 50-100 ml.

And that was statistically significant in day1 postoperative (P value less than 0.05), while it was non-significant in the following 72 hours.

Conclusion

Uncomplicated gall stone diseases can be treated by laparoscopic cholecystectomy without need for drain with reasonable safety by an experienced surgeon. With no usage of drain, it is significantly advantageous in terms of post-operative pain, use of analgesics, infection and hospital stay.

In our study we compare drain insertion post laparoscopic cholecystectomy in matter of post-operative pain, port site infection, hospital stay and post-operative collection.

It revealed that there is significant reduction in postoperative pain in patients without drain than in those with drain.

Moreover regarding postoperative wound infection it was also lower in patient with no drain.

There were also statistically significant reduction in postoperative hospital stay in patients without drain.

Furthermore, postoperative intrabdominal collection is significantly lower in patients with drain in first 24 hours, while after that there were no significant difference regarding intrabdominal collection.

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الملخص العربي

يوفر استئصال المرارة بالمنظار علاجًا آمنًا وفعالًا للمرضى الذين يعانون من حصوات المرارة لأنه يقلل من آلام ما بعد الجراحة مع ندبات قليلة جدًا، وإقامة قصيرة في المستشفى والعودة إلى العمل مبكرًا.

تستخدم المصارف الوقائية في جراحة البطن على نطاق واسع إما للكشف عن المضاعفات مبكرًا، مثل نزيف ما بعد الجراحة أو تسرب الصفراء، أو لتصريف التجمعات التي قد تكون سامة، مثل الصفراء. ومع ذلك، فإن البيانات القائمة على الأدلة لا تدعم استخدام الصرف الوقائي في غالبية جراحات البطن.

لا تزال حصوات المرارة من أكثر الحالات شيوعًا بالعيادات الخارجية الجراحية. أثبت استئصال المرارة بالمنظار، بعد ظهوره في عام 1987، نفسه بسرعة باعتباره العلاج القياسي الذهبي لحصوات المرارة. في عام 1913 تم وصف استئصال المرارة بدون تصريف، ومنذ ذلك الحين تم تقسيم الجراحين سواء كانوا سيستخدمونها أم لا في حالات غير معقدة. يواصل معظم الجراحين استخدام التصريف الروتيني خوفًا من تسرب أو نزيف الصفراء. تحدث مثل هذه المضاعفات دائمًا على الرغم من وضع التصريف. لذا فإن هناك حاجة للدراسة، سواء لوضع الصرف أم لا، ونتائجها.

من بين 100 مريضًا شملتهم هذه الدراسة، كان 70 من الإناث (70%) و30 من الذكور (30%)، مع نسبة الإناث إلى الذكور 3.1:1 تقريبًا. تم توزيع المرضى بشكل عشوائي على مجموعتين: المجموعة الأولى (أ) خضعوا لعملية استئصال المرارة بالمنظار مع إدخال التصريف، وتتكون هذه المجموعة من 50 مريضًا، 12 منهم من الذكور (24%) و38 من الإناث (76%) بينما خضعت المجموعة الثانية (ب) أيضًا لاستئصال المرارة بالمنظار ولكن بدون إدخال تصريف وتتكون هذه المجموعة من 50 مريضًا، 18 ذكور (36%) و32 إناث (64%).

تشمل دراستنا المرضى الذين يعانون من التهاب حصوي مراري مزمن والذي يشار إليه في استئصال المرارة بالمنظار الاختياري.

❖ فيما يتعلق بألم ما بعد الجراحة:

كان أعلى في المرضى في المجموعة أ (مع الصرف) منه في المجموعة ب (بدون صرف)، وكان ذلك ذا دلالة إحصائية (قيمة P أقل من 0.05).

❖ فيما يتعلق التهاب أماكن دخول المنظار الجراحي:

تمت متابعة جميع المرضى المشمولين في هذه الدراسة بعد الجراحة لعدوى أماكن دخول المنظار الجراحي (حتى اليوم السابع من إزالة الغرز) ووجدنا أن حدوث عدوى في أماكن دخول المنظار الجراحي كان أعلى في المجموعة أ (مع الصرف) في الموقع حيث تم إدخال الصرف ، وكانت ذات دلالة إحصائية (قيمة P أقل من 0.05).

❖ فيما يتعلق الإقامة في المستشفى بعد الجراحة:

كانت الإقامة في المستشفى أقل بكثير في المرضى الذين ليس لديهم تصريف (المجموعة ب) حيث تم خروج 42 مريضاً (84%) في نفس يوم العملية من المستشفى وهو ذو دلالة إحصائية (قيمة P أقل من 0.05).

❖ فيما يتعلق بالتجمعات ما بعد الجراحة:

تابعنا التجمعات ما بعد الجراحة في المجموعتين في اليوم الأول والثاني والثالث وتبين أن التجمعات في اليوم الأول بعد العملية كانت أقل في المجموعة أ (مع استنزاف)، حيث 80% من المرضى لم يطوروا تجمعات بالبطن، بينما طور 16% من المرضى تجمعات أقل من 50 مل، بينما طور 4% من المرضى تجمعات 50-100 مل.

