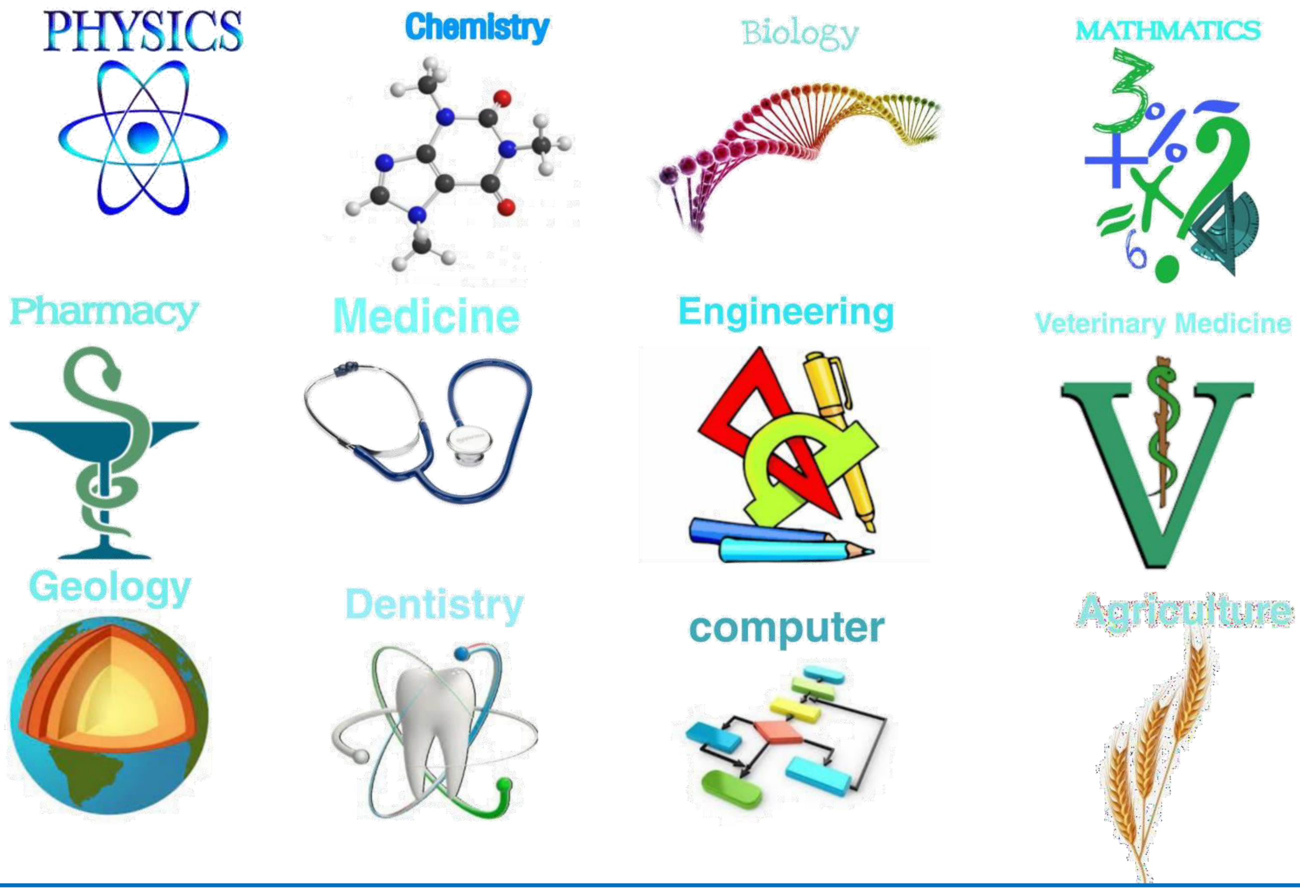
ISSN (Print): 2958-8995

ISSN (Online): 2958-8987

No: 3 Vol: 2/ October/ 2023

Journal of Natural and Applied Sciences URAL

A Quarterly Multidisciplinary Scientific Journal Issued by European Academy for Development and Research / Brussels and Center of Research and Human Resources Development Ramah- Jordan



Focus & Scope:

Journal of Natural and Applied Sciences URAL

Journal welcomes high quality contributions investigating topics in the fields of Biology, physics, computer science, Engineering, chemistry, Geology, Agriculture, Medicine, Mathematics, Pharmacy, Veterinary, Nursing, Dentistry, and Environment.

Publication specializations in the journal	
Biology	**Chemistry**
Physics	**Geology**
Computer	**Agriculture**
Engineering	**Mathematics**
Medicine	**Pharmacy**
Veterinary	**Dentistry Veternity,**
Environment	**Nursing**

Administrative Title of the Journal:
Amman\ Jordan\ Wasfi Al-Tal \ Gardens
Phone: +962799424774

Index

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

Enhancing Hybrid System Based Mixing AES and RSA Cryptography Algorithms

Ali Mahmood Khalaf [1]
Research Scholar
Department of Computer Science
Gujarat University
Ahmadabad, Gujarat, India
alikhalaf@gujaratuniversity.ac.in

Dr. Kamaljit Lakhtaria [2]
Associate Professor
Department of Computer Science
Gujarat University
Ahmadabad, Gujarat, India
kamaljit.lakhtaria@gujaratuniversity.ac.in

Enhancing Hybrid System Based Mixing AES and RSA Cryptography Algorithms

Ali Mahmood Khalaf
Research Scholar
Gujarat University
Department of Rollwala Computer Science
Ahmadabad, Gujarat, India
alikhalaf@gujaratuniversity.ac.in

Dr. Kamaljit Lakhtaria
Associate Professor
Gujarat University
Department of Rollwala Computer Science
Ahmadabad, Gujarat, India
kamaljit.lakhtaria@gujaratuniversity.ac.in

ABSTRACT

Information security is an important matter, especially with the increased demand for information due to the advent of the Internet, as this information has entered many scientific, commercial, and military fields, and this has become widely circulated as the need to protect this information from penetration has arisen by developing techniques to encrypt and preserve information. In this research, a system based on mixing was developed, taking advantage of the strengths of both algorithms, to ensure information protection and more reliability, and to add an additional level of security, where the modified symmetric encryption algorithm, which works with the public key AES, was combined with the modified asymmetric encryption algorithm, which works with the private key, RSA algorithm. In the AES algorithm, there was an increase in speed and a new security prefix by adding Four-S Boxes to the generation key, as well as adding Four-S Boxes to the encryption algorithm to increase the computational complexity of the difficulty of breaking by the attacker. In the RSA algorithm, an additional level of security was added to the modified algorithm by increasing the complexity of (n), which depends on the values of the initial numbers (p, q), in addition to adding other layers of complexity by calculating the value of e. Among the most prominent findings of the study is that hybrid algorithm(AES+RSA), compared with previous studies, has a high level of security, strength, and speed at the time of encryption and decryption, as this hybrid system algorithm was faster than the RSA algorithm and slightly slower than the AES algorithm, as the throughput was high compared to other studies.

Keywords: Cryptography, Cryptography Algorithms, Mixing AES and RSA, Execution Time, Throughput

1. INTRODUCTION

The matter of protecting and preserving information from penetration is a major concern as a result of the increased demand for this information, which is represented by (texts, images, audio files, and video files) that have entered many areas, including wireless networks, and engineering, medical, and military fields, where this information is exchanged through an open environment that is easy to penetrate, as it called on scientists to develop techniques to hide this information, and among these technologies are cryptography algorithms (Zong & Natgunanathan, 2014)(Abood, 2017).

Cryptography is of Greek origin from two words first, crypto means "hidden secret"; The second is graphein, meaning "covered writing". Encryption is an important science and it is one of the techniques of concealment science. It is an important and necessary technique to protect information from external attacks. Through this technique, the original message is converted into a secret encrypted message between the sender and the receiver using encryption algorithms. There are two types of cryptography algorithms: symmetric key algorithms are also known as the private key cipher system, where this one has one private key for encryption and is itself private for decryption, while the second type of algorithm is known as the asymmetric key cipher system is also known as the public key cipher system, where this type of algorithm has two keys, the first is private for encryption and the second key is private for decryption(Rajkamal & Zoraida,2014)(Bokhari & Shallal,2016)(Timilsina & Gautam,2019), as shown in Figure 1.

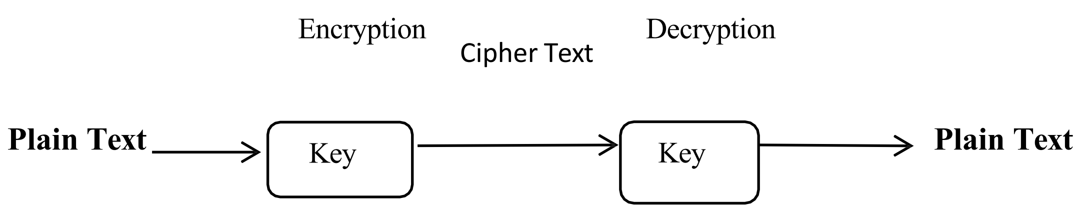


Figure 1: cryptography model

The main term of cryptography can be described as follows (Stallings,2006):

- Plain Text: An original plain message or data, it's nourished into an encryption algorithm.
- Encryption Algorithm: The encryption algorithm converts plain text into ciphertext by using performs various substitutions and transformations.
- Key: The secret value independent of the plain text and encryption algorithm. It's also input into the encryption algorithm. The exact substitutions and transformations performed by the algorithm depend on the secret key.
- Cipher Text: The ciphertext is a random stream of data, it's the output incomprehensible of a scrambled message. It depends on the plaintext and secret key.
- Decryption Algorithm: The decryption algorithm run in the opposite. It takes the ciphertext and secret key and produced the plain text(original format),as shown in figure 2.

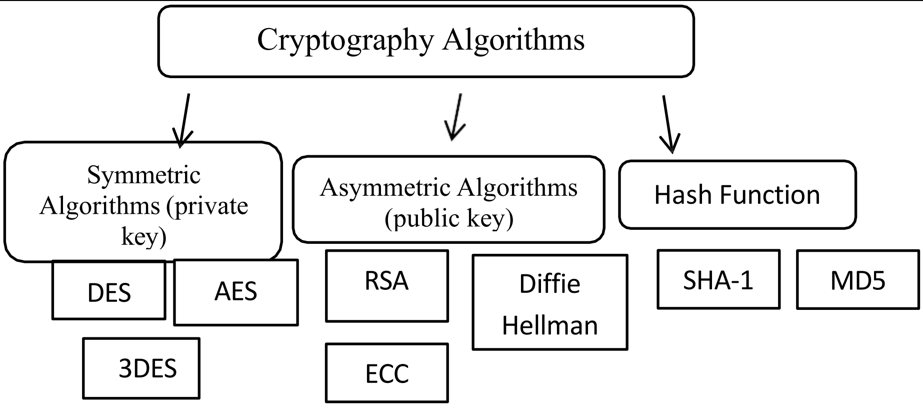


Figure 2: Cryptography Algorithms

2. LITERATURE REVIEW

(Bokhari et al.,2018) "Hybrid Blowfish and RSA Algorithms to Secure Data between Cloud Server and Client." In this paper, a hybrid algorithm is proposed to encrypt and decrypt data during transmission between the server and the client in cloud computing (CCS and CCC). Applying the HMAC feature to the ciphertext that was produced by the fish algorithm, the results of the proposed system were good in comparison with previous literature.

(Abd Zaid &Hassan,2018) "Lightweight RSA Algorithm Using Three Prime Numbers." In this study, a novel strategy is utilized to obtain (n) with the same length as the usual RSA but with fewer bits for prime numbers by using three prime numbers instead of two prime numbers. This method uses three prime numbers and the Chinese Reminder Theory (CRT) to increase speed for both the regular RSA key generation side and the decryption side. Research results indicate that the average speed improvement is 80% in the key generation process, 96% in the decryption process, and only 4% in the encryption process.

(Carlo et al.,2019) “Modified Key Generation in RSA Algorithm”. In this paper, the RSA algorithm is modified based on modulo and public key. , the public key was modified to be a hidden key through the random selection of the collected values and converting it to a different value. From the results of this research, the modification of the RSA algorithm based on modulo and the public key gave a new model consisting of two levels of the encryption process and the decryption process. One of the conclusions of this research is that the new model has the factors to hide the private key.

(Isiaka et al.,2019) “Hybridization of RSA and Blowfish Cryptography Algorithms for Data Security on Cloud Storage”.In this research, a hybrid system is proposed that is able to use the BLOWFISH symmetric encryption algorithm and the other asymmetric RSA, where the algorithms are designed in such a way that one of them authenticates the authorized user and the other provides confidentiality and security for the data stored on the cloud. One of the most prominent results of this research was a high improvement in data security in cloud storage.

(Ezekiel Bala et al.,2019) “Hybrid Data Encryption and Decryption using RSA and RC4”.In this study, a hybrid system based on two algorithms was designed to add very high security to the public and private keys. The first is private key encryption based on a straightforward symmetric algorithm, and the second is public key encryption based on a linear block cipher. Compared to other encryption algorithms, this one offers a more reliable and secure authentication system. Data is transferred utilizing keys with symmetric encryption to accomplish hybrid encryption. Public key cryptography has been implemented for symmetric random key encryption. Once the symmetric key is retrieved the recipient can use the public key encryption method to decrypt the symmetric key. In comparison to earlier tests, the research's findings indicate a significant improvement in data security and a general improvement in the system's performance. This system was programmed using the C# programming language.

(Alegro et al.,2019) "Hybrid Schnorr, RSA, And AES Cryptosystem.". This study develops a hybrid Schnorr Authentication Algorithm-based authentication algorithm that confirms the identity of the message's sender. When a message is sent from the sender to the receiver and vice versa, algorithms with RSA and AES encryption methods are combined to increase security and lessen the impact of a man-in-the-middle attack on the system. by including further encryption techniques.

(Timilsina et al.,2019) "Performance analysis of hybrid cryptosystem-A technique for better security using blowfish and RSA”.In this research, a hybrid system was created by combining two algorithms, AES and RSA. By combining them with each other, their performance is analysed based on five parameters, which are throughput, encryption time, decryption time, total execution time, and plaintext size to the ratio of ciphertext size with key size. Various for the Blowfish algorithm range from 32-bit-448-bit. As a result of this research, we found that Blowfish RSA with a key size of 448 bits has better performance than all other bit sizes.

(Abd Zaid & Hassan,2019) "Modification advanced encryption standard for design lightweight algorithms.". In this paper, AES-128 encryption has been analysed and made lightweight with respect to power consumption. In the modified AES algorithm, it is proposed to implement the AES mix columns operation and combine the round key addition operation with the mix columns to perform one cycle, and the shift row operation is modified into shift rows and shift columns and the number of rounds is reduced to only 6 rounds for the modified AES. The results of the research were that the modified algorithm excelled and was faster and had a safety ratio of 6 rounds due to the modification in the operations of mix columns and transformation rows higher than the standard algorithm due to its success through the set of statistical tests.

(Gupta &Sanghi,2021) Matrix Modification of RSA Public Key Cryptosystem and its Variant” .In this paper an RSA public key cipher system is proposed using $h \times h$

square matrices. Also, a variant of RSA using the model coefficient $p^r q$ with a matrix with a modified matrix has been proposed.

(Chaloop, & Abdullah,2021) "Enhancing Hybrid Security Approach Using AES And RSA Algorithms." In this study, a hybrid encryption method is introduced to safeguard sensitive data shared between individual users, businesses, organizations, or cloud applications, among other things. During data transmission over the network. Firstly, the algorithm was designed by merging two algorithms, AES and RSA, and secondly, the work of this algorithm was evaluated in comparison with other hybrid algorithms based on its efficiency based on time analysis. In comparison to earlier studies, the experiment findings demonstrated that this hybrid method is more secure.

(Guru & Ambhaikar,2021). "AES and RSA-based Hybrid Algorithms for Message Encryption & Decryption." In order to address security issues, lack of complexity, time, and other issues, a hybrid encryption method that combines the AES and RSA algorithms is developed in this study. According to the research's experimental findings, the hybrid encryption algorithm RSA and AES may not only encrypt files but also improve the technique's efficiency and security.