في حين أن المجموعة ب (بدون تصريف) 16% من المرضى لم يطوروا تجمعات بالبطن، بينما طور 76% من المرضى تجمعات أقل من 50 مل، بينما طور 8% من المرضى تجمعات 50-100 مل.

وكان ذلك ذا دلالة إحصائية في اليوم الأول بعد الجراحة (قيمة P أقل من 0.05) ، بينما لم تكن ذات دلالة في الـ 72 ساعة التالية.

The impact of utilizing alveolo-paste on the healing of soft tissue after dental extraction

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Abstract

Aim:

The aim of the current research is to evaluate the effectiveness of Local application of alveogyl paste on soft tissue healing after lower molars extraction.

Material and methods:

The research included a total of 40 healthy patients between the ages of 18-35 years. Patients were randomly divided into two groups; alveolopaste was applied to alveogyl group. while no material was applied in the control group . Patients were assessed at the first and seventh postoperative days and the scores done using numerical periodontal probe to evaluate soft tissue healing of post extraction socket.

Results:

Bucco-lingual width of soft tissue margins of the socket, were highest on the first postoperative day and decreased gradually in both control and alveogyl group in the seven postoperative days. There were statistically significant differences in bucco-lingual width between control and alveogyl groups in the seven postoperative days

Conclusions:

Local application of alveogyl paste after teeth extraction accelerate soft tissue healing.

تأثير مادة الفيوجيل على شفاء الأنسجة الرخوة بعد قلع الأسنان

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الخلاصة

هدف:

الهدف من البحث الحالي هو تقدير مدى فعالية التطبيق الموضعي لمعجون الفيوجيل على شفاء الأنسجة الرخوة بعد قلع أضرار الفك السفلي.

المواد والطرق: شمل البحث إجمالي 20 مريضاً أصحاء تتراوح أعمارهم بين 18-35 سنة وكان لديهم أسنان بدون أعراض عند الخلع.

تم تقسيم المرضى بشكل عشوائي إلى مجموعتين. تم تطبيق الفيوباست على مجموعة الفيوجيل. بينما لم يتم تطبيق أي شيء في المجموعة الضابطة. تم تقييم المرضى في

اليوم الأول والسابع بعد العملية الجراحية وسجل على مسبار اللثة العددي.

نتائج:

كان العرض الدهليزي اللساني هو الأعلى في اليوم الأول بعد العملية الجراحية، ثم انخفض تدريجياً في كل من مجموعة السيطرة ومجموعة الفيوجيل وفي اليوم السابع بعد العملية الجراحية. كانت هناك فروق ذات دلالة إحصائية في العرض الشدق اللساني بين مجموعات التحكم ومجموعات alveogyl في الأيام السبعة بعد العملية الجراحية

الاستنتاجات: أظهرت نتائج هذه الدراسة أنه بعد الاستئصال الجراحي للأسنان الفيوجيل يمكن أن يسرع من شفاء الأنسجة الرخوة.

1. Introduction

Exodontia or dental extraction (also referred to as tooth extraction , exodontist , is removal of teeth from dental alveolus (socket) in the alveolar bone[1]. Extractions are performed for a wide variety of reasons, but most commonly to remove teeth which have become unrestorable through dental caries , periodontal disease, or dental trauma, especially when they are associated with pulpitis . Sometimes impacted wisdom teeth cause recurrent infections of the gum (pericoronitis), and may be removed when other conservative treatments have failed (cleaning, antibiotics and operculectomy). In orthodontics, if the teeth are crowded, healthy teeth may be extracted (often bicuspid) to create space so the rest of the teeth can be straightened. [2]

Simple extractions are performed on teeth that are visible in the mouth, usually with the patient under local anesthetic, and require only the use of instruments to elevate and/or grasp the visible portion of the tooth. Typically the tooth is lifted using an elevator, and using dental forceps, specific tooth movements are performed expanding the tooth socket. Once the periodontal ligament is broken and the supporting alveolar bone has been adequately widened the tooth can be removed. Typically, when teeth are removed with forceps, slow, steady pressure is applied with controlled force. Surgical extractions involve the removal of teeth that cannot be easily accessed or removed via simple extraction, for example because they have broken under the gum or because they have not erupted fully, such as an impacted wisdom [3] tooth Surgical extractions almost always require an incision. In a surgical extraction we may elevate the soft tissues covering the tooth and bone, and may also remove some of the overlying and/or surrounding jaw bone with a drill or, less commonly, an instrument called an osteotome. Frequently, the tooth may be split into multiple pieces to facilitate its removal from the socket . [3]

Wound healing, is a normal biological process in the human body, [4] wound-healing process consists of four highly integrated and overlapping phases: hemostasis, inflammation, proliferation, and tissue remodeling[5]. For a wound to heal successfully,

all four phases must occur in the proper sequence and time frame. Many factors can interfere with one or more phases of this process.

Multiple factors can lead to impaired wound healing. In general terms, the factors that influence repair can be categorized into local and systemic. Local factors are those that directly influence the characteristics of the wound itself, while systemic factors are the overall health or disease state of the individual that affect his or her ability to heal (Table 1), Many of these factors are related, and the systemic factors act through the local effects affecting wound healing. [5]

The process of the wound being closed by clotting. Happens very quickly. Starts when blood leaks out of the body then blood vessels constrict to restrict the blood flow The platelets aggregate and adhere to the sub-endothelium surface within seconds of the rupture of a blood vessel's epithelial wall [6, 7].

After that, the first fibrin strands begin to adhere in about sixty seconds. As the fibrin mesh begins, the blood is transformed from liquid to gel through pro- coagulants and the release of prothrombin. The formation of a thrombus or clot keeps the platelets and blood cells trapped in the wound area [8, 9]. The thrombus is generally important in the stages of wound healing but becomes a problem if it detaches from the vessel wall and goes [10, 11].

Dry socket” refers to a post-extraction socket where some or all of the bone within the socket, or around the occlusal perimeter of the socket, is exposed in the days following the extraction, due to the bone not having been covered by an initial and persistent blood clot or not having been covered by a layer of vital, persistent, healing epithelium [12]. The patient may not be able to prevent food particles or the tongue from mechanically stimulating the exposed bone, which is acutely painful to touch, resulting in frequent acute pain [13]. All parts of a dry socket lesion, except the exposed bone, can be gently touched with a periodontal probe or an irrigation needle tip without causing

acute pain. Dry socket lesions occur in approximately 1% to 5% of all extractions and in up to 38% of mandibular third molar extractions [14]

A dry socket lesion may show exposed bone located superior to the projected location of the occlusal surface of the socket after the socket heals. This bone may be a protruding septum of bone or may be located on the socket occlusal perimeter. This superiorly-located exposed bone would be the last aspect of the socket to be covered by epithelium [15].

since the bone, protruding superiorly to the projected occlusal surface of the healed socket, would be exposed to food particles or mechanical trauma that may erode epithelium growing over that bone. This bone, if mechanically stimulated, would be a source of acute pain until the end of the healing period.[16]

The dentist may anesthetize the patient and use a football diamond bur with copious irrigation to trim this bone to approximately (1mm) inferior to the projected occlusal surface of the healed extraction socket. Such trimming can result in the bone becoming immediately coverable by a blood clot or medicament, thereby reducing the total number of days that this hyper-sensitive bone is exposed and helping to ensure that epithelium will systematically grow over the remaining exposed bone of the dry socket.

If the protruding bone is located on the socket occlusal perimeter, the dentist can reduce the bone to a level that is inferior to the occlusal aspect of the gingival tissue located just lateral to the protruding bone. If the gingiva on the socket occlusal perimeter is superior to all of the socket bone, a socket blood clot or dry socket medicament is more likely to cover the bone [13-15-17].

Alveogyl is a dry-socket treatment and post-extraction dressing which every dental office should have on hand. Alveogyl is a one-step, self-eliminating treatment which rapidly alleviates pain and provides a soothing effect throughout the healing process. It's fibrous consistency allows for easy filling of the socket and good adherence during the entire healing process.[11]

Indications:

- Dry socket treatment
- Post Extraction Dressing

Analgesic action due the soothing effect of eugenol on the alveolar tissues. Very easy to apply because of its fibrous consistency. Easily maintained in the alveolus. There is no need for Suturing or special attention.

2. Materials and Methods

Forty medically healthy patients were randomly assigned, their ages ranged from 18 to 35 years and included males only.

The diagnosis of the tooth was based on clinical examination and standard intraoral periapical radiographs [18].

2.1 Inclusion criteria included:

1. Free of inflammation and infection of tissue at the time of the surgical procedure.
2. Medically fit, not allergic, not taking any medication that could interfere with the study drugs.

2.2 Exclusion criteria included:

1. History of compromised medical health, history of allergic reactions, or hypersensitivity to the medications used in the operative work.
2. Patients receiving chemotherapy or radiation therapy.
3. Patients needing total extraction or with severe periodontitis, and patients who had any other oral pathology.
4. Patients' rejection of being involved in the research or those who could not commit to follow-up visits or those who used other drugs during the research period.

The patients were arbitrarily allocated to one of the two treatment groups: group I included 20 patients allocated to alveogyl paste

Group II included 20 patients without alveogyl paste

Indicated teeth lower first and second molars were extracted under local anesthesia gained by inferior alveolar nerve, lingual and long buccal nerve block injections using 1.8ml of 2% lidocaine with 1:80,000 adrenaline.

All patients were instructed to

1. Eat soft and cold diet for the first 24 hour after operation
2. Do not rinse for 24 h.
3. Do not smoke for 72 h

Directions for Use:

After measuring the buccolingual width of the extraction socket using periodontal probe (Figure 1), Using a pair of tweezers apply Alveogyl to adequately cover the bottom of the socket and pack gently into place. After 1 week, the width was measured utilizing periodontal probe as in (Figure 2).



Figure 1: Buccolingual width of the socket measurement



Figure 2: Healing after 1 week.