(Abroshan,2021) "Enhancing Hybrid Security Approach Using AES And RSA Algorithms".In this paper an effective encryption system is proposed to improve security in cloud computing. Improvements have been made to the hybrid algorithm (Blowfish, ECC). In order to improve security and performance, Blowfish will encrypt the data and the elliptical curve technique will encrypt the key. Moreover, digital signature technology is used to ensure data integrity, and the results show improvement in throughput, execution time, and memory consumption parameters.

(Sahin,2023) “Memristive chaotic system-based hybrid image encryption application with AES and RSA algorithms” .In this paper, we propose a two-stage image encryption model, the first stage is the logistic map, chaotic Lorenz system and memristor-based super similar system, and the second stage with AES and RSA encryption algorithms applies the scheme to improve the security of encrypted images. The results of this research show the effectiveness of the proposed image encryption scheme in terms of security, speed, and reliability and provide valuable insights for the development of chaos-based encryption systems in the future. This research was evaluated through statistical tests and compared with previous studies.

1. PRINCIPLES OF ALGORITHMS AND TECHNIQUE

1.1 AES Symmetric Algorithm

It is a symmetric algorithm that was replaced by the DES algorithm in 1991. The AES algorithm supports three key sizes 128,192,256. The AES algorithm is an analog algorithm that uses a single key for encryption and decryption. The AES algorithm gives more security and high confidence in the encryption of information, as AES 10 passes Rounds for the 128-bit key, 12 rounds for the 192-bit key, and 14 rounds for the 256-bit key[Stallings,2006][Chowdhury et al.,2010]. The AES algorithm goes through four stages [Mandal et al.,2012]:-

1- First Stage (Substitute Byte)

In this stage. The AES algorithm contains a 128-bit data block, which means that each data block is 16 bytes, in this type, every byte (8 bits) of one block of data is converted to another block using an (8-bit) square known as Rijndael Sbox.

2- Second Stage (Shift of Rows)

In this stage and depending on the location of the row, the data in the three rows of the case is shifted periodically, a circular left shift of 1 byte is made, and a circular left shift of 2 bytes is performed, for the third and fourth rows.

3- Third Stage (Mix Columns)

In this process polynomial bytes are taken instead of numbers as the fix matrix is multiplied by each of them as polynomial vectors.

4- Fourth Stage (Add Round Key)

In this stage. It is a single XOR between 128 bits of the current state and 128 bits of the round key. Figure 3 shows the steps of AES algorithm.

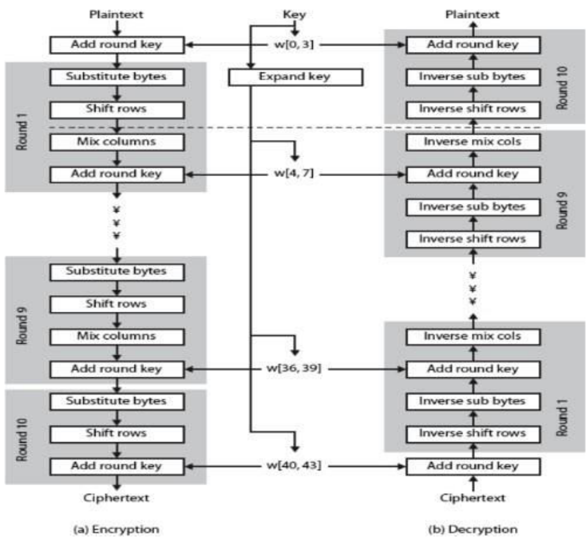


Figure 3: AES Encryption and Decryption Diagram

1.2 RSA Asymmetric Algorithm

This algorithm was published in 1977 by scientists (Rivest-Shamir-Adleman), which is an encryption algorithm used to encrypt information and increase its security, and this type of algorithm is asymmetric as it consists of the public key that is for encryption and the private key that is for decryption, simply the RSA algorithm is slow, and the calculation is RSA is of integer modulo n=p*q, where this algorithm requires a key of at least 1024 bits to increase the security of information encryption, the larger the key size such as 2048, 4096, the more secure the information encryption[Sadkhan & Sattar,2014]. To create public and private keys, follow these steps [Chuang et al.,2016]:

Steps of RSA Asymmetric Algorithm

Step-1: consider two large prime numbers p and q.

Step-2: compute $n=p*q$

Step-3: compute φ(pq)=(*p*-1)*(*q*-1)
Step-4: select integer *e* such that GCD *(φ(n),e)=1; 1<e< φ(n),* then get public key: KU={e,n}using for encryption.
Step-5: Calculate d=e^{-1}(mod φ(*n*)), then get private key: KR={d,n} using for decryption
Step-6: Calculate cipher text C from plain text M such that (C=M^e mod *n*) for encryption, then calculate plain text M from cipher text (M=C^d mod *n*) for decryption. Figure 4 shows the steps of RSA algorithm.

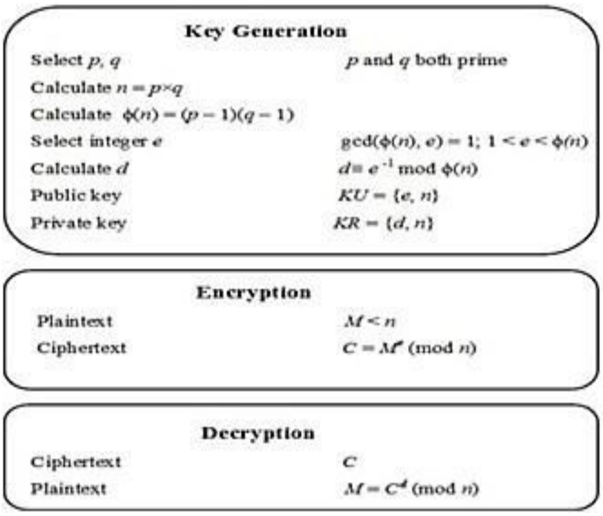


Figure 4: RSA Encryption and Decryption Diagram

3. METHODOLOGY

In this paper, a hybrid system based on three steps was developed. The first step is the encryption and decryption of information using the modified AES algorithm with a symmetric key, the second step is the encryption and decryption of information using the modified RSA algorithm with two keys, and the third step is the combination of the two algorithms to produce A new algorithm takes advantage of the strengths of the two algorithms called mixing AES and RSA cryptography algorithms represented in speed, security and computational complexity. The steps are divided into the following:

3.1 First Step (AES Algorithm Modification)

The first stage of the research methodology is the modification of Advanced Encryption Standard (AES) algorithm with one symmetric key AES with a key size of 128 bits. The purpose of developing the algorithm is to increase the percentage of security and speed and add complexity to the key and encryption to be four times higher than the standard algorithm by adding (Four S- Boxes) to generate the key and (Four S-Boxes) to encrypt the original data so that it is difficult for the attacker to break it and access the original information.

Encryption Process
Inputs: Plaintext data block.

Output: Ciphertext data subblock right.

- Use a latch selector to select the right block of 128 bit from the input data block.
- Store the plaintext in 2d 4x4 state matrix $S_{4\times4}$.
- Select the round key of 128 bit.
- For key expansion, generate the word0 of key$k[0]$ from

$$\boldsymbol{K[n]: w[0] = K[n-1]: w[0] \oplus SubByte(K[n-1]: w[3] \gg 8) \oplus Recon[i]}$$

- Generate the remain words from $\boldsymbol{K[n]: w[i] = K[n-1]: w[i] \oplus k[n]: w[i-1]}$
- Store the round key in 2d 4x4 key matrix $K_{4\times4}$.
- Add round key matrix to the plaintext using xor-function $\acute{S}_{4\times4} = S_{4\times4} \oplus K_{4\times4}$.
- Select two bits, $(Byte\ 3.2)\ mod\ 2$ and $(Byte\ 3.3)\ mod\ 2$ of key matrix $K_{4\times4}$.
- The selected two bits determine one of four sub byte (s-box table).
- Each value of produced state matrix $\acute{S}_{4\times4}$ replaced with the corresponding value in the selected S-box to produce $SB_{4\times4}$.
- Each row in $SB_{4\times4}$ is moved over (shifted) 0,1, 2, or 3 spaces over the right depending on the row to produce $SR_{4\times4}$.
- Product the state matrix $SR_{4\times4}$ by mixing columns matrix $M_{4\times4}$ to produce $CM_{4\times4} = M_{4\times4} \times SR_{4\times4}$.
- Repeat the above steps 10 times from Add round key.
- The ciphertext of 128-bit produced $CM_{4\times4}$

$$k1: \begin{matrix} w[0]: w_{00} & w_{01} & w_{02} & w_{03} \\ w[1]: w_{10} & w_{11} & w_{12} & w_{13} \\ w[2]: w_{20} & w_{21} & w_{22} & w_{23} \\ w[3]: w_{30} & w_{31} & w_{32} & w_{33} \end{matrix}$$

$Where\ w_{ij}\ 2\ hexadecimal\ digits\ for\ word\ w[0]$

Decryption Process
Inputs: Ciphertext data subblock right.
Output: Plaintext data block.

- Product the state matrix $SR_{4\times4}$ by mixing of Inverse Columns Matrix $M_{4\times4}$ to produce $ICM_{4\times4} = M_{4\times4} \times SR_{4\times4}$.
- Each row in $ICM_{4\times4}$ is moved over (shifted) 0,1, 2, or 3 spaces over the left depending on the row to produce $ISR_{4\times4}$.
- Each value of produced state matrix $ISR_{4\times4}$ replaced with the corresponding value in the selected S-box to produce $ISB_{4\times4}$.
- Apply inverse for the selected two bits to determine one of four sub-bytes (s-box table).
- Select two bits, $(Byte\ 3.2)\ mod\ 2$ and $(Byte\ 3.3)\ mod\ 2$ of key matrix $K_{4\times4}$.
- Add inverse round key matrix to the plaintext using xor-function $\acute{S}_{4\times4} = S_{4\times4} \oplus K_{4\times4}$.

- Store the plaintext in 2d 4x4 state matrix $S_{4\times4}$.
- Select the round key of 128 bit.
- For key expansion, generate the word0 of key$k[0]$ from

$$\boldsymbol{K[n]:w[0] = K[n-1]:w[0] \oplus SubByte(K[n-1]:w[3] \gg 8) \oplus Recon[n]}$$

- Generate the remain words w's from $\boldsymbol{K[n]:w[i] = K[n-1]:w[i] \oplus k[n]:w[i-1]}$
- Store the round key in 2d 4x4 key matrix $K_{4\times4}$.
- Repeat the above steps 10 times from Add round key to find deciphered text.

AES Modified Encryption and Decryption, as shown in figure 5.

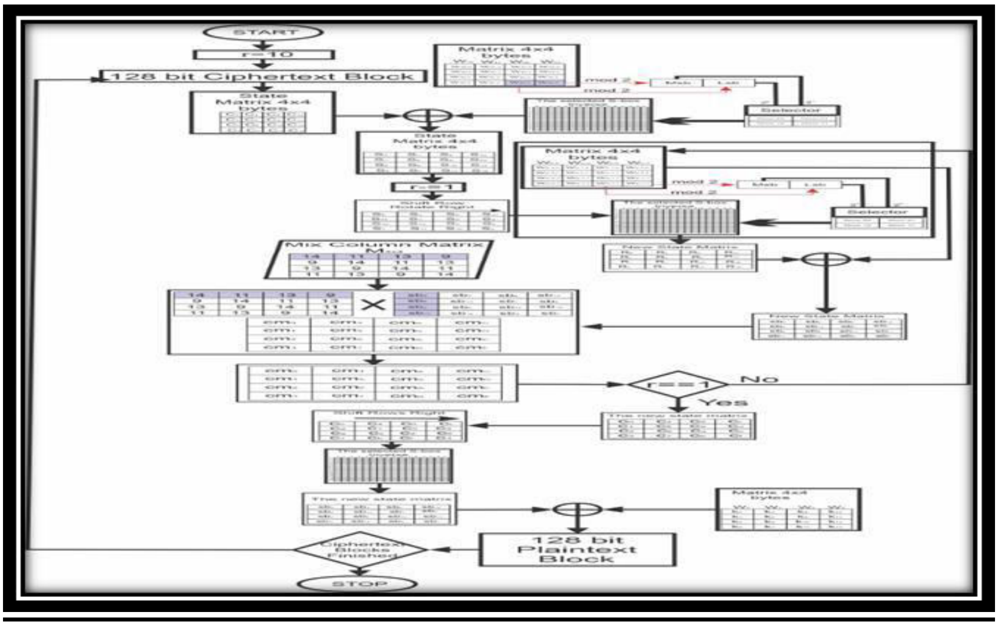


Figure 5: AES Modified Encryption and Decryption

3.2 Second Step (RSA Algorithm Modification)

The second stage of the research methodology is the modification of the standard asymmetric algorithm with two keys. The purpose of developing the algorithm is to increase the security rate and add complications to the clear text by dividing it into a matrix with dimensions (h*h) and calculating its determinant, in addition to adding complexity to the key by giving large values to (p,q) and thus The value of (n, N) increases, in addition to adding other complications to the algorithm, so that it is difficult for the attacker to break it and access the original information.

Encryption Process

- Select p, q, where p, q both prime, p ≠ q.
- Calculate $n = p \times q$.
- Construct $(M)_{h\times h}$ matrix from plaintext block into.
- Calculate determinate $|M|$.
- Calculate $N = p(p^h - 1) \times (q^h - 1)$
- Calculate the Greatest Common Divisor $gcd(|M|, n)$.

- Select integer e where $gcd\ (e, N) = 1;\ 1 < e < N$
- Select integer k, where $0 < k < e;\ \ d|e$.
- Calculate $d = \frac{k \times N+1}{e}$, where: $d \equiv e^{\ -1}\ mod\ N$
- Select r, where $r \geq 2$.
- Calculate $x = r + 2e$
- Calculate $y = 2n - r$
- Public Key $PU = \ \{x, y, r\}$
- Private Key $RP = \ \{d, y, r\}$
- Calculate ciphertext $C = M^{\frac{x-r}{2}}\ mod[\frac{y+r}{2}])$

Decryption Process

- Calculate decipher text $M = C^d\ mod[\frac{y+r}{2}])$

RSA Modification Algorithms Encryption and Decryption, as shown in figure 6.

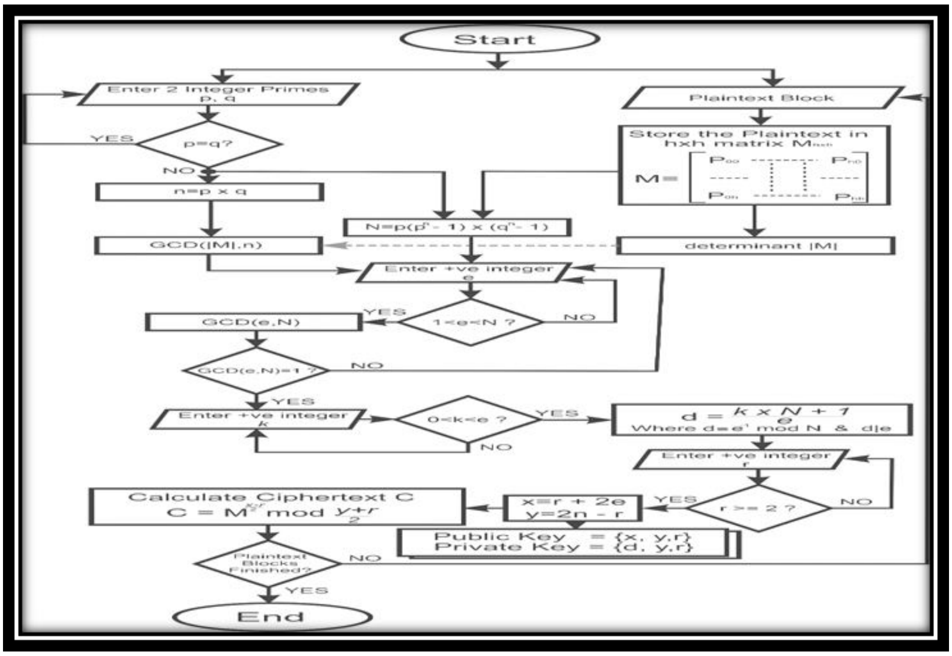


Figure 6 : RSA Modification Algorithms Encryption and Decryption

3.3 Third Step (Mixing AES and RSA Cryptography Algorithms)

In the third step, the files are encrypted and decrypted using the hybrid or combined system by merging the two modified algorithms AES and RSA, where the file is divided into two parts, one part works with the modified AES algorithm with a 128-bit key, and the other part works with the RSA algorithm. . That works with a 128-bit key. Thus, after the encryption process between the two algorithms, the encrypted file is obtained, through the decryption algorithm of this mixing algorithm, the original file is obtained. The primary purpose of mixing two algorithms is to take advantage of the strengths of security, speed, reliability, and throughput of each form of encryption. The second primary purpose, by combining public and private key cryptographic systems, is to overcome some of the drawbacks of each algorithm. The block diagram of mixing AES and RSA cryptography algorithms, as shown in figure 7.