3. Results

The first table summarizes the sample sizes (N), means, standard deviations (StDev), and standard errors (SE Mean) for the two groups: "Alveogyl Group." and "Without Alveogyl Group." The second table shows the results of the two-sample t-test, with a T-Value of -2.51 and a statistically significant P-Value of 0.018, indicating a significant difference in means between the two groups (Table 1 and 2). This suggests that the presence or absence of "Alveogyl Group." impacts the measured outcome (Figure 3).

Table 1: Comparing After Alveogyl Group. vs. After Without Alveogyl Group .

Group	N	Mean	StDev	SE Mean
Alveogyl .Group.	20	3.975	0.993	0.22
Without. Alveogyl .Group.	20	4.600	0.503	0.11

Statistic	Value
T-Value	-2.51
P-Value	0.018
Hypothesis Tested	Difference = 0 (vs \neq)

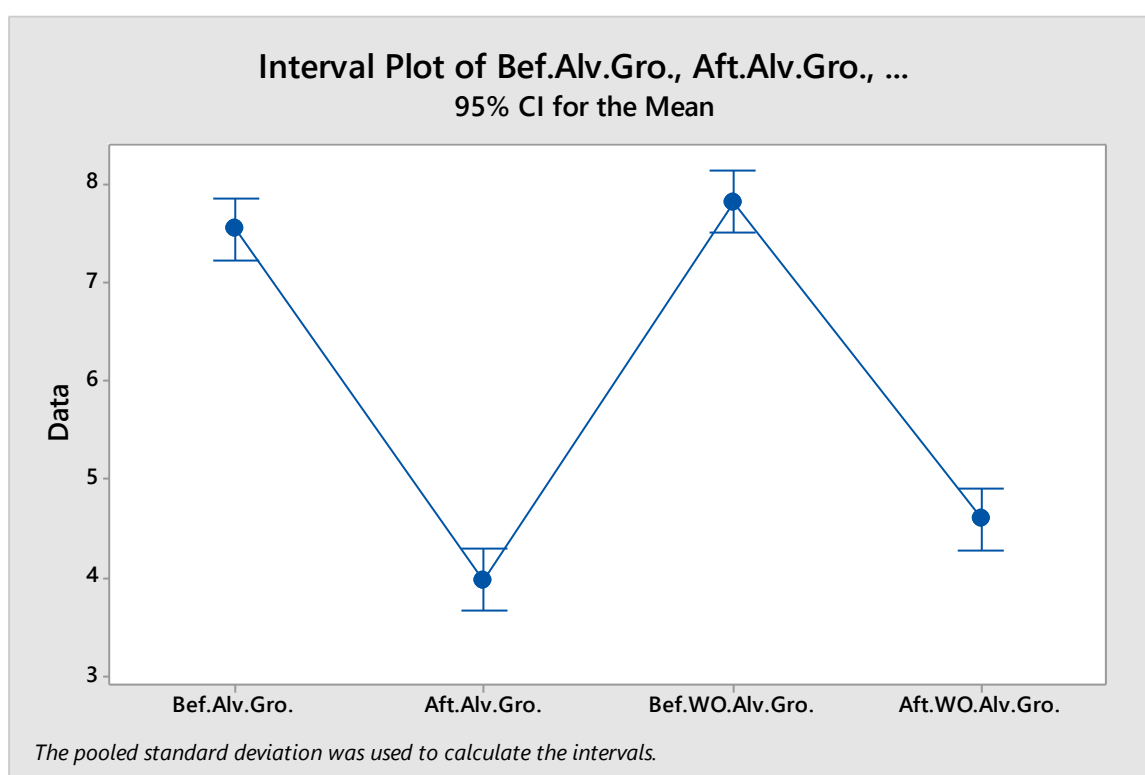


Figure 3: Blot between the groups.

Table 3: Descriptive Statistics for Before Alveogyl Group and After Alveogyl Group

Group	N	Mean	StDev	SE Mean
Before Alveogyl Group.	20	7.550	0.510	0.11
After Alveogyl Group.	20	3.975	0.993	0.22

Two-Sample T-Test Results for Before Alveogyl Group vs. After Alveogyl Group .

Statistic	Value
T-Value	14.32
P-Value	0.0006
Hypothesis Tested Difference = 0 (vs \neq)	

The first table summarizes the sample sizes, means, standard deviations, and standard errors for the groups "Before Alveogyl Group." and "After Alveogyl Group ." The second table provides the t-test results, showing a T-Value of 14.32 and a highly significant P-Value of 0.0006. This indicates a substantial decrease in the measured outcome after the intervention involving "Alveogyl Group."

4. Discussion

The purpose of this study to measure the efficacy of alveogyl paste on the wound after tooth extraction, alveogyl paste can reduce the probability of dry socket and accelerate the healing process, we aimed to measure the effects of alveogyl paste on soft tissue healing after tooth extraction [16].

There were many obstacles we had to overcome in this study, including the patients don't commit to the follow up date.