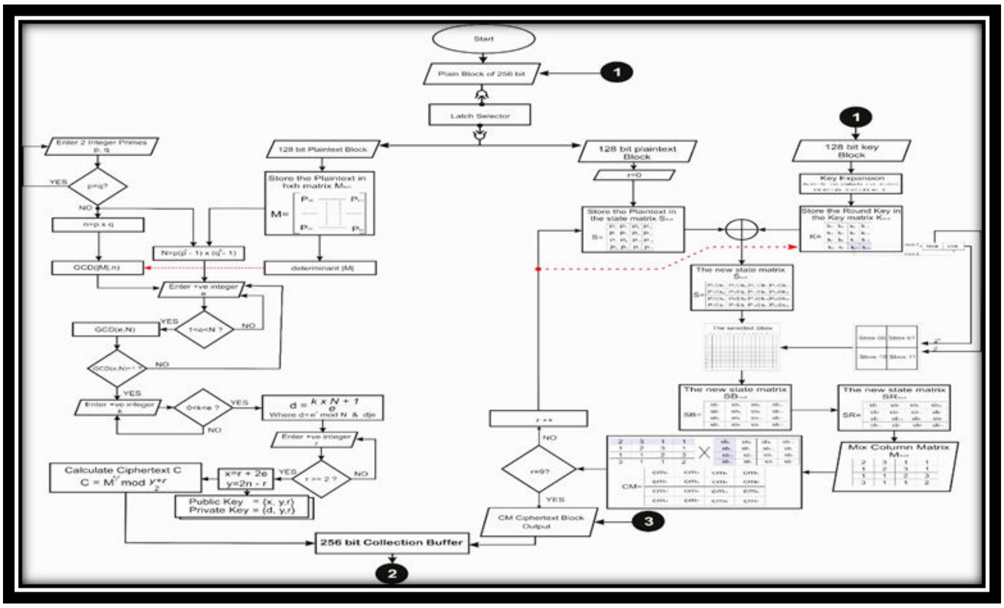


Figure 7: Block Diagram of Mixing AES and RSA Cryptography Algorithms

4 QUANTITATION ANALYSIS

There are many performance measures, used to measure the performance which is used to enhance hybrid system-based AES-RSA Algorithms and the hopping technique, and they are as follows [Isiaka et al.,2019]:

4.1 Time of Encryption: It takes to convert a plain text to a cipher text.

Time of Encryption = recording time after Encryption - recording time before Encryption ….(1)

4.2 Time of Decryption: It takes to convert a cipher text to a plain text.

Time of Decryption = Record time after Decryption – Record time before Decryption …(2)

4.3 Time of Execution: Is the summation the encryption time and the decryption time.

Time of Execution = Encryption Time + Decryption Time ...... (3)

4.4 Throughput: Is the rate at which data or file is transferred. It is the size of the file uploaded divided time it takes to recover the file.

Throughput = Total size of the file uploaded /Total Evaluation Time of Algorithm……(4)

4.5 File Size: Is the size of the file uploaded to the server.

5 RESULTS EVALUATION

In this research, two algorithms, AES and RSA, were modified and a third integrated algorithm called Hybrid Algorithm(AES+RSA) was produced. Where these algorithms were evaluated in terms of speed in encryption and decryption time as well as different file sizes for information, as well as calculating the throughput of each algorithm, through the use of the platform Microsoft Visual Studio Community 2022 (64-bit), Version 17.6.5 Visual Basic language to construct the algorithms, under Windows 10 64-bit, The CPU Intel(R) Core(TM) i5-3230M CPU @ 2.60GHz, RAM 8 GB DDR3 and HARD 320 GB are the device specifications used. This paper uses ten files with sizes of different (1.19 MB, 5.384 MB, 11.804 MB, 21.4 MB, 35.350 MB, 42.8 MB, 46.4 MB, 50 MB, 59.809 MB, 106 MB). In this paper calculates the encryption and decryption time for (AES, RSA, and Hybrid System), and compares this study with previous studies. The results in this paper depend on two approaches as shown below.

5.1 Securing Data

In this research, three algorithms were developed and tested on ten files format (.txt, .jpg, .mp3, .mp4, .pdf) of different sizes, where the encryption and decryption time measured (in seconds) were calculated for each of the AES, RSA, Hybrid Algorithms (AES+RSA). The results showed that the hybrid algorithm provides reliability and has a high level of security of the transmitted the data when comparing the algorithms RSA and AES, as shown in table 1 and figure 8.

Table 1: Time for each Algorithm in second

No of File	Plain file size (MB)	Modification Algorithms				Mixing Algorithms	
		AES		RSA		AES+ RSA	
		Encryption Time	Decryption Time	Encryption Time	Decryption Time	Encryption Time	Decryption Time
1.	1.19	0.011	0.028	0.1	0.2	0.01	0.19
2.	5.384	0.09	0.07	0.2	0.9	0.1	0.3
3.	11.804	0.18	0.12	0.25	0.95	0.15	0.45
4.	21.4	0.3	0.33	0.35	1.55	0.25	0.7

5.	35.350	0.55	0.61	0.65	2.85	0.35	0.95
6.	42.8	0.79	0.8	0.83	4.57	0.5	1.1
7.	46.4	0.88	1.01	1.1	5.1	0.75	1.35
8.	50	1.04	1.14	1.25	5.65	0.87	1.43
9.	59.809	1.1	1.19	1.3	6.2	1.02	1.48
10.	106	2.18	1.35	2.38	7.62	1.5	2.1

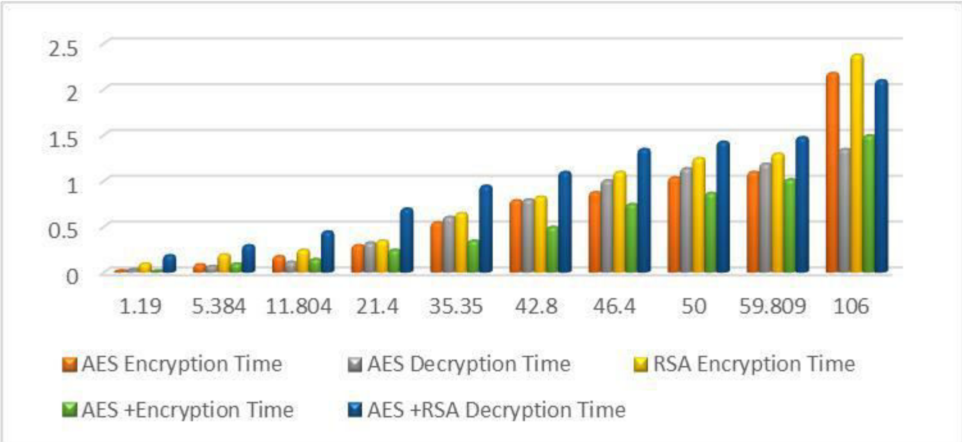


Figure 8:Time in Second for each algorithm

By calculating the total execution time in table 2 of the three algorithms for ten files of different sizes, it is shown in figure 9. Because compared to other algorithms, the hybrid algorithm produces outcomes that are better and has higher levels of information security. And that the speed of the hybrid algorithm in the overall execution is faster much slower than RSA algorithm and much slower than AES algorithm

Table 2: Total Time in Seconds for Each Algorithm

No of File	Plain file size (MB)	Modification Algorithms		Hybrid Algorithms
		AES Total Time (Second)	RSA Total Time (Second)	AES+ RSA Total Time (Second)
1.	1.19	0.039	0.3	0.2
2.	5.384	0.16	1.1	0.4
3.	11.804	0.3	1.2	0.6
4.	21.4	0.63	1.9	0.95
5.	35.350	1.16	3.5	1.3
6.	42.8	1.59	5.4	1.6
7.	46.4	1.89	6.2	2.1

8.	50	2.18	6.9	2.3
9.	59.809	2.29	7.5	2.5
10.	106	3.53	10	3.6

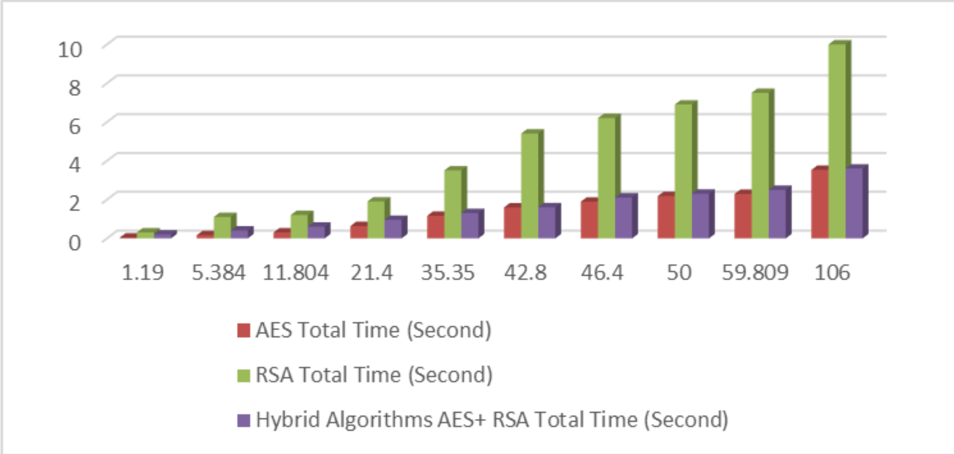


Figure 9: Total Time Execution Time for Three Algorithms

The results in this research showed that the hybrid algorithm in this research was faster 23.31% execution time compared with previous studies (Chaloop, & Abdullah,2021) (Ghaly & Abdullah, 2021) and table 3 shows that.

Table 3: Compare Hybrid Algorithms with Existing Systems

No of File	Plain file size (MB)	Existing Systems (Second)	Hybrid Algorithms AES+RSA (Second)
1.	1.19	0.53	0.2
2.	5.384	0.67	0.4
3.	11.804	0.90	0.6
4.	21.4	1.26	0.95
5.	35.350	1.73	1.3
6.	42.8	2.15	1.6
7.	46.4	2.45	2.1
8.	50	2.78	2.3
9.	59.809	2.95	2.5
10.	106	4.26	3.6

Figure 10 shows the execution time of the hybrid algorithm with execution time for previous studies

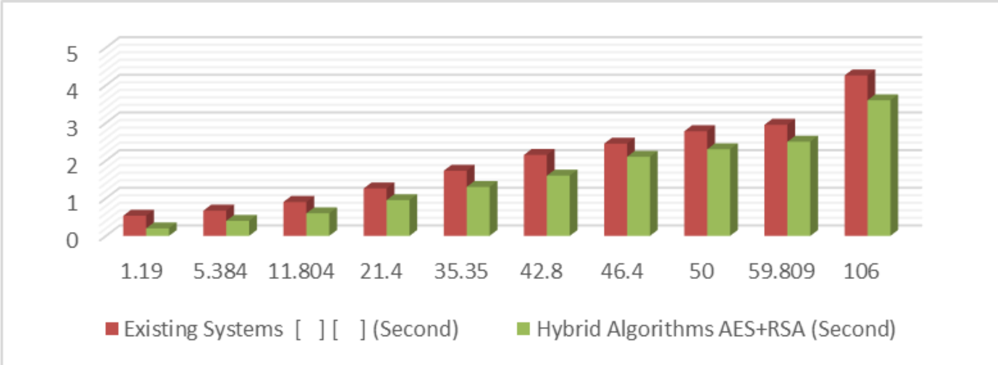


Figure 10: Execution Time for Hybrid Algorithms with Existing Systems

5.1 Throughput

Throughput means the sum of file sizes different divided by the average execution time of the algorithm. The table shows the values of throughput of the three algorithms (MB/second). Analysis of throughput shown in table 4 and figure 11.

Table 4: Throughput values for the algorithms

Total Files Sizes (MB)	RSA Throughput	Hybrid Algorithms Throughput	AES Throughput
380.137	86.395	244.461	276.082

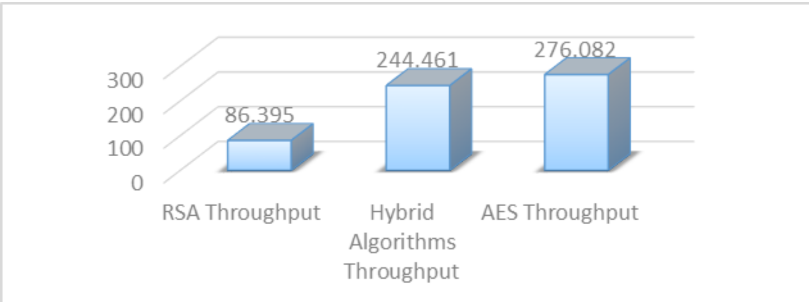


Figure 11: Throughput values for the algorithms

The results of this research , through table 5 showed that the throughput of the hybrid algorithm was higher according to the size of the files and compared with the throughput of previous studies (Chaloop, & Abdullah,2021) (Ghaly & Abdullah, 2021) , and the figure 12 shows the throughput analysis.

Table 5: Values of Throughput of Hybrid Algorithms with Existing System

Total File Size (MB)	Existing System Throughput	Hybrid Algorithms Throughput
380.137	193.110	244.461

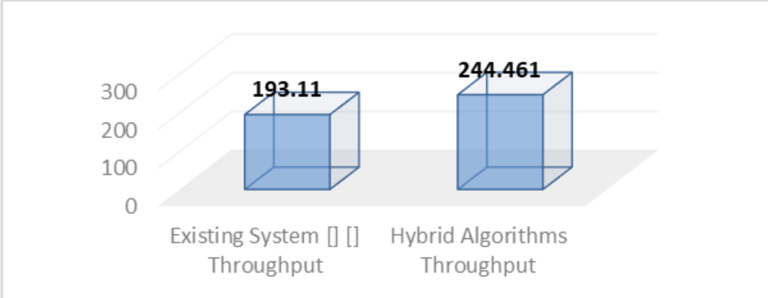


Figure 12: Values of Throughput of Hybrid Algorithms with Existing System

6 Conclusion

In this research, a hybrid system based on the combination of the symmetric and asymmetric encryption algorithm AES-RSA has been improved. Where the purpose of this research was to improve encryption performance, enhance data security, store keys, calculate encryption and decryption time, execution time, and throughput for the standard algorithms and the hybrid algorithm, and compare it with previous studies.

Among the most important findings of this research is that the improved hybrid AES-RSA algorithm is 63.15% faster than RSA Algorithm, and 36.85% slower than the AES algorithm.

References

Abd Zaid, M., and Soukaena Hassan. (2019) .Modification advanced encryption standard for design lightweight algorithms. *J. Kufa Math. Comput.* 6.1: 21-27.

Abd Zaid, Mustafa M., and Soukaena Hassan. (2018) .Lightweight RSA Algorithm Using Three Prime Numbers. *Int. J. of Engineering & Technology* 7.4.36 :293-295.

Abood, M. H. (2017). An efficient image cryptography using hash-LSB steganography with RC4 and pixel shuffling encryption algorithms. 2017 Annual Conference on New Trends in Information & Communications Technology Applications (NTICT). doi:10.1109/ntict.2017.7976154.

Abroshan, Hossein. (2021) .A hybrid encryption solution to improve cloud computing security using symmetric and asymmetric cryptography algorithms. *International Journal of Advanced Computer Science and Applications* 12.6: 31-37.