Dry sockets are more common after surgical or traumatic extraction. Similarly traumatic extraction can also lead to traumatic thrombosis of vessels in socket resulting in delayed healing and wound infection.[12-19]. The main aim in the treatment of dry sockets is to relieve pain. Various materials have been placed in extraction sockets for this purpose [15]. Alveogyl was the one of the most commonly used materials. Alveogyl contains eugenol which has a soothing effect and relieves the pain, these properties are often desirable in the presence of inflammation to reduce postoperative pain.[20]

The results of our study come in acceptance with result by previous studies done by Ahmed salem et al. who concluded that Alvogyl showing higher pain relief and healing capability, as evidenced by clinical signs of improvement attributed to its components. [21]

5. Conclusion

Alveogyl administration after tooth extraction reduce pain and accelerate socket healing and soft tissue closure. Alveogyl use decrease the probability of dry socket.

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The Effect of Organic Dye Concentration on the Properties of Dye-Sensitized Solar Cells With Titanium Dioxide

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Abstract

Dye-sensitized solar cells were fabricated by depositing a layer of nanostructured titanium dioxide (TiO_2) on ITO substrates and using an electrolyte solution of potassium iodide/iodine. The dye used was extracted from local beetroot, purified, and prepared at different concentrations of 0:100, 20:80, 40:60, 60:40, and 80:20 dye to distilled water. The absorption spectrum of the dye showed high absorption at a wavelength of 524 nm. Current-voltage (I-V) measurements indicated that the dye-sensitized solar cell achieved the best efficiency at the 40:60 concentration, with a short-circuit current density (J_{sc}) of 0.1 mA/cm^2 , an open-circuit voltage (V_{oc}) of 214 mV, and a power conversion efficiency of 0.082%.

Keywords: Dye-sensitized solar cells, beetroot dye, dye concentration

تأثير تركيز الصبغة العضوية على خصائص الخلايا الشمسية الصبغية لثاني أكسيد التيتانيوم

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الخلاصة

تم تصنيع الخلايا الشمسية الصبغية عن طريق ترسيب طبقة من ثاني أكسيد التيتانيوم النانوي على ركائز من ITO وكذلك استخدام محلول الكتروليتي من يوديد البوتاسيوم/اليود . اما الصبغة المستخدمة فقد تم استخلاصه من الشمندر المحلي و تنقيتها و تحضيرها بتركيزات مختلفة 0:100, 20:80, 40:60, 60:40, 80:20 صبغة : ماء مقطر, و دراسة طيف الامتصاصية لهذا الصبغة والذي بين الامتصاصية العالي للصبغة عنده الطول الموجي 524 نانومتر. اظهرت قياسات (كثافة تيار- جهد) حصولنا على خلية صبغية لها افضل كفاء عنده التركيز 40:60 حيث كان كثافة تيار الدائرة القصيرة $J_{sc} = 0.1$ مللي أمبير/سم² و جهد دائرة مفتوحة $V_{oc} = 214$ مللي فولت، وكفاءة تحويل 0.082 .

الكلمات المفتاحية :- الخلايا الشمسية الصبغية , صبغة الشمندر , تركيز الصبغة

المقدمة :

الخلايا الشمسية الصبغية DSSCs ، وهي نوع من تكنولوجيا الطاقة الشمسية ذات الأغشية الرقيقة، تستفيد من الأصباغ العضوية لالتقاط ضوء الشمس وتحويله إلى كهرباء. يعد تركيز هذه الأصباغ العضوية في طبقة أشباه الموصلات من أكسيد التيتانيوم TiO_2 عاملاً رئيسياً يؤثر على الأداء العام وكفاءة [1]. عند دراسة تأثير تركيز الصبغة العضوية على الخلايا الشمسية الصبغية ، يأتي التفاعل بين العوامل المختلفة في الاعتبار. أولاً، تتأثر بشكل ملحوظ كفاءة امتصاص الضوء للخلية الشمسية مع زيادة تركيز الصبغة العضوية حيث يصبح المزيد من جزيئات الصبغة متاحة لامتصاص أشعة الشمس. يؤدي التركيز المتزايد إلى توسيع نطاق امتصاص الضوء، مما قد يؤدي إلى تحسين شامل في كفاءة الخلية الشمسية. ترتبط أيضاً حركية حقن الإلكترون ومعدلات إعادة التركيب بشكل معقد بتركيز الصبغة العضوية. تُعد التركيزات المثالية أمراً حاسماً لتحقيق التوازن، مما يضمن حقن الإلكترون بكفاءة من