Akash Kumar Mandal, Chandra Parakash and Mrs. Archana Tiwari (2012). Performance Evaluation of Cryptographic Algorithms: DES and AES. IEEE Students' Conference on Electrical, Electronics and Computer Science, pp. 1-5.

Alegro, Jhoanne Kris P., et al. (2019) .Hybrid Schnorr, RSA, And AES Cryptosystem. *Int. J. Sci. Technol. Res* 8.10: 1777-1781.

Bokhari, Mohammad Ubaidullah, and Qahtan Makki Shallal (2016). A review on symmetric key encryption techniques in cryptography. *International Journal of Computer Applications* 147.10.

Bokhari, Mohammad Ubaidullah, Qahtan Makki Shallal, and Md Zeyauddin.(2018). Hybrid Blowfish and RSA Algorithms to Secure Data between Cloud Server and Client.

Carlo A. Intila, Bobby D. Gerardo, Ruji P. Medina. (2019). Modified Key Generation in RSA Algorithm" International Journal of Recent Technology and Engineering (IJRTE) ISSN: 2277-3878 (Online), Volume-8 Issue-2, July.

Chaloop, Samir G., and Mahmood Z. Abdullah. (2021). Enhancing Hybrid Security Approach Using AES And RSA Algorithms. *Journal of Engineering and Sustainable Development* 25.4 : 58-66.

Ezekiel Bala, Ajibola Aminat, and Ebelogu Christopher. (2019).Hybrid Data Encryption And Decryption Using RSA And RC4. International Journal of Scientific & Engineering Research Volume 10, Issue 10, ISSN 2229-5518, October.

Ghaly, S., & Abdullah, M. Z. (2021). Design and implementation of a secured SDN system based on hybrid encrypted algorithms. *TELKOMNIKA (Telecommunication Computing Electronics and Control)*, *19*(4), 1118-1125.

Guru, Mr Abhishek, and Asha Ambhaikar.(2021) .AES and RSA-based Hybrid Algorithms for Message Encryption & Decryption. *Information Technology in Industry* 9.1: 273-279.

Isiaka, O. S., et al.(2019). Hybridization of RSA And Blowfish Cryptography Algorithms for Data Security on Cloud Storage.", International Journal of Engineering Technologies and Management Research, ISSN: 2454-1907 DOI: 10.5281/zenodo.3595252, December.

Rajkamal, M., and B. S. E. Zoraida (2014). Image and Text Hiding using RSA & Blowfish Algorithms with Hash-Lsb Technique. *Int. J. Innov. Sci. Eng. Technol* 1.6.

S.C. Gupta and Manju Sanghi.(2021) .Matrix Modification of RSA Public Key Cryptosystem and its Variant. ISSN No. (Print): 0975-8364 ISSN No. (Online): 2249-3255, International Journal on Emerging Technologies 12(1): 76-79.

Sadkhan A. M., Sattar B. (2014). Multidisciplinary Perspectives in Cryptology and Information Security: Advances in Information Security, Privacy, and Ethics", Book, IGI Global, ISBN: 978- 1466658097.

Sahin, M. Emin. (2023). Memristive chaotic system-based hybrid image encryption application with AES and RSA algorithms." *Physica Scripta* 98.7: 075216.

Stallings, William (2006) .Cryptography and Network Security: Principles and Practice. Pearson Education/Prentice Hall, 5th Edition.

Stallings, William (2006). Cryptography and network security, 4/E. Pearson Education India.

Timilsina, Suresh, and Sarmila Gautam. (2019) .Performance analysis of hybrid cryptosystem-A technique for better security using blowfish and RSA. *Journal of Innovation in Engineering Education* 2.1.

Ting-Wei Chuang, Chaur-Chin Chen and Betty Chien (2016). Image Sharing and Recovering based on Chinese Remainder Theorem. Proceedings of International IEEE Symposium on Computer, Consumer and Control, pp. 487- 494.

Zilhaz Jalal Chowdhury, Davar Pishva and G. G. D. Nishantha, (2010). AES and Confidentiality from the Inside Out", the 12th International Conference on Advanced Communication Technology (ICACT), pp. 1587-1591.

Zong, T., Xiang, Y., &Natgunanathan, I. (2014). Histogram shape-based robust image watermarking method. 2014 IEEE International Conference on Communications (ICC). doi:10.1109/icc.2014.6883430.

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

Semi-QUASI HAMSHER MODULES

Nada, K. Abdullah
Department of Mathematics,
College of Education for Pure Sciences,
Tikrit University, Iraq
nada.khalid@tu.edu.iq

Semi-QUASI HAMSHER MODULES

Nada, K. Abdullah
Department of Mathematics,
College of Education for Pure Sciences,
Tikrit University, Iraq
nada.khalid@tu.edu.iq

Abstract:

This study presents a semi-quasi Hamsher module that each non-zero Artinian submodule has a semi-maximal submodule, which is a generalized of Hamsher module. And semi-quasi Loewy module that each non-zero Noetherian submodule has a semi-maximal submodule, which is a generalized of Loewy module. This article introduces some properties of semi-quasi Hamsher and semi-quasi Loewy modules .

Keywords: semi-quasi Hamsher module, semi-quasi Loewy module, semi-maximal submodule and semi-socle submodule .

Introduction:

Throughout rings and modules are unitary. We use the terminology and notations of Anderson and Fuller[1]. Faith [2] defined a module X is Hamsher if each non-zero submodule of X has a maximal submodule. In[3] we see that X has finite length if and only if X is Hamsher and Artinian. Weimin[4] generalized to quasi-Hamsher module X if every non-zero Artinian submodule of X has a maximal submodule. A ring R is said to be right maximal if each non-zero right R-module has a maximal submodule[2].This class of rings includes right perfect rings. In this paper, we characterize semi-quasi-Hamsher module(for short; S.Q.Ham.Mod) if each non-zero Artinian submodule has a semi-maximal submodule(for short; s-max.sub).

1-S.Q.Ham.Mod: The class of S.Q.Ham.Mod is closed under submodules, also closed under extensions, direct products, and direct sums as we see in the following propositions .

Proposition(1.1)*Let* $0 \to X_1 \xrightarrow{f} X \xrightarrow{g} X_2 \to 0$ be an exact sequence of modules. If X_1 and X_2 are S.Q.Ham.Mod, then so is X.

Proof: Let L≠0 be an Artinian submodule of X. If g(L) # 0, being an Artinian submodule of the S.Q.Ham.Mod X_2, g(L) has a s-max.sub N. Then $L \cap g^{-1}(N)$ is a s-max.sub of L. If $g(L) = 0, L \subseteq \ker(g) = Im(f) \cong X_1$, so L has a s-max.sub since X is S.Q.Ham.Mod .

Proposition (1.2) Let {Xi}S.Q.Ham.Mod be a family of modules, then the following statements are equivalent :

1- Each Mi is S.Q.Ham.Mod
2- $\prod_{i \in I} X_i$ *is* S.Q.Ham.Mod

3-$\bigoplus_{i\in I} X_i$ *is* S.Q.Ham.Mod

Proof: (1) ⇒ (2) Let L≠0 be an Artinian submodule of $\prod_{i\in I} X_i$, and let f_i : $\prod_{i\in I} X_i$ →X_i be a canonical projections. We have X_i such that f_i(L)≠ 0. Then f_i (L) is an Artinian submodule of S.Q.Ham.Mod Xi so f_i(L) has a s-max.sub N. Thus L∩f^{-1}(N) is a s-max.sub of L[6], therefore $\prod_{i\in I} X_i$ is S.Q.Ham.Mod .

(2) ⇒ (3) ⇒ (1) These are obvious because the class of S.Q.Ham.Mod is closed under submodules. Cai and Xue[5] called a module X is strongly Artinian if each of its proper submodule has finite length. It is easy to see that a non-zero strongly Artinian module has finite length if and only if it has a maximal submodule if and only if it is finitely generated. And since every maximal submodule is a semi-maximal submodule, thus we can see that a module X is semi-strongly Artinian if every of its proper submodule has finite length[6]. So we can say that a non-zero semi-strongly Artinian module has finite length if and only if it has a semi-maximal submodule if and only if it is finitely generated . Proposition(1.3) the following statements are equivalent :

1- X is S.Q.Ham.Mod;
2-Each Artinian submodule of X has finite length
3-Each Artinian submodule of X is finitely generated
4-Each semi-strongly Artinian submodule of X is finitely generated
5-Each non-zero semi-strongly Artinian submodule of X has a semi-maximal submodule, so it has finite length .

Proof: (1) ⇒ **(2)**Let L be a non-zero Artinian submodule of X. Since each submodule of L is still Artinian, L is an Artinian Hamsher module, which has finite length.

(2) ⇒ **(3)** ⇒ **(4)** ⇔ **(5)** and **(3)** ⇒ **(1)** Thes are obvious .

(3) ⇒ (2) If L is an Artinian submodule of X and L has infinite length, then the non-empty family { N ⊆ L | N has an infinite length } has a minimal member, say N. It is easy to see that N is strongly Artinian and N has infinite length .

As a generalization of maximal module the module X is semi-maximal if it is semi-simple[7], and [8] defined quasi-maximal module if Rad(ann_R X) is semi-maximal ideal of a ring R. Also a ring R is said to be right semi-maximal if each non-zero right R-module has semi-maximal submodule[9], and we call a ring R is right semi quasi maximal if every right R-module is semi quasi Hamsher. The next characterizations of right semi quasi maximal rings follow immediately from the above proposition.

Theorem(1.4) The following statements are equivalent :
1-R is right semi quasi maximal ring
2-Every non-zero strongly Artinian right R-module has a semi-maximal submodule
3-Every (strongly) Artinian right R-module has finite length;
4-Every (strongly) Artinian right R-module is finitely generated.

Camillo and Xue [3] called a ring R right quasi-perfect if every Artinian right R-module has a projective cover. Using Th.(1.4) and [3], we see that a ring R is right quasi perfect if and only if it is semi perfect and right quasi semi-maximal[3]

Proposition(1.5) If R is commutative semi perfect ring with nil J(R), then R is semi-quasi maximal ring .

A ring R is right maximal if and only if R/J(R) is right semi-maximal and J(R) is right T-nilpotent[5]. The ring R is a local commutative ring with nil J(R) which is not T-nilpotent. Hence R is not maximal[3], but R is semi-quasi maximal (Prop.1.5). Therefore there is a semi-quasi-Hamsher R-module which is not Hamsher. We conclude that semi-quasi-Hamsher modules and right semi-quasi-maximal rings are proper generalizations of Hamsher modules and right semi-maximal rings, respectively.

Example(1.6) Let V be a division ring. Let R be the ring of all countable infinite upper triangular matrixes over V with constant on the main diagonal and having non-zero entries in only finitely many rows above the main diagonal. Then R is a local right perfect ring which is not left perfect. Miller and Turnidge [10] constructed and Artinian left R-module X which is not Noetherian. Hence R is not left semi-quasi maximal. This shows that the notion of semi-quasi maximal rings is not left-right symmetric.

In view of the above example and prop.(1.5), we mention the following result .

Proposition(1.7) Let R be a semi perfect ring with nil J(R). If J(R) is of bounded index n, i.e.($j^n = 0$) for each $j \in J$ (R), then R is semi-quasi-maximal or semi-quasi-perfect.

Modifying the proof of [2] we have an analogous result.

Theorem(1.8): The following statements are equivalent

1-R is right quasi-maximal ring

2-The category Mod-R has a cogenerate G which is S.Q.Ham.Mod

3-The injective envelope E(X) of X is S.Q.Ham.Mod for each simple right module X.

Proof: (1) ⇒ (2) This is obvious.

(2) ⇒ (3) Since G is a cogenerator there is a monomorphism E(X) G for each simple right R-module X. Hence E(X) must be S.Q.Ham.Mod, since G is.

(3) ⇒ (1) Let X range over all simple right R-modules. Then ⊕E(X) is a co-generator of Mod-R and ⊕E(X) is S.Q.Ham.Mod by prop.(1.2) Let A be a non-zero Artinian right R-module. We have a non-zero homo. f:L→ ⊕E(X). Since f(L) is a non-zero Artinian submodule of E(X), which is S.Q.Ham.Mod, f(L) has a semi-maximal submodule of N. Then $f^{-1}(N)$ is a semi-maximal submodule of L .

2-Semi-Quasi Loewy Modules : [12]Recall that a module M is called Loewy if every non-zero factor module of M has non-zero socle. And a module M is called quasi-Loewy if every non-zero Noetherian module of M has non-zero socle[4]. A module M has finite length if and only if M is Loewy and Noetherian [1]. A module X is semi-local if $\frac{X}{Rad(X)}$ is semi-simple[13]. In this section we interduce a concept that a module X is semi-quasi Loewy module(for short; S-Q Loy. Mod.) if every non-zero Noetherian module of X has non-zero semi-socle. The next two propositions show that the class of S-Q Loy. Mod. is closed under extensions and direct sums.

Proposition(2.1) $Let\ 0 \to X_1 \xrightarrow{f} X \xrightarrow{g} X_2 \to 0$ be an exact sequence of modules. If both X_1 and X_2 are S-Q Loy. Mods., then X is S-Q Loy. Mod.

Proof: $Let\ \frac{X}{L} \neq 0\ be\ a\ factor\ module\ of\ X.$

$$We\ have\ an\ exact\ sequence\ \ 0 \to \frac{X_1}{L_1} \to \frac{X}{L} \to \frac{X_2}{L_2} \to 0$$

$$If\ \frac{X_1}{L_1} \neq 0\ and\ soc\left(\frac{X_1}{L_1}\right) \neq 0, then\ soc\left(\frac{X}{L}\right) \neq 0.$$

$$If\ \frac{X_1}{L_1} = 0, \frac{X_2}{L_2} \cong \frac{X}{L} \neq 0. Then\ soc\left(\frac{X_2}{L_2}\right) \neq 0\ and\ soc\left(\frac{X}{L}\right) \neq 0.$$

Proposition(2.2) Let $\{X_i\}_{i\epsilon I}$ be a family of modules. W is S-Q Loy. Mod. if and only if every X_i is S-Q Loy. Mod..

Proof: The class of S-Q Loy. Mod. is closed under factor modules .
Conversily; $let\ f_i : X_i \to \bigoplus_{i\epsilon I} X_i$ be a canonical injection.

$If\ \frac{\bigoplus_{i\epsilon I} X_i}{L}$ is a non-zero (Noetherian) factor module of $\bigoplus_{i\epsilon I} X_i\ , then\ there\ is\ i\epsilon I\ such\ that\ 0 \neq g_i : X_i \to \frac{\bigoplus_{i\epsilon I} X_i}{L}$

$where\ g{:}\bigoplus_{i\epsilon I} X_i \to \frac{\bigoplus_{i\epsilon I} X_i}{L}$ is the natural epimorphism.

Since $Im(g_{ji}) \neq 0$ which is isomorphic to a (Noetherian) factor module of $X_i\ , we\ have\ 0\ \#\ soc(Im(gji)) \subseteq soc\left(\frac{\bigoplus_{i\epsilon I} X_i}{L}\right)$.

$If\ R = \prod_{i=1}^{\infty} P_i$ is an infinite product of the fields P_i

then R is not a Loewy module [8]. Since every P_i is a Loewy module, this shows that the class of Loewy modules is not closed under direct products. We do not know if the class of S-Q Loy. Mod. is closed under direct products.
A module is called strongly Noetherian if each of its proper factor module has finite length[14],[15]. It is easy to see that a non-zero strongly Noetherian module has finite length if and only if it has non-zero semi-socle if and only if it is finitely cogenerated[6] .
Proposition (2.3) The following statements are equivalent:

1-X is S-Q Loy. Mod.

2- Each Noetherian factor module of X has finite length
3- Each Noetherian factor module of X is finitely cogenerated
4- Each strongly Noetherian factor module of X is finitely cogenerated
5- Each non-zero strongly Noetherian factor module of X has non-zero semi-soc and finite length

Proof: (1) ⇒(2) $Let\ \frac{X}{L} \neq 0$ be a Noetherian factor module of X. Since each factor module of $\frac{X}{L}$ is still Noetherian, thus $\frac{X}{L}$ has finite length .

(2) ⇒(3) ⇒(4) ⇔ (5) and (3) ⇒(1) These are obvious .

(5) ⇒(2) $If\ \frac{X}{L}$ is a Noetherian factor module of X and $\frac{X}{L}$ has infinite length, then the non-empty family $\{L \subseteq L` \subseteq X \mid \frac{X}{L}\ has\ infinte\ length\}$.

$has\ a\ maximal\ member\ say\ L`.Thus\ \frac{X}{L`}\ is\ strongly\ Noetherian\ and$

has infinite length .

A ring R is called right S-Q Loy. if every right R-module is S-Q Loy. . The next characterizations of right S-Q Loy. rings follow immediately from the above proposition.

Theorem(2.4) The following statements are equivalent :
1-R is right S-Q Loy. ring

2- Each non-zero (strongly) Noetherian right R-module has non-zero semi-socle
3- Each (strongly) Noetherian right R-module has finite length

4- Each (strongly) Noetherian right R-module is finitely co-generated

It follows from Th.(1.4) and Th.(2.4) that the rings studied by Tanabe [11] are precisely left semi-quasi maximal and left S-Q Loy. rings. An analogous result of Th.(1.8) is the following

Theorem (2.5) A ring R is right S-Q Loy. if and only if Mod-R has a generator C which is S-Q Loy.

Proof: If X is a Noetherian right R – module $X \cong \frac{C^n}{L}$.

C^n is S-Q Loy. prop.(2.2), so $\frac{C^n}{L}$ has finite length prop.(2.3). Hence R is right S-Q Loy. th.(2.4) .

The convers is clear

The next proposition gives a class of commutative S-Q Loy. rings.

Proposition (2.6) If R is a commutative semi-perfect ring with nil J(R) then R is S-Q Loy. Mod. ring .

Proof. By Th.(2.5), it suffices to show that R is a S-Q Loy. Mod. . Let A be an ideal of R such that $\frac{R}{A}$ is a Noetherian R-module .

Then the commutative semi − perfect Noetherian ring $\frac{R}{A}$ has nil $J\left(\frac{R}{A}\right)$.

Hence $\frac{R}{A}$ is an Artinian ring. Then $\frac{R}{A}$ has finite length as an R-module .

R is right Loewy ring if every right R-module is Loewy[4], its mean every non-zero right R-module has non-zero socle, equivalently, the right R-module R_R is Loewy. Every left perfect ring is right Loewy. In [16] R is right Loewy if and only if $\frac{R}{J(R)}$ R is right Loewy, and J (R) is left X-nilpotent. The ring R in [3] is a local commutative ring with nil J (R) which is not X-nilpotent. Hence R is not Loewy, prop.(2-6) but R is semi-quasi. Therefore there is a S-Q Loy. Mod. which is not Loewy. We conclude that S-Q Loy. Mod. and S-Q Loy. rings are proper generalizations of Loewy modules and right Loewy rings, respectively.
Let R be the ring in ex.(1.6) Then R is a local right perfect ring which is not left perfect[17]. Miller and Turnidge [6] constructed a Noetherian right module X which is not Artinian. Hence R is not right S-Q Loy. Mod. .This shows that the notion of S-Q Loy. rings is not left-right symmetric. In view of this fact and prop.(2.6), we state the next result, which follows from [11] .

Proposition (2.7) Let R be a semiperfect ring with nil J(R). If J (R) is of bounded index n then R is (two-sided) S-Q Loy. ring .

Since a commutative regular ring need not be Loewy (see $R = \prod_{i=1}^{\infty} P_i$ preceding prop. 2.3),

Proposition (2.8) Every strongly regular ring R is a (two-sided) S-Q Loy. ring .

$Proof: let \sum_{i=1}^{n} x_i R \ be\ a\ Noetherian\ right$

$R - module. It\ suffices\ to\ show\ X$

has finite length, we have

$x_i R \cong \frac{R}{A}$ for some ideal A of R, since $\frac{R}{A}$ $is\ a\ right$

Noetherian regular ring it is semi − simple,

and $\frac{R}{A} \cong x_i R$ has finite length .

3-Semi-Quasi Hamsher Rings(S-Q Ham. Rings)

Morita duality. A bimodule $_GI_R$ defines a Morita duality if $_GI_R$ is faithfully balanced and both I_R and $_GI$ and I_R are injective co-generators. In this case, both R and G are semi-perfect rings. In [18] we can see a presentation of Morita duality, by using properties of Morita duality .

Proposition(3.1) Let $_GI_R$ define a Morita duality. If X_R is a I-reflective right module then

1- X_R is S-Q Low. Mods if and only if the left G-module $_G\mathrm{Horn}_I(X_R , {_GI_R})$ is S-Q Low. Rings .
2- X_R is S-Q Ham. Mods if and only if the left G-module $_G\mathrm{Horn}_I(X_R , {_GI_R})$ is S-Q Ham. Rings .

Theorem(3.2)If $_GI_R$ defines a Morita duality, then the following statements are equivalent:

1- R is right semi-quasi maximal
2- G is left semi-quasi maximal

3- R is right semi-quasi Loewy

4- G is left semi-quasi Loewy.

Discussion and conclusion: The aim of this manuscript is to introduced a new generalized of Hamsher module which is semi-quasi Hamsher module that each non-zero Artinian submodule has a semi-maximal submodule. This class of module is closed under extension, direct product and direct sum. Furthermore; we introduce a new generalized of Loewy module which is semi-quasi Loewy module that each non-zero

Noetherian submodule has a semi-socle submodule. This class of module is closed under extension, direct product and direct sum .

REFERENCES

1- F. W. Anderson and K. R. Fuller(1992):Rings and *Categories* of Modules, 2nd edition, Springer, New York .
2- C. FAITH: Rings whose modules have maximal submodules, Publ. Mate. 39 (1995), 201-214.
3- V. P. Camillo and K. R. Fuller(1974): On Loewy length of rings, Pacific J. Math. 53, 347-354 .
4- Welimin, X.(1997) Quasi-Hamsher Modules And Quasi-Max Rings, Math. J. Okayama Univ. 39, 71-79 .
5- Faith: (1995), Rings whose modules have maximal submodules, Publ. Mate. 39, 201-214 .
6- Tony, J. Puthenpurakal (2023) On The Loewy Length Of Modules Of Finite Projective Dimention-II. Vol.1022, ar.Xiv:2305.
7- Inaam, M. and Alaa, A. (2019): Semi-T-Maximal Submodules, vol. 60 . No. 12 .
8- Bothaynah, N. and Hatam, Y. (2010): On Quasi-Maximal Modules. Vol. 13(4), 205-210 .
9- Jerzy, M. and Edmund, R. (2016): On The Intersection Graphs of Modules and Rings. arXiv: 1606, vol. 01647
10- R. W. Miller and D. R. Turnidge: Some examples from infinite matrix rings, Proc. Amer. Math. Soc. 38 (1973), 65—67.
11- K. Tanabe: On rings whose artinian modules are precisely noetherian modules, Comm. Algebra 22 (1994), 4023—4032 .

12- - Luigi, S. and Paolo, Z. (2004) Loewy Length of Modules over almost perfect domains, Vol. 280, Issue 1, 207-218 .
13- Firas, N. and Isria, S. (2021): Some Properties of Local Modules. Vol. 1818, No. 012167 .
14- Y. Cay and W. XUE: Strongly noetherian modules and rings, Kobe J. Math. 9 (1992), 33-37.
15- Tugba, Y. and Sevgi, H. (2020): Modules of Finite Length, Math. Reports 22(72), 121-131 .8i
16- C. NASTASESCU and N. POPESCU: Anneaux semi-artiniens, Bull. Soc. Math. France 96 (1968), 357-368 .
17- Mohammad, R. and Abdoljawad, T. (2017) Characterizing Local rings via perfect and coperfect modules. Vo. 16, No. 04, 1750066 .
18- W. XUE: Rings with Morita Duality(1992), Lect. Notes Math. Vol. 1523, Springer, Berlin .

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

Investigation of Radiation Effect Assessment of Five Minerals by Graph Method

Summer W. Omar

Department of mathematics

College of Education for Pure Sciences Tikrit University/ Iraq

E-mail: samar.watheq@tu.edu.iq

Investigation of Radiation Effect Assessment of Five Minerals by Graph Method

Summer W. Omar

Department of mathematics

College of Education for Pure Sciences Tikrit University/ Iraq

E-mail: samar.watheq@tu.edu.iq

Abstract:

The aim of this study is to understand the interaction between different minerals and radiation, including how they respond when exposed to expected levels or doses of radioactive sources. The study will advance understanding of the consequences radiation exposure and provide insight into mineral use at relevant contexts in nuclear, industrial etc. The principle objective is to investigation on five type o four different minerals in radiation effects by using the plotting method. The main steps of the survey included choice for 5 minerals analysis include Iron, thorium and lead, calcite and pyrite or vice versa where mineral samples were irradiated by cesium radiation source or x-rays. Then the study is done by using plotting method which consists of evaluating dosimetric effects and consequent from radiation exposure. Part of this technique includes constructing impact curves for each mineral as a function of mass attenuation coefficient and its physical or chemical properties when irradiated. In addition, the study includes modeling data from the plots with specific and related mineral composition parameters that vary under different levels of radiation. The ultimate purpose of this investigation is to assess the radiation-induced effects on each mineral based on the results and ascertain how radioactivity may affect physical-chemical properties for each particular type.

Keyword: Radiation effects, Mineral, Thorium, Calcite, Pyrite.

1. Introduction

Biological networks include protein-protein interaction (PPI) network, gene regulatory network and metabolic pathway where Graph theory is extensively used in the field of biology to analyze various biological systems that ultimately help us in understanding an organization of a biological system. One of the most relevant studies on this latter matter is an investigation by Barabási and Oltvai (2004), which explores how graph theory can play a role in studying the architecture of biological networks [1]. Almaas (2007) reviews work in network analysis techniques and their application to systems biology [1]. In addition, this theory is critical to investigate molecular structures, chemical reactions and identify the features of atoms and bonds. Molecular entities may be depicted" in the form of graphs, in which atoms constitute nodes and the chemical bonds between them are edges. In the research of Trinajstić (1992), explores the applications of graph theory in chemical graph theory [3]. Furthermore, graph theory is essential to the world of electrical engineering for analysis of communication networks; routing algorithms and coding theory. Since networks have topologies (or structures) that govern the way nodes interact with each other, it is natural to use graph-based models and algorithms for creating and improving network building.

Deo (2005) covers many different uses of graph theory in electrical engineering [4]. Graph theory also is the cornerstone for numerous computer science algorithms and data structures. It is applicable to different fields like network analysis, social networking analysis and algorithmic design optimization. An alternative and more detailed treatment of graph theory and its use within Computer Science is presented in the book by West (2001) [5]. In addition, Graph theory plays an important role in the area of operational investigation for dealing with optimization challenges including scheduling and transportation networks and supply chain management. Graph-based algorithms can be used to model and analyze complex systems in order to improve the allocation of resources, as well help decision making parties. Ahuja et al. Applications of graph theory in operations research [6]. (1993). The device is employed in various scientific areas and state-of-art information & computer technologies. A graph, denoted by Q = (M,N), is formed as a collection of vertices (M(H)) and edges(N(H) in the graph H.

In the realm of graph theory, a cut-set denotes a collection of edges that, upon elimination, causes the graph to become disconnected. The vertex connectivity of the correspondence graph H is characterized as the minimal quantity of vertices that, upon removal, result in a disconnected residual graph [7-9]. significantly, Haregeweyn and Yohannes utilized the non-agricultural pollution model (AGNPS) to gauge drainage basin pollution in Ethiopia [11], whereas the focus was on second-generation computer software for internal dose evaluation in nuclear medicine [10]. Ilyas et al. concentrated on the estimation and comparison of diffuse solar radiation distribution throughout Pakistan [12]. Arshad Ali delved into temperature gradients for thermal systems utilizing the spectral methodology [13]. Moreover, investigations were carried out concerning skin dose evaluations and methodologies for estimating radiation dose to the skin throughout fluoroscopically assisted practices [8].
Within the scope of this manuscript, we delve into the visual approach for evaluating the influence of mineral radiation on five particular mineral types. This work extends prior research efforts, which encompass the utilization of recognized methodologies to assess radiation dosage on the skin during fluoroscopy, as explored in the work of [16].

2- Create a mathematical model using hr to estimate the estimate of $\mathrm{M}(hr)$

Graph theory.

In an empirical setting, the present study employed a graph-based methodology to predict the amount of minerals and evaluate the influence of radiation on a specific group of individuals. Through exploring all possible situations, we investigated different likelihood of mineral radiation among a randomized sample consisting of 11 individuals. Thereafter, we performed a thorough examination of the aforementioned situations, analyzing and assessing the various outcomes to determine the utmost precise calculations. This study presents three options as outlined:
2-1 Using the graph-based technique on the interval [3, 23] in Table 2-1, our objective was to evaluate the average mineral content in urea samples for the selected group of individuals. With respect to this aspect, illustrates the central mineral rate, which signifies the count of mineral radiation within the subset of individuals. The analysis conducted led us to derive the following equation:

$$M_l (hr) = M_l (hr) + \frac{m(hr_2) - m(hr_1)}{(hr_2) - (hr_1)} hr \quad (1)$$

It is a useful equation used to calculate the average mineral concentration for a particular interval based on figures of radiation rate derived from minerals.

Table 2-1. $M(hr)$ to hr and compare with the obtained experimental value using graph.

No	Name	Rate of mineral Ml	Class of People Exp. $M(hr)$	Class of People Det. $M(hr)$	Absolute Error
1	Iron	3	3.11	3.09	0.15
2	Thorium	9	3.48	3.41	0.041
3	Lead	14	3.52	3.49	0.048
4	Calcite	17	3.73	3.66	0.109
5	Pyrite	23	3.90	3.81	0.199
					$\sum$ 0.547

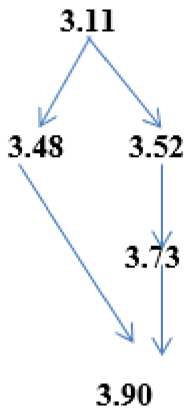


Fig. 2-1 The function $M(hr)$ is determined based on the variable hr, and it represents the relationship between hr and the corresponding values of the mineral M.

Through the application of a graph-based approach within the [4.26] interval in Table 2-2, our objective was to approximate the mean mineral content in urea samples for a specific study population. In this scenario, denotes the focal point for mineral $M_l (hr)$, reflecting the mineral radiation tally in the sample of individuals. From our examination, we arrived at the subsequent equation:

$$M_i (hr) = M_2 (hr) + \frac{m(hr_3) - m(hr_2)}{(hr_3) - (hr_2)} hr \quad (2)$$

Table 2-2. M(hr) to hr and compare with the obtained experimental value using graph.

No	Name	Rate of mineral Ml	Class of People Exp. M(hr)	Class of People Exp. M(hr)	Absolute Error
1	Iron	4	4.23	4.23	0.07
2	Thorium	11	4.45	4.45	0.037
3	Lead	17	4.72	4.72	0.051
4	Calcite	21	4.89	4.89	0.089
5	Pyrite	26	5.01	5.01	0.125
					$\sum$ 0.372

Fig 2-2. The function M(hr) is determined based on the variable hr, and it represents the relationship between hr and the corresponding values of the mineral M.

Employing the graph-based approach for the interval in Table 2-3, our objective was to assess the average mineral content in urea samples for a specific study population. In this scenario, denotes the central measure for mineral rateM_l (hr), signifying the mineral radiation count within the sample population. From our examination, the equation formulated is as follows:

$$M_i(hr) = M_3(hr) + \frac{m(hr_4) - m(hr_3)}{(hr_4) - (hr_3)} hr \quad (3)$$

Table 2-3. M(hr) to hr and compare with the obtained experimental value using graph.

No	Name	Rate of mineral Ml	Class of People Exp. M(hr)	Class of People Det. M(hr)	Absolute Error
1	Iron	5	5.19	5.16	0.12
2	Thorium	9	5.31	5.28	0.041
3	Lead	18	5.36	5.31	0.049
4	Calcite	26	5.39	5.33	0.057
5	Pyrite	31	5.51	5.44	0.082
					$\sum$ 0.349

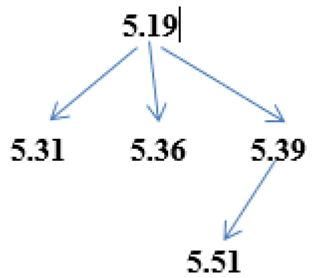


Fig. 2-3 The function M(hr) is determined based on the variable hr, and it represents the relationship between hr and the corresponding values of the mineral M.