الصبغة إلى شبه موصل (أوكسيد التيتانيوم) مع تقليل خسائر إعادة التركيب [2]. قد يؤدي التركيز المنخفض للغاية إلى عدم كفاية حقن الإلكترون، في حين أن التركيزات العالية بشكل مفرط قد تزيد من إعادة التركيب، وكلاهما يمكن أن يؤثر على الكفاءة الإجمالية للخلية الشمسية. يتأثر تيار الدائرة القصيرة I_{sc} وجهد الدائرة المفتوحة V_{oc} ، وهما معلمتان محورتان في أداء الخلايا الشمسية بتركيز الصبغة العضوية. يعد ضبط هذا التركيز أمرًا ضروريًا لتحسين قيم I_{sc} و V_{oc} ، حيث يمكن أن يؤدي التطرف إلى نتائج غير مثالية، مما يؤثر على كفاءة تحويل الطاقة الإجمالية. إن عامل التعبئة، الذي يمثل كفاءة نقل الشحنة وتجميعها داخل الخلية الشمسية، حساس لتركيز الصبغة العضوية. يعد تحقيق التوازن الصحيح في تركيز الصبغة أمرًا بالغ الأهمية لتحقيق عامل تعبئة عالٍ، مما يضمن التحويل الفعال للضوء الممتص إلى طاقة كهربائية. يعد استقرار وطول عمر الخلايا الشمسية الصبغية أمرًا بالغ الأهمية للتطبيقات العملية. وهنا، يمكن أن يؤثر تركيز الصبغة العضوية على استقرار الخلية الشمسية. قد تؤدي التركيزات المفرطة إلى مشكلات مثل امتصاص الصبغة، مما قد يؤثر على استقرار الجهاز وموثوقيته على المدى الطويل [3,4]. علاوة على ذلك، لا يمكن تجاهل الجانب الاقتصادي. يؤثر تركيز الصبغة العضوية بشكل مباشر على تكلفة تصنيع الخلايا الشمسية الصبغية. يعد العثور على التركيز الأمثل عملية دقيقة، حيث يتطلب الموازنة بين اعتبارات التكلفة بهدف تعظيم كفاءة الخلية الشمسية. باختصار، يعد تركيز الصبغة العضوية في الخلايا الشمسية الصبغية معلمة مهمة تؤثر بشكل معقد على جوانب مختلفة من أداء الخلية الشمسية، بما في ذلك كفاءة امتصاص الضوء، وحركية الإلكترون، والمعلومات الحرجة مثل I_{sc} و V_{oc} ، وعامل التعبئة، والاستقرار، واعتبارات التكلفة. يقوم الباحثون باستمرار باستكشاف وتجريب تركيبات مختلفة لتحقيق التوازن الذي يحقق أعلى مستويات الكفاءة والاستقرار للتطبيقات العملية [5,6]. يمثل دمج صبغة الشمندر في مجال الخلايا الشمسية الصبغية استكشافًا رائعًا لاستخدام المركبات الطبيعية لتحويل الطاقة الشمسية. يحتوي الشمندر، المعروف بلونه الأحمر المميز، على أصباغ البيتاين التي يمكن أن تكون بمثابة محسسات فعالة في بناء خلايا الشمسية الصبغية. يقدم دمج صبغة الشمندر بعدًا صديقًا للبيئة ومستدامًا لتكنولوجيا الخلايا الشمسية، فإنه يجلب حلولاً لمشاكل الاستقرار وطول عمر الأصباغ الطبيعية. ويشارك الباحثون بنشاط في مواجهة هذه التحديات، بهدف تعزيز كفاءة وموثوقية مراكز الخلايا الشمسية الصبغية باستخدام الصبغات الطبيعية [7,8]. في هذا العمل، نسعى لتصنيع خلية شمسية صبغية ذات طبقة من أوكسيد التيتانيوم النانوي ومن ثم دراسة تأثير تركيز صبغة الشمندر على كفاءة ومعلومات هذه الخلية.

الجانب العملي:

تم استخراج صبغة الشمندر من خلال اخذ 100 غرام من الشمندر الطازج وتنظيفها وتقسيرها ومن ثم هرسها وغليها في 500 مل من الماء المقطر عند درجة حراره 100 درجة مئوية ولمده ساعة وتركها لمدة يوم كامل في وعاء مغلق بعدها تم تصفية المحلول باستخدام ورق الترشيح لحصول على صبغة الشمندر. تم تخفيف الصبغة المركزه التي حصلنا عليها باضافة الماء المقطر وكما مبين في الجدول

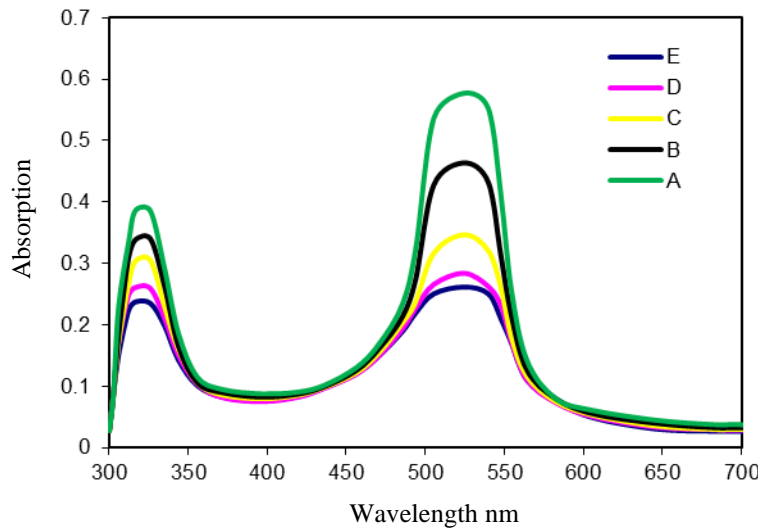
جدول 1 . رمز العينات و نسبة التخفيف لمحلول الصبغة