Conclusion

The study investigated how radiation affects the physical properties of particular minerals and questioned which among them are more likely to be damaged if exposed to radiation. The research specifically covered five minerals: iron, thorium, lead, calcite and pyrite. The researchers tested this assumption by exposing samples of these minerals to radiation such as cesium radiation or X-rays and observing the physical and chemical changes that ensued.

The results of this study contribute to the understanding on the consequences of minerals radiation exposure. This knowledge has additional importance for diverse nuclear or industrial scenarios, where insights on proper use of minerals paramagnetic in radiation conditions come at hand.

The research utilized the plot approach to gauge and interpret the impacts of radiation interaction on the minerals. Through plotting influence curves for each mineral derived from the measured alterations in their physicochemical attributes, valuable data was obtained. Subsequently, the data was scrutinized to discern the variations in specific mineral characteristics, contingent on the degree of radiation exposure.

The research also included use of graph method which was used to determine the mineral radiation level in a given sample of individuals. The introduction of ten alternative methods for assessing mineral radiation was therefore considered. A comparison among these potentials was performed to determine the most accurate estimations.

The research findings showed that the best estimate is in interval [3,23] which provides minimum absolute error rate over all evaluated periods. Conversely, the smallest range [5,31] also result in worst case for absolute error rate. Thus, it is suggested to choose the path with lowest absolute error rate so that estimation of mineral radiation could be done more accurately.

References:

[1]A. L. Barabási and Z. N. Oltvai, "Network biology: Understanding the cell's functional organization," in IEEE Reviews in Biomedical Engineering, vol. 5, no. 2, pp. 101-113, 2004.

[2]E. Almaas, "Biological impacts and context of network theory," in IEEE/ACM Transactions on Computational Biology and Bioinformatics, vol. 4, no. 4, pp. 594-597, Oct.-Dec. 2007. doi: 10.1109/TCBB.2007.1036.

[3] N. Trinajstić, "Chemical graph theory," CRC Press, 1992.

[4] N. Deo, Graph Theory with Applications to Engineering and Computer Science. Prentice Hall, 2005.

[5] D. B. West, "Introduction to Graph Theory," Prentice Hall, 2001.

[6] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, "Network Flows: Theory, Algorithms, and Applications," Prentice Hall, 1993.

[7] R. Balakrishnan and K. Ranganathan, "A Textbook of Graph Theory," 2nd ed., Springer, New York, 2012.

[8] S. Balter, D. W. Fletcher, H. M. Kuan, D. Miller, D. Richter, H. Seissl, and T. B. Shope, "Skin Dose Measurements: AAPM Techniques to Estimate Radiation Dose to Skin during Fluoroscopically Guided Procedures α," in Proceedings of the IEEE International Symposium on Biomedical Imaging: From Nano to Macro, July 2002, pp. 1-10.

[9] N. Deo, "Graph Theory with Applications to Engineering and Computer Science," Prentice Hall, New Jersey, 2000.

[10] A. Dharwadker and S. Pirzada, "Applications of graph theory," J. Korean Soc. Ind. Appl. Math. (KSIAM), vol. 11, no. 4, 2007.

[11] N. Haregeweyn and F. Yohannes, "Testing and evaluation of agricultural non-point source pollution model (AGNPS) on Augucho catchment, Western Hararghe, Ethiopia," Agric. Ecosyst. Environ., vol. 99, pp. 201-212, 2003.

[12] S. Z. Ilyas, Sh. M. Nasir, and Sdik Kakac, "Estimation and Comparison of Diffuse Solar Radiation Over Pakistan," J. Alternative Energy and Ecology, vol. 3, no. 47, pp. 109-111, 2007.

[13] A. A. Kadhem, "Temperature estimation of EXDRA and SSUMI dwarf Nova systems from spectroscopic data," Iraqi J. Phys., vol. 13, no. 27, pp. 31-35, 2015.

[14] S. Ahmed, "Applications of Graph Coloring in Modern Computer Science," Int. J. Comp. Inf. Tech., vol. 3, no. 2, pp. 1-7, 2012.

[15] S. G. Shirinivas, S. Vetrivel, and M. Elango, "Applications of Graph Theory in Computer Science Review," Int. J. Eng. Sci. Tech., vol. 2, no. 9, pp. 4610-4621, 2010.

[16] M. G. Stabin, R. B. Sparks, and E. Crowe, "OLINDA/EXM: The Second-Generation Personal Computer Software for Internal Dose Assessment in Nuclear Medicine," J. Nucl. Med., vol. 46, pp. 1023-1027, 2005.

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

V-Constant Type of Conharmonic Tensor of Vaisman-Gray Manifold

Abdulhadi Ahmed Abd
Directorate General of Salahuddin Education
ba4117063@gmail.com

V-Constant Type of Conharmonic Tensor of Vaisman-Gray Manifold.

Abdulhadi Ahmed Abd
Directorate General of Salahuddin Education
ba4117063@gmail.com

Abstract. In this work we will study geometric conharmonic curvature tensor characteristics Viasman- Grey menifold ,and the constant of conharmonic type Vaisman-Gray Manifold conditions are obtained when the Viasman- Grey menifold is a manifold conharmonic constant type (V). Also,we will prove that M Vaisman-Gray Manifold of point wise constant holomorphic sectional conharmonic (PHKm(X)) – curvature) curvature tensor if the components of holomorphic sectional (HS- curvature) curvature tensor in the adjoined G-structure space that satisfies condition.

Keywords: Constant type, Vaisman-Gray Manifold, Pointwise holomorphic sectional.

1. Introduction

The Hermitian manifold is one of the most crucial topics of the Comparative geometry. This subject classified into various elements in trying to precisely determine its specifications and features. Then appeared important matter is the classification of the different classes of almost Hermitian manifold according to specific features. Many researchers studied the almost Hermitian manifold and they found many important geometrical properties. One of them is Russian scholars called Kirichenko, when he used G-structure space to study the almost Hermitian manifold that does not depend on a manifold itself but on a sub principle of all complicated frames' collective fiber bundle is known as the adjoined G-structure space[8]. We used this method to study the projective tensor of the class Vaisman-Gray manifold (VG-manifold). This class $W_1 \oplus W_4$,denotes this, where W_1 and W_4 corresponding to the neärly kohler menifold as well as the locälly conformäl kohler manefold (LCK-manifold)[2].

In 1994, Kirichenko and shchipkova, studied the class $W_1 \oplus W_4$ under the name Vaisman-Gray manifold. They found its structure equation in the adjoined G-structure space [6]. In 1996, Kirichenko and Eshova studied the conformal invariant of the class $W_1 \oplus W_4$ [7].

There are many researchers studied the geometry properties of the curvature tensors on almost Hermitian manifold. Ali Shihab [1] studied the geometry of conhormonic curvature tensor of almost Hermitian manifold. One of these curvature tensor is conharmonic tensor. kirichenk & shechepkova fund the equations of proper VG-manifold with respect to the structured & vertual teasers [3]. In particular, M. Vaisman-Gray was prove of point wise constant holomorphic sectional conharmonic (PHKm(X)) – curvature) curvature tensor if the components of holomorphic sectional (HS- curvature) curvature tensor in the adjoined G-structure space that satisfies condition.

2. Preliminaries

Make X(M) the smooth surface. vector field module of M. C^{∞}(M) be a set of operations on M. The Hermitain manifold (H M) be a set {M, J, g=⟨.,.⟩} where M is 2n-dimensional (n>1) smooth manifold; J is a tangent space endomorphism. T_P(M) with $(J_P)^2$=id and g=<.,.> Metric Riemann such that on M. ⟨JZ,JW⟩=⟨Z,W⟩;Z,W∊Z(M)[9].The basis $\{e_1 \dots, e_n \dots Je_1 \dots . Je_n\}$ is named an authentic competent AH- structural basis { J, g }, by using this basis, the new as a basis be constructed as follow $\{i_1, \dots i_n, \dots \bar{i}_1, \dots \bar{i}_n\}$.Where i_a=σ(e_a) and i_a= $\overline{\sigma}$ (e_a), this basis is known as an almost structure basis or almost basis. The equivalent of the frame is $\{P, \dots i_1, \dots, i_n \dots, \bar{i}_1, \dots, \bar{i}_n\}$ This is known as an A-frame.

The indicators u, g, l and p in the vicinity 1,…,2n and the indexes m, q , o, p, n, r, s and d We shall use the numbers 1,2,...,k. employ the markings $\{i_{\hat{1}} = \bar{i}_1, \dots . i_{\hat{n}} = \bar{i}_n\}$ where $\hat{a} = a + n$.than form can be used to write a-frame$\{p, i_1 \dots, i_n \dots, i_{\hat{1}}, \dots i_{\hat{n}}\}$. The components matrices of the complex structure f and y of adjoined the following forms of Q-structure space are:

$$(\langle JX, JY\rangle J^i_j) = \begin{pmatrix} \sqrt{-1}\, I_n & 0 \\ 0 & -\sqrt{-1}\, I_n \end{pmatrix}, \quad (g^i_j) = \begin{pmatrix} 0 & I_n \\ I_n & 0 \end{pmatrix},$$

Where I_n is the rank n unit matrix[8].

Definition 2.1 [10]:

A tensor of type (2,0) which is defined as is $r_{ij} = R^k_{ijk} = g^{kl} R_{kijl}$ called a Ricci tensor.

Definition 2.2 [4]:

In The adjoined G-structure space , the components of Ricci tensor of Viasman-Grey manifold are given as the following forms:

1- $r_{ab} = \frac{1-n}{2}(\alpha_{ab} + \alpha_{ba} + \alpha_a + \alpha_b)$

2- $r_{\hat{a}b} = r^a_b = 3B^{cah}B_{cbh} + A^{ca}_{cb} + \frac{n-1}{2}(\alpha^a\alpha_b - \alpha^h\alpha_h) - \frac{1}{2}\alpha^h{}_h\delta^a_b + (n-2)\alpha^a{}_b$

Definition 2.3 [4]:

Let (M,J, g) be a Vaisman- Gray Manifold .The Conharmonic curvature tensor of AH- manifold M of type (4, 0) which is defined as the following form:

$$T_{ijkl} = R_{ijkl} - \frac{1}{2(n-1)}[r_{il}g_{jk} - r_{jl}g_{ik} + r_{jk}g_{il} - r_{ik}g_{jl}] \qquad (1)$$

Where r, R and g are respectively Ricci tensor, Riemannian curvature tensor and Riemannian metric. And satisfies all the properties of algebraic curvature tensor:

$$\left.\begin{array}{l} 1)\ T(X,Y,Z,W) = -T(Y,X,Z,W); \\ 2)\ T(X,Y,Z,W) = -T(X,Y,W,Z); \\ 3)\ T(X,Y,Z,W) + T(Y,Z,X,W) + T(Z,X,Y,W) = 0 \\ 4)\ T(X,Y,Z,W) = T(Z,W,X,Y); \\ \qquad \forall X,Y,Z,W \in X(M) \end{array}\right\} \quad (2)$$

Theorem 2.4 [12]:

In the adjoined G-structure space, the components of Conharmonic tensor of VG-manifold are given by the following forms:

i) $T_{abcd} = 2(B_{ab[cd]} + \alpha_{[a}B_{b]cd})$;

ii) $T_{\hat{a}bcd} = 2A^{a}_{bcd} + \frac{1}{2(n-1)}(r_{bd}\delta^{a}_{c} - r_{bc}\delta^{a}_{d})$;

iii) $T_{\hat{a}\hat{b}cd} = 2\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d})$;

iv) $T_{\hat{a}bc\hat{d}} = A^{ad}_{bc} + B^{adh}B_{hbc} - B^{ah}{}_{c}B_{hb}{}^{d} - \frac{1}{(n-1)}(r^{(a}_{(c}\delta^{d)}_{b)})$;

1. Main results

Definition 3.1 [11]:

Suppose that $\lambda(X,Y,Z,W) = \lambda(X,Y,Z,W) - \lambda(X,Y,JZ,JW)$

Consider the following tensor $\lambda(X,Y) = \lambda(X,Y,Y,X)$

We say that an AH- manifold M is of constant type at $p \in M$

Provided that for all $X \in T_p(M)$

$$\lambda(X,Y) = \lambda(X,Z) \qquad (3)$$

Remark 3.2 [11]:

1- If (3) holds for all $p \in M$ then the manifold M has pointwise constant type.

2- If (3) is constant function, then (M,J,g) has a globally constant type .

Definition 3.3 [11]:

An AH- manifold M^{2n}n is conharmonic constant type (V- constant type)

If $X,Y,Z,W \in X(M^{2n})$. That

$$\lambda(X,Y,Z,W) = \lambda(X,Y,Z,W) - \lambda(X,Y,JZ,JW)$$

Consider the following tensor $\lambda(X,Y) = \lambda(X,Y,Y,X)$

We say that an AH- manifold M is of constant type at p

Provided that for all $X \in T_p(M)$

$\lambda(X,Y) = \lambda(X,Z)$.

Theorem 3.4

If M is Viasman-Grey manifold of the conharmonic tensor then M is manifold conharmonic constant if and only if

$$\lambda(X,Y) = \lambda(X,Z) = 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$$

Proof:

Suppose that M is Viasman-Grey manifold of conharmonic tensor,we find the following result:

By using definition (3.3) it follows :-

1- $\lambda(X,Y) = T(X,Y,Y,X) - T(X,Y,JY,JX)$

Let M^{2n} Viasman manifold to compute the $\lambda(X,Y,Y,X)$and $\lambda(X,Y,JY,JX)$ on the space of the adjoined G-structure

(i)

$T(X,Y,Y,X) = T_{ijkl}X^iY^jY^kX^l = T_{abcd}X^aY^bY^cX^d + T_{\hat{a}bcd}X^{\hat{a}}Y^bY^cX^d + T_{a\hat{b}cd}X^aY^{\hat{b}}Y^cX^d$ ·
$T_{ab\hat{c}d}X^aY^bY^{\hat{c}}X^d + T_{abc\hat{d}}X^aY^bY^cX^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^cX^d + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^bY^{\hat{c}}X^d + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^b$
$T_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}Y^{\hat{c}}X^d + T_{a\hat{b}c\hat{d}}X^aY^{\hat{b}}Y^cX^{\hat{d}} + T_{a\hat{b}\hat{c}\hat{d}}X^aY^{\hat{b}}Y^{\hat{c}}X^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}Y^{\hat{b}}Y^{\hat{c}}X^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^aY$
$T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Y^{\hat{b}}Y^{\hat{c}}X^d + T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Y^{\hat{b}}Y^cX^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Y^bY^{\hat{c}}X^{\hat{d}}$

By using the properties of conharmonic tensor equation (2), we get:

$T(X,Y,Y,X) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^bY^cX^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^bY^{\hat{c}}X^d + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^cX^d + T_{a\hat{b}c\hat{d}}X^aY^{\hat{b}}Y^cX$
$T_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}Y^{\hat{c}}X^d + T_{ab\hat{c}\hat{d}}X^aY^bY^{\hat{c}}X^{\hat{d}}$
(4)

(ii) $T(X,Y,JY,JX)$

In the space of the adjoined G-structure space

$T(X,Y,JY,JX) = T_{ijkl}X^iY^j(JY)^k(JX)^l = T_{abcd}X^aY^b(JY)^c(JX)^d + T_{\hat{a}bcd}X^{\hat{a}}Y^b(JY)^c(JX$
$T_{a\hat{b}cd}X^aY^{\hat{b}}(JY)^c(JX)^d + T_{ab\hat{c}d}X^aY^b(JY)^{\hat{c}}(JX)^d + T_{abc\hat{d}}X^aY^b(JY)^c(JX)^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y$
$T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^b(JY)^{\hat{c}}(JX)^d + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^b(JY)^c(JX)^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^aY^{\hat{b}}(JY)^{\hat{c}}(JX)^d + T_{a\hat{b}c\hat{d}}X^aY$
$T_{a\hat{b}\hat{c}\hat{d}}X^aY^{\hat{b}}(JY)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}Y^{\hat{b}}(JY)^{\hat{c}}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^aY^b(JY)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Y$
$T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Y^{\hat{b}}(JY)^c(JX)^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Y^b(JY)^{\hat{c}}(JX)^{\hat{d}}$

By using the properties of conharmonic tensor equation(2), we get:

$$T(X,Y,JY,JX) = T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}(JY)^{c}(JX)^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}(JY)^{\hat{c}}(JX)^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}(JY)^{c}($$
$$T_{a\hat{b}\hat{c}d}X^{a}Y^{\hat{b}}(JY)^{\hat{c}}(JX)^{d} + T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}(JY)^{c}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Y^{b}(JY)^{\hat{c}}(JX)^{\hat{d}}$$
(5)

According to the properties $(JX)^{a} = \sqrt{-1}\,X^{a}$ and $(JX)^{\hat{a}} = -\sqrt{-1}\,X^{\hat{a}}$ we get:

$$T(X,Y,JY,JX) = -T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}Y^{\hat{c}}X^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}Y^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X$$
$$T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}Y^{c}X^{\hat{d}} - T_{ab\hat{c}\hat{d}}X^{a}Y^{b}Y^{\hat{c}}X^{\hat{d}}$$

Making use of the equation (4) and (5), we get :

$$T(X,Y,Y,X) - T(X,Y,JY,JX) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}Y^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}Y^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d} +$$
$$T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}Y^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Y^{\hat{b}}Y^{\hat{c}}X^{d} + T_{ab\hat{c}\hat{d}}X^{a}Y^{b}Y^{\hat{c}}X^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d} - T_{\hat{a}b\hat{c}d}X^{\hat{a}}Y^{b}$$
$$T_{\hat{a}bc\hat{d}}X^{\hat{a}}Y^{b}Y^{c}X^{\hat{d}} - T_{a\hat{b}\hat{c}d}X^{a}Y^{\hat{b}}Y^{\hat{c}}X^{d} - T_{a\hat{b}c\hat{d}}X^{a}Y^{\hat{b}}Y^{c}X^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Y^{b}Y^{\hat{c}}X^{\hat{d}}$$
$$= 4T_{\hat{a}\hat{b}cd}X^{\hat{a}}Y^{\hat{b}}Y^{c}X^{d}$$

This is the V_4 of theory (2.4) equation (iii) and the compensation, we get:

$$= 4\left(2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\,)\right)$$

$$= 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right) \qquad (6)$$

2- $\lambda(X,Z) = T(X,Z,Z,X) - T(X,Z,JZ,JX)$

Let M^{2n} Viasman manifold to compute the $\lambda(X,Z,Z,X)$ and $\lambda(X,Z,JZ,JX)$ on the space of the adjoined G-structure

(i)

$$T(X,Z,Z,X) = T_{ijkl}X^{i}Z^{j}Z^{k}X^{l} = T_{abcd}X^{a}Z^{b}Z^{c}X^{d} +$$
$$T_{ab\hat{c}d}X^{a}Z^{b}Z^{\hat{c}}X^{d} + T_{abc\hat{d}}X^{a}Z^{b}Z^{c}X^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}$$
$$T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}\hat{d}}X^{a}Z^{\hat{b}}Z$$
$$T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Z^{b}Z$$

By using the properties of conharmonic tensor equation (2), we get:

$T(X,Z,Z,X) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z$
$T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}}$ (7)

(ii) $T(X,Z,JZ,JX)$

In the space of the adjoined G-structure space

$T(X,Z,JZ,JX) = T_{ijkl}X^{i}Z^{j}(JZ)^{k}(JX)^{l} = T_{abcd}X^{a}Z^{b}(JZ)^{c}(JX)^{d} + T_{\hat{a}bcd}X^{\hat{a}}Z^{b}(JZ)^{c}(JX)$
$T_{a\hat{b}cd}X^{a}Z^{\hat{b}}(JZ)^{c}(JX)^{d} + T_{ab\hat{c}d}X^{a}Z^{b}(JZ)^{\hat{c}}(JX)^{d} + T_{abc\hat{d}}X^{a}Z^{b}(JZ)^{c}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}$
$T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}(JZ)^{\hat{c}}(JX)^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}(JZ)^{c}(JX)^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}$
$T_{a\hat{b}\hat{c}\hat{d}}X^{a}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}\hat{d}}X^{\hat{a}}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}(JZ)^{\hat{c}}(JX)^{\hat{d}} + T_{\hat{a}\hat{b}\hat{c}d}X^{\hat{a}}Z^{\hat{b}}$
$T_{\hat{a}\hat{b}c\hat{d}}X^{\hat{a}}Z^{\hat{b}}(JZ)^{c}(JX)^{\hat{d}} + T_{\hat{a}b\hat{c}\hat{d}}X^{\hat{a}}Z^{b}(JZ)^{\hat{c}}(JX)^{\hat{d}}$

By using the properties of conharmonic tensor equation (2), we get:

$T(X,Z,JZ,JX) = T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}(JZ)^{c}(JX)^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}(JZ)^{\hat{c}}(JX)^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}(JZ)^{c}(J$
$T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}(JZ)^{\hat{c}}(JX)^{d} + T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}(JZ)^{c}(JX)^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}(JZ)^{\hat{c}}(JX)^{\hat{d}}$

According to the properties $(JX)^{a} = \sqrt{-1}\,X^{a}$ and $(JX)^{\hat{a}} = -\sqrt{-1}\,X^{\hat{a}}$ we get:

$T(X,Z,JZ,JX) = -T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} + T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X$
$T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} - T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}}$ (8)

Making use of the equation (7) and (8), we get :

$T(X,Z,Z,X) - T(X,Z,JZ,JX) = T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} + T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z^{b}Z^{\hat{c}}X^{d} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X$
$T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}} + T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d} - T_{\hat{a}b\hat{c}d}X^{\hat{a}}Z$
$T_{\hat{a}bc\hat{d}}X^{\hat{a}}Z^{b}Z^{c}X^{\hat{d}} - T_{a\hat{b}\hat{c}d}X^{a}Z^{\hat{b}}Z^{\hat{c}}X^{d} - T_{a\hat{b}c\hat{d}}X^{a}Z^{\hat{b}}Z^{c}X^{\hat{d}} + T_{ab\hat{c}\hat{d}}X^{a}Z^{b}Z^{\hat{c}}X^{\hat{d}}$
$= 4T_{\hat{a}\hat{b}cd}X^{\hat{a}}Z^{\hat{b}}Z^{c}X^{d}$

This is the V_4 of theory (2.4) equation (iii) and the compensation, we get:

$$= 4\left(2(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\,)\right)$$

$$= 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right) \qquad (9)$$

From equation (6) and (9) it follows:

$$\lambda(X,Y) = \lambda(X,Z) = 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$$

Thus by definition (3.3) we get:

M is constant type if and only if

$$\lambda(X,Y) = \lambda(X,Z) = 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$$

Lemma 3.5 [5]:

An AH- manifold M is a zero constant type if, and only if, M is Kahler manifold.

Corollary 3.6:

If M is VG-manifold of conharmonic tensor , then M is not Kahler manifold.

Proof:

Let that M is VG-manifold of conharmonic tensor

By using Theorem (3.4) we get:

M is constant type

$$\lambda(X,Y) = \lambda(X,Z) = 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$$

By using Lemma (3.5) it follows

M is not Kähler manifold .

Conclusions

1- Prove that if M is Viasman-Grey manifold of the conharmonic tensor then M is manifold conharmonic constant if and only if

$$\lambda(X,Y) = \lambda(X,Z) = 8\left(-B^{abh}B_{hcd} + \alpha^{[a}_{[c}\delta^{b]}_{d]}\right) - \frac{1}{(n-1)}\left(r^{[a}_{d}\delta^{b]}_{c} - r^{[b}_{c}\delta^{a]}_{d}\right)$$

2- Prove that if M is VG-manifold of conharmonic tensor , then M is not Kahler manifold.

References

[1] Ali Shihab A. Geometry of the Conharmonic curvature tensor of almost hermitian manifold. Mat. Zametki. 2011;90(1):87-103.

[2] Gray A. Sixteen classes of almost Hermitian manifold and their linear invariants. Ann. Math. Pure and Appl. 1980;123(3):35-58.

[3] Ignatochkina L., New aspects of geometry of Vaisman-Gray manifold, Ph.D. thesis, Moscow State Pedagogical University, Moscow, 2001.

[4] Ishi Y., On conharmonic transformation, Tensor N. S. 7, (1957), 73-80.

[5] Kirichenko V.F. and Vlasova L.I., "Concircular geometry of nearly Kähler manifold " ,Sb. Math. 193(5) (2002), 685-707.

[6] Kirichenko V. F. and Shipkova N. N., On Geometry of Vaisman-Gray manifold, YMN. V. 49(2) (1994), 161-162.

[7] Kirichenko V. F., Ishova N. A., Conformal invariant of Vaisman-Gray manifold, UMN. 51(2) (1996), 331-332.

[8] Kirichenko VF. New results of K–spaces theory. Ph. D. thesis, Moscow state University; 1975.

[9] Kobayashi S, Nomizu K. Foundations of differential geometry, Vol. I, John Wily and Sons.1963.

[10] Rachevski P. K., Riemannian geometry and tensor analysis, Uspekhi Mat. Nauk, V. 10, Issue 4(66)(1955), 219-222.

[11] Saadan M.J.,"Complex Concircular Curvature Tensor of Certain Classes of Almost Hermition Manifold", (Master thesis). University of Basrah, 2011.

[12] Yasir Ahmed Abdulameer (2016) "On Geometry of Almost Hermitian Manifold". (Master thesis) .University of Basrah.

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

Some Grill of Nano Topological Space

Ekram A. Saleh[1, *], Leqaa M. Saeed Hussein[2], Taha Hameed Jasim Al-Douri[3]

[1]Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Mosul, Iraq.
[2]Department of Mathematics, College of Basic Education, University of Telafer, Mosul, Iraq.
[3]Department of Mathematics, College of Computer Sciences and Mathematics, University of Tikrit, Tikrit, Iraq
*Corresponding author: ekram.math@uomosul.edu.iq

Some Grill of Nano Topological Space

Ekram A. Saleh[1, *], Leqaa M. Saeed Hussein[2], Taha Hameed Jasim Al-Douri[3]

[1]Department of Mathematics, College of Computer Sciences and Mathematics, University of Mosul, Mosul, Iraq.
[2]Department of Mathematics, College of Basic Education, University of Telafer, Mosul, Iraq.
[3]Department of Mathematics, College of Computer Sciences and Mathematics, University of Tikrit, Tikrit, Iraq
*Corresponding author: ekram.math@uomosul.edu.iq

Abstract. In this work, we made a concept game of $\mathfrak{S}$ -nano g-open sets "employing the notion of grill nano topological space, or $G(NT_i.\mathfrak{S})$, where $i = \{0.1.2\}$. The relationships between various kinds of games have been researched with the use of numerous figures and propositions while providing similar examples.

Keywords. $\mathfrak{S}Ng$-closed set, $\mathfrak{S}Ng$-open set, $G(NT_0.\mathfrak{S})$, $G(NT_1.\mathfrak{S})$and $G(NT_2.\mathfrak{S})$.

1. Introduction

Choquet [1] studied grill ($\mathfrak{S}$) on a topological space $(X.\tau)$that has already been explored. In [2] a nano topological space was defined using lower, upper, and boundary conditions. In [3] a game was studied and the concepts of grill Ti -space where $i = \{0.1.2\}$ and denoted by $G\ (Ti.X)$. In [4] introduced grill g-open set on the game of the generalized , grill-g-closed set and insert $\mathfrak{S} - g - Ti$-space with $i = \{0.1.2\}$ were examined, and a game $G(T_i.\mathfrak{S})$ was defined. In [5] introduced the game denoted by (G) between "two "players Åand $\mathcal{B}$, the range of options $J_1.J_2.J_3 \dots J_n$ for every Player. These possibilities are referred to as moves.In [6,7] studied a game is defined as alternating when one of the Players Å chooses one of the options $J_1.J_2.J_3.\dots J_n$. Can be chosen by $\mathcal{B}$ when the choices of Å are Known. In alternating games, the player must determine who starts the game. In this paper provided the sorts of games through a given set. The gaining and losing strategy of any player $\mathcal{P}$ in the game G , if $\mathcal{P}$ has a gaining strategy in G denoted by ($\mathcal{P} \hookrightarrow G$). On the other hand, if $\mathcal{P}$doesn't have a gaining strategy denoted by ($\mathcal{P} \not\hookrightarrow G$). if $\mathcal{P}$ has a losing strategy denoted by ($\mathcal{P} \hookleftarrow G$) and if $\mathcal{P}$ doesn't have a losing strategy denoted by ($\mathcal{P} \not\hookleftarrow G$).

2. Preliminaries

Definition 2.1 [2] Let R be an equivalence relation on U known as the "indiscernibility relation," and let U be a non-empty finite set of objects termed the universe. Then different equivalence classes for U are created. It is argued that elements in the same equivalence class are indistinguishable from one another.

- "The approximation space is referred to as the pair $(U.R)$. Let $X \subseteq U$". The set of all objects that can be categorically identified as X with regard to R is the lower approximation of X with respect to R, and it is denoted by "$L_R(X)$. To put it another way, L_R (X)=_ $\cup_{x\in U}\{R(x) : R(x) \subseteq X\}$, where R(x) stands for the equivalence class established by x U.

- According to $U\_R(X)$, "the set of all objects that can possibly be classified as X with respect to R and" it is the upper approximation of X with respect to R. This is,
  $U_R\ (X) = \cup_{x \in U} \{R(x): R\ (x) \cap X \neq \varphi\ \}$
- The collection of all objects that cannot be classified as either X or not-X with regard to R is known as the boundary region of X with respect to R and is indicated by the symbol $B_R(X)$ thus $B_R(X) = U_R\ (X) - L_R\ (X)$. is defined.

Definition 2.2 [2] The set of all objects that may conceivably be categorized as X with respect to R and it, as denoted by $U\_R(X)$, is the upper approximation of X with regard to R. That is, suppose U is a"universe, R be an equivalence relation on U and" $\tau_R\ (X) = \{\varphi. U. L_R\ (X). U_R\ (X). B_R\ (X)\}$where X agree with the following axioms.

- $U. \varphi \in \tau_R(X)$
- The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
- The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in τ_R(X).

Then $\tau_R(X)$ is called the Nano topology on U with respect to X. The space $(U. \tau_R(X))$ is the Nano topological space. The elements of $\tau_R(X)$ are called Nano open sets.

Definition 2.3 [1,8] A nonempty collection $\mathfrak{S}$ of nonempty subsets of a topological space $\mathfrak{X}$ is named a grill if

- $A\ \in \mathfrak{S}\ and\ A\ \subseteq\ B \subseteq \mathfrak{X}$ then $B \in \mathfrak{S}$.
- $A\ .B \subseteq \mathfrak{X}$ and $A\ \cup\ B \in \mathfrak{S}$ then $A\ \in \mathfrak{S}\ or\ B \in \mathfrak{S}$ [6].

Let $\mathfrak{X}$ be a nonempty set. Then the following families are grills on $\mathfrak{X}$. [1,67]

Definition 2.4 [2] In space $(\mathfrak{X}, T, \mathfrak{S})$, let $D \subseteq \mathfrak{X}$. D is named to be grill-g- closed set denoted by " $\mathfrak{S}$ - g-closed", if (D- U) $\notin$ $\mathfrak{S}$ then, $(cl(D)\ -\ U) \notin \mathfrak{S}$ where, $U \subseteq \mathfrak{X}$ and $U \in t$ Now, D^c is a grill-g- open set denoted by $\mathfrak{S}$ -g-open". The family of all " $\mathfrak{S}$ - g –closed" sets denoted by $\mathfrak{S}gC(\mathfrak{X})$. The family of all " $\mathfrak{S}$ -g-open" sets denoted by $\mathfrak{S}$ $\mathcal{Y}$O ($\mathfrak{X}$)

Definition 2.5 [4] The space $(\mathfrak{X}, T, \mathfrak{S})$is a $\mathfrak{S}$ -g-$\mathcal{T}_0$-space shortly $\mathfrak{S}$ -g-$\mathcal{T}_0$-space" if for each $m \neq \mathfrak{o}$ and $m. \mathfrak{o}\ \in X$, there exist $U \in\ \mathfrak{S}\ gO(\mathfrak{X}\ )$ whenever, $m\ \in U$ and $\mathfrak{o} \notin U$ or $m \notin U$ and $\mathfrak{o}\ \in U$.