رمز العينة	نسبة الصبغة : الماء المقطر
A	0:100
B	20:80
C	40:60
D	60:40
E	80:20

تم فحص طيف الامتصاصية لكل عينة باستخدام مقياس الطيف الضوئي (SHIMADZU UV-1650) . تم تحضير قطب الأنود الضوئي الماص للضوء بالاعتماد على الصبغة من خلال استخدام مسحوق ثاني أكسيد التيتانيوم النانوي، حيث خلط نصف غرام منه مع 3 مل من حامض الخليك وترك تحت التحريك المستمر لمدة ساعتين للحصول على محلول معلق لثاني اوكسيد التيتانيوم النانوي . تم صنع غشاء منه على ركيزه من زجاج ال-ITO بطريقة طلاء الشرائط (bar-coating method) و وضع الاغشية المحضرة في فرن لمدة نصف ساعة عنده درجة حرارة 200 مئوي . بعدها وضع قطرات من الصبغة المحضرة على الغشاء وتركها لتجف . من جهة اخرى تم تحضير القطب الثاني للخلية باستخدام ركيزه من زجاج ال-ITO ايضا ولكن مطلية بطبقة من الكرافيت. بعدها تم جمع القطبين وجهه لوجه وتنبيتهما باستخدام الايبوكسي وترك ثقب صغير لحقن المحلول الالكتروليتي المتكون من يوديد البوتاسيوم/اليود (KI/I_2) ومن ثم اغلاق الثقب. خصائص الخلية المصنعة قيست من خلال قياس كثافة التيار كدالة للجهد المسلط تحت إضاءة 100 ملي وات/سم² . ثم تم حساب كل من كثافة تيار الدائرة القصيرة J_{sc} وجهد الدائرة المفتوحة V_{oc} وعامل التعبئة $F.F$ والكفاءة η

النتائج والمناقشة:

يظهر الشكل (1) تباين الامتصاصية لمحاليل صبغة الشمندر بتراكيز مختلفة مع الطول الموجي للضوء الساقط. يشير الشكل إلى زيادة الامتصاصية للمحاليل المحضرة مع زيادة تركيز الصبغة وتظهر هناك قمنا احدهما عنده منطقة الأشعة فوق البنفسجية عنده طول موجي 326 نانومتر و اخرى في منتصف المنطقة المرئية عند الطول الموجي 524 نانومتر وخصوصا عنده التراكيز المرتفعة للصبغة . وهذا السلوك يُعزى لصبغة البيتانين $C_{24}H_{26}N_2O_{13}$ الموجوده في الشمندر وهذه القمم تعد من الصفات المميزة لهذه الصبغة والتي تتفق مع الكثير من الدراسات السابقة دومبرافا أنكا وجماعته 2012 [9] و غارسيا سالياس وجماعته 2019 [10].



شكل (1). طيف الامتصاص لصبغة الشمندر بتركيز مختلفة .

الشكل (2) يبين منحنيات (كثافة التيار - جهد) للخلايا الشمسية المصنعة وتأثير تركيز صبغة الشمندر عليها ومنها تم حساب معلمات الخلايا والمبينه في الجدول رقم 2 . من الشكل والجدول نلاحظ ان التيار الضوئي او ما يسمى بتيار الدائرة القصيرة يزداد مع زيادة تركيز صبغة الشمندر ومن ثم يتناقص مع استمرار زيادة تركيز الصبغة بينما نلاحظ السلوك غير المنتظم الجهد الدائرة المفتوحة مع تغير تركيز صبغة الشمندر. عامل التعبئة تم احتسابه من المعادلة [11]

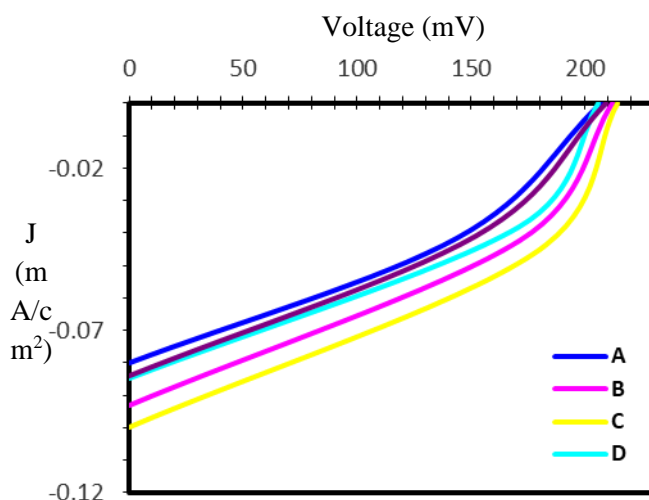
$$F.F = \frac{P_{max}}{V_{oc} \cdot I_{sc}} = \frac{V_{max} \cdot I_{max}}{V_{oc} \cdot I_{sc}}$$

P_{max} هو الحد الأقصى للطاقة الكهربائية التي تم الحصول عليها؛ V_{oc} و I_{sc} هما تيار الدائرة القصيرة وجهد الدائرة المفتوحة، على التوالي [12]. يتم تعرف كفاءة الخلية الشمسية η على أنها أقصى نسبة لتحويل الطاقة الشمسية الساقطة على الخلية الى طاقة كهربائية ويتم احتسابها حسب المعادلة التالية [11]

$$\eta = \frac{V_{oc} \cdot I_{sc} \cdot FF}{P_{in}}$$

تعتمد الطاقة القصوى التي تولدها الخلية الشمسية على عامل التعبئة، وبالتالي فهو معلمة مهمة لتحديد كفاءة تحويل الطاقة للخلية الشمسية الصبغية. وهناك عدة عوامل يمكن أن تؤثر بشكل كبير على عامل التعبئة ، وتتفاعل هذه العوامل مع بعضها البعض بشكل معقد للغاية. ولهذا السبب، فإن الفهم العميق لعامل التعبئة أمر صعب للغاية [13]. ومن تلك العوامل [14] هو انخفاض الجهد بسبب المقاومة التسلسلية للخلية الشمسية R_s ، وكذلك بسبب تيار التسرب والمسمى بمقاومة التحويل R_{sh} للخلية الشمسية. من الجدول 2 لوحظ أن عامل التعبئة وكفاءة التحويل قد

كانتا ذا سلوك غير منتظم مع زيادة تركيز صبغة الشمندر حيث كانت العينة C ذات التركيز 40:60 صبغة : ماء مقطر , لها أعلى قيمة لعامل التعبئة و كفاءة التحويل اما عند التراكيز الأقل والاكثر من هذا التركيز فقد كانت قيم عامل التعبئة و الكفاءة وكفاءتها أقل وهذا يعزى الى اعتماد الخلايا الشمسية الصبغية على الصبغة الحساسة للضوء لالتقاط ضوء الشمس وتحويله إلى طاقة كهربائية. العلاقة بين كفاءة DSSC وتركيز الصبغة معقدة ويمكن أن يعتمد على عوامل مختلفة, منها امتصاص الضوء , حيث تمتص الصبغة الموجودة في DSSC الضوء وتولد الإلكترونات, وتبدأ عملية توليد الكهرباء. يمكن أن تؤدي زيادة تركيز الصبغة إلى امتصاص أعلى للضوء, خاصة في نطاق الطول الموجي حيث تكون الصبغة نشطة. غالبًا ما تؤدي تراكيز الصبغة الأعلى إلى زيادة معامل الامتصاص, مما يعني امتصاص المزيد من الفوتونات لكل وحدة طول من طبقة الصبغة. يمكن أن يساهم ذلك في الاستخدام الأكثر كفاءة لأشعة الشمس الساقطة. في حين أن زيادة تركيز الصبغة يعزز بشكل عام امتصاص الضوء, إلا أن هناك نقطة تشبع لا يؤدي بعدها إضافة المزيد من الصبغة إلى زيادة الكفاءة بشكل ملحوظ. وذلك لأن جزيئات الصبغة يمكن أن تبدأ في التجمع أو التفاعل بطرق لا تساهم بشكل فعال في توليد الإلكترون. حيث يضمن تركيز الصبغة الأمثل تغطية فعالة لسطح TiO_2 دون تراكم مفرط. الكثير من الصبغة قد تؤدي إلى التجميع وتعيق حقن الإلكترون في TiO_2 , من جهة أخرى يمكن أن تؤدي تراكيز الصبغة الأعلى إلى زيادة إعادة تركيب الشحنة, حيث تتحد الإلكترونات الناتجة عن الصبغة مع جزيئات الصبغة المؤكسدة. وهذا يمكن أن يقلل من الكفاءة الإجمالية لـ DSSC [16,15].



شكل (2). منحنيات (تيار - جهد) للخلايا المصنعة عنده تراكيز مختلفة لصبغة الشمندر .

الجدول 2 معلمات الخلية الشمسية و تأثير تركيز الصبغة عليها

Sample	V_{oc} (mV)	J_{sc} (mA/cm ²)	V_{max} (mV)	J_{max} (mA)	P_{max} (mW/cm ²)	F.F	$\eta\%$
A	207	0.08	144	0.041	5.96	0.36	0.059

B	212	0.093	171	0.042	7.23	0.36	0.072
C	214	0.1	177	0.046	8.22	0.38	0.082
D	206	0.085	171	0.038	6.53	0.374	0.065
E	209	0.084	151	0.040	6.21	0.35	0.062

الاستنتاجات

من العمل اعلاه نستنتج بانه يمكن تصنيع خلية شمسية صبغية باستخدام صبغة الشمندر وان تركيز صبغة الشمندر له تأثير واضح على معلمات الخلية المصنعة و كفاءتها, حيث احتواء صبغة الشمندر على صبغة البيتانين اعطت لها القدرة على الامتصاصية العالية للضوء المرئي عند الطول الموجي 524 نانومتر وهذا بدور صفة جيدة في الية عمل الخلية الصبغة وفي تحسين معلمات هذا الخلية وكذلك تبين من العمل وجود تركيز محدد هو الامثل لعمل الخلية المصنع حيث كان التركيز C هو افضل تركيز حصلنا من خلال على اعلى كفاء للخلية المصنعة .

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