Definition 2.6 [4] The space $(\mathfrak{X}. T. \mathfrak{S})$is a $\mathfrak{S}$ g $\mathcal{T}_1$,-space shortly $\mathfrak{S}$ g-$\mathcal{T}_1$-space" if for each m, $\mathfrak{o} \in X$ and $m \neq \mathfrak{o}$. Then there are $\mathfrak{S}$ g-open sets U_1, U_2 whenever $m \in U_1$, $\mathfrak{o} \notin U_1$, and $\mathfrak{o} \in U_2, m \notin U_2$.

Definition 2.7 [4] The space $(\mathfrak{X}. T. \mathfrak{S})$ is a $\mathfrak{S}g$ $\mathcal{T}_2$-space shortly " $\mathfrak{S}$ g $\mathcal{T}_2$-space" if for each $m \neq \mathfrak{o}$. There are $\mathfrak{S}$ $\mathcal{Y}$-open sets U_1, U_2 whenever me $m \in U_1$, $\mathfrak{o} \in U_2$, $U_1 \cap U_2$ =$\emptyset$

3. Grill Nano g-open –on Game

Definition 3.1 Let $(U,\tau_R(x),\mathfrak{S})$ be grill Nano topological space and $E \subseteq U$,E is called "grill Nano -g-closed set" denoted by $\mathfrak{S}Ng$-closed if $(E - G) \notin \mathfrak{S}$ thans $(CL(E) - G) \notin \mathfrak{S}$ where $G \subseteq U$ and , $G \in \tau_R(x).E^c$ as "grill Nano g -open set denoted by $\mathfrak{S}Ng$ –open" . The family of all "grill Nano -g-closed set denoted by $\mathfrak{S}NgC(U)$ .The family of all "grill Nano -g-open set" denoted by $\mathfrak{S}NgO(U)$.

Example 3.2 Let (U,$\tau_R(x).\mathfrak{S}$) be grill Nano topological space
$U = \{a_1 . a_2 . a_3\}$. U /R= $\{\{a_1\}. \{a_1 . a_3\}\}$ $X = \{a_1 . a_3\} \subseteq U$.
$\tau_R(x) = \{\emptyset . U. \{a_1\}. \{a_2\} \quad \{a_1 . a_2\}\}$
$\tau_R(x) - closed = \{\emptyset . U. \{a_2 . a_3\}. \quad \{a_1 . a_3\}. \{a_3\}\}$
$\mathfrak{S} = \{U. \{a_1\}. \{a_1 . a_2\}. \{a_1 , a_3\}\}$
Then $\mathfrak{S}NgC(U)$)= P(X) / $\{\emptyset\}$ and $\mathfrak{S}NgC(U)$ is $(\mathfrak{S}NgO(U))^c$

Remark 3.3 For any $(U,\tau_R(x).\mathfrak{S})$ then

- Each Nano closed set is a $\mathfrak{S}Ng$- closed set
- Each Nano open set is a $\mathfrak{S}Ng$-open set.

Convers above Remark is not true. Shows from exam 3.2

- $\{a_1\}$ is $\mathfrak{S}NgC$ but $\{ a_1\}$ is not Nano closed set.
- $\{a_1 . a_3\}$ is $\mathfrak{S}NgO$ set but $\{a_1 . a_3\}$ is not Nano open.

Definition 3.4 Let $(U,\tau_R(x).\mathfrak{S})$ be grill Nano $g - T_0$space denoted by $\mathfrak{S}Ng - T_0$-space if for every i≠j and i≠j∈ $U . \exists\, G \in \mathfrak{S}NgO(U)$ whenever $i \in G and\, j \notin G\, or\, i \notin G and\, j \in G$.

Definition 3.5 Let $(U, \tau_R(x).\mathfrak{S})$be grill Nano $g - T_1$ space denoted by $\mathfrak{S}Ng - T_1$ -space if for every i≠j and i,j∈ $U . \exists\, \mathfrak{S}NgO\, sets\, G_1 . G_2$ whenever $i \in G_1 and\, j \notin G_1\, and\, i \notin G_2 . j \in G_2$

Definition3.6 Let $(U,\tau_R(x).\mathfrak{S})$ be grill Nano $g - T_2$ space denoted by $\mathfrak{S}Ng - T_2$ -space if for every i≠j then are $\mathfrak{S}NgO\, sets\, G_1 . G_2$ whenever $i \in G_1 and\, j \in G_2\, and\, G_1 \cap G_2 = \emptyset$.

Definition 3.7 Let $(U, \tau_R(x),\mathfrak{S})$ be a grill Nano topological space, $G(NT_0.\mathfrak{S})$ is a game that is defined as follows :In the m-th inning, the two players A and B will play an inning for each natural number., the prime race, A will select $a_m \neq b_m$, whenever a_m, b_m belong to U. Next B choose NG_m belong to $\mathfrak{S}NgO(U)$ such that a_m belong to $NG_m$and b_m, not belong to NG_m, B get in the game, whenever $\mathcal{B}$ =\{ $NG_1, NG_2, \dots\dots, NG_m \dots$\} satisfies that for all $a_m \neq b_m$in U ∃ NG_m belong to P such that a_m belong to NG_m and $b_m \notin NG_m$. Other hand A gets.

Example 3.8 Let $G(NT_0.\mathfrak{S})$ be a game $U = \{a_1 . a_2 . a_3\}$ and $U/R = \{\{a_2\}. \{a_1 . a_3\}\}$
$X = \{a_1 . a_2\} \subseteq U$ then $\tau_R(x) = \{\emptyset . U, \{a_2\}. \{a_1 . a_3\}\}$
$\tau_R(x)$ –closed= $\{\emptyset . U, . \{a_1 . a_3\}. \{a_2\}\}$, $\mathfrak{S} = \{U, \{a_1\}. \{a_1 . a_2\}. \{a_1 . a_3\}\}$

- Then $\mathfrak{S}NgC(U) = \{U , \emptyset . \{a_1\}. \{a_2\}. \{a_1 , a_2\}. \{a_1 . a_3\}\}$
- $\mathfrak{S}NgO(U) = \{\emptyset . U, \{a_2 . a_3\}. \{a_1 . a_3\}. \{a_3\}. \{a_2\}\}$

Then in the first race A shall choose $a_1 \neq a_2$ whenever $a_1 . a_2 \in U$ following B choose $\{a_2 . a_3\} \in \mathfrak{S}NgO(U)$ such that $a_2 \in \{a_2 . a_3\}$ and $a_1 \notin \{a_2 . a_3\}$in the other race A shall choose $a_1 \neq a_3$ whenever $a_1 . a_3 \in U$ following B choose $\{a_2 . a_3\} \in \mathfrak{S}NgO(U)$such that $a_2 \in \{a_2 . a_3\}$ and $a_1 \notin \{a_2 . a_3\}$in the tertiary race , A shall choose $a_2 \neq a_3$whenever $a_2 . a_3 \in U$ following B choose $\{a_1 . a_3\} \in \mathfrak{S}NgO(U)$ such that $a_3 \in \{a_1 . a_3\}$and $a_2 \notin \{a_1 . a_3\}$ B get in the game ,whenever $\mathcal{B} = \{\{a_2 . a_3\} . \{a_2 . a_3\}\}$ satisfies that for all $a_m \neq b_m$ in $U \exists NG_m \in \mathcal{B}$ such that $a_m \in NG_m$ and $b_m \notin NG_m$ whenever $NG_m \in \mathfrak{S}NgO(U)$ so B is the getter of the game .

Theorem 3.9 Let $(U, \tau_R(x), \mathfrak{S})$ be $\mathfrak{S}NgT_0$- space if and only if B $\hookrightarrow G(NT_0 . \mathfrak{S})$
Proof. since $(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_0$- space, then any choice for the primary player A in the m-th inning $a_m \neq b_m$ whenever $a_m . a_m \in U$. The other it is possible to locate player B. $NG_m \in \mathfrak{S}NgO(U)$ so $\mathcal{B} = \{ NG_1 . NG_2 \ldots \ldots NG_m \ldots\}$is the gaining strategy for B . Contrary to lucid .

Theorem 3.10 The space(U $\tau_R(x), \mathfrak{S})$ is a $GNgT_0$- space if and only if. there is a $\mathfrak{S}Ng - closed$ set containing only one of the items $a \neq b$.
Proof. Suppose that two points are a and b. belong to U with $a \neq b$ since U is $\mathfrak{S}NgT_0$- space . $\exists G$ *is a* $\mathfrak{S}Ng - open$ set contain only one of them ,therefore $(U - G)$ is $\mathfrak{S}Ng - closed$ set contain the other one .Contrary to Suppose that a and b are two points belong to U with $a \neq b$. $\exists H$ *is a* $\mathfrak{S}Ng - closed$ set contain only one of them ,therefore $(U - H)$is $\mathfrak{S}Ng - open$ set contain the other one .

Corollary 3.11 Let $(U, \tau_R(x) . \mathfrak{S})$ be a grill Nano topological space, B $\hookrightarrow G(NT_0 . \mathfrak{S})$ if and only if, for each $a \neq b$ of U $\exists H \in \mathfrak{S}NgO(U)$ such that $a \in H$ *and* $b \notin H$.
Proof. Suppose that $a \neq b$ with $a . b \in U$.since B $\hookrightarrow G(NT_0 . \mathfrak{S})$ then by Theorem 3.9 the space$(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_0$- space therefore theorem 3.10 is applicable. Contrary to: by theorem 3.10 the grill Nano topological-space $(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_0$- space, therefore Theorem 3.9 is applicable.

Theorem 3.12 $(U, \tau_R(x), \mathfrak{S})$ be not a $\mathfrak{S}NgT_0$- space iff $A \hookrightarrow G(NT_0, \mathfrak{S})$.
Proof. Of the m-th race A of $G(NT_0 . \mathfrak{S})$ choose $a_m \neq b_m$ whenever $a_m . b_m \in U$. B of $G(NT_0 . \mathfrak{S})$ cannot be founder G_m *is a* $\mathfrak{S}Ng - open\ set$ contain only one point of them, because ($U, \tau_R(x) . \mathfrak{S})$ be not a $\mathfrak{S}NgT_0$- space then $A \hookrightarrow G(NT_0 . \mathfrak{S})$ Contrary to lucid .

Definition 3.13 Let (U $\tau_R(x) . \mathfrak{S})$ be a grill Nano "topological space", and describe the game $G(NT_1 . \mathfrak{S})$ as follows: the two players A and B compete in a race for all natural numbers, with the m-th race, the prime round, being the most difficult. A shill picks $a_m \neq b_m$, whenever $a_m . b_m$. belong to U. Therefore B choose G_m *and* H_m, belong to $\mathfrak{S}NgO(U)$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$,B get in the game whenever $\mathcal{B} = \{\{G_1 - H_1\}, \{G_2 - H_2\}, \ldots, \{G_m - H_m\}, \ldots\}$ satisfies that for all $a_m \neq b_m$ of U $\exists \{G_m . H_m\} \in \mathcal{B}$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$, other hand A get .

Example 3.14 From Example 3.8
$\mathfrak{S}NgC(U) = \{U, \emptyset . \{a_1\}, \{a_2\}, \{a_1 . a_2\}, \{a_1 . a_3\}\}$

$\mathfrak{S}NgO(U) = \{\emptyset, U, \{a_2 . a_3\}, \{a_1 . a_3\}, \{a_3\}, \{a_2\}\}$

Then in the prime race A shall choose $a_1 \neq a_2$ whenever a_1 *and* $a_2 \in U$ therfore B choose $\{a_2 . a_3\}$and$\{a_1 . a_3\} \in \mathfrak{S}NgO(U)$) such that $a_1 \in (\{a_1 . a_3\}$-$\{a_2 . a_3\})$ and $a_2 \in (\{a_2 . a_3\}$-$\{a_1 . a_3\})$in the other race A shall choose $a_1 \neq a_3$ whenever a_1 *and* $a_3 \in U$ therfore B can't find $G_m . H_m \in \mathfrak{S}NgO(U)$, such that $a_1 \in (G_m - H_m)$ and $a_3 \in (H_m - G_m)$ then A get in the game .

Theorem 3.15 $(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space if and only if $B \hookrightarrow G(NT_0 . \mathfrak{S})$.

Proof. Suppose that $(U, \tau_R(x) . \mathfrak{S})$ be a grill Nano topological space in the prime run A shall select $a_1 \neq b_1$ whenever a_1 *and* $b_1 \in U$,therefore ,since $(U \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space B can be founder $G_1 . H_1 \in \mathfrak{S}NgO(U)$ such that $a_1 \in (G_1 - H_1)$ and $b_1 \in (H_1 - G_1)$ in the other race A shall choose $a_2 \neq b_2$whenever a_2 *and* $b_2 \in U$ therefore can be founder $G_2 . H_2 \in \mathfrak{S}NgO(U)$ such that $\notin_2$ $(G_2 - H_2)$ and $b_2 \in (H_2 - G_2)$ in the m-th race , A shall choose $a_m \neq b_m$ whenever a_m *and* $b_m \in U$ therefore B can be founder $G_m . H_m \in \mathfrak{S}NgO(U)$ such that $a_m \in (G_m - H_m)$ and $b_m \in (H_m - G_m)$. So $\mathcal{B} = \{\{G_1 . H_1\}, \{G_2 . H_2\}, \ldots, \{G_m . H_m\}, \ldots\}$ is the gaining strategy for B. Contrary to lucid

Theorem 3.16 $(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_1$ - space if and only if for every point $a \neq b$ $\exists$ two $\mathfrak{S}Ng - closed$ sets $K_1 and\ K_2$ such that $a \in (K_1 - K_2)$ and $b \in (K_2 - K_1)$.

Proof Suppose that a and b are two points of U with $a \neq b$ since U is a $\mathfrak{S}NgT_1$ - space then $\exists\ G_1 and\ G_2\ are\ \mathfrak{S}Ng - open$ sets such that $a \in (G_1 - G_2)$ and $b \in (G_2 - G_1)$.then $\exists$ $\mathfrak{S}Ng - closed$ sets $(U - G_1)$ and $(U - G_2)$, such that $a \in \{(U - G_2) -(U - G_1)\}$ and $b \in \{(U - G_1) -(U - G_2)\}$ whenever $(U - G_2) = K_1$ and $(U - G_1) = K_2$. Then $\exists$ two $\mathfrak{S}Ng - closed$ sets $(K_1 and\ K_2)$ satisfy $a \in (K_1 \cap K_2^c)$ and $b \in (K_2 \cap K_1^c)$ then $a \in (K_1 - K_2)$ and $b \in (K_2 - K_1)$. contrary to suppose that a and b are two points of U with $a \neq b$ $\exists$ two $\mathfrak{S}Ng - closed$ sets $K_1 and\ K_2$ satisfy $a \in (K_1 \cap K_2^c)$ and $b \in (K_2 \cap K_1^c)$ then $\exists \mathfrak{S}NgO(U - K_1)$ and $(U - K_2)$ whenever a $\in \{(U - K_2) -(U - K_1)\}$ and $b \in \{(U - K_1) -(U - K_2)\}$ whenever $(U - K_2) = G_1$and $(U - K_1) = G_2$.

Corollary3.17 Let $(U, \tau_R(x) . \mathfrak{S})$ be space, B $\hookrightarrow G(NT_1 . \mathfrak{S})$ if and only if for each $a_1 \neq a_2$ of U $K_2, K_1 \in \mathfrak{S}NgO(U)$, such that $a_1 \in (K_1 - K_2)$ and $a_2 \in (K_2 - K_1)$.

Proof. suppose that $a_1 \neq b_1$ with $a_1 . b_1 \in U$ since B $\hookrightarrow G(NT_1 . \mathfrak{S})$ so by-theorem 3.15 the space $(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space. So, Theorem 3.16 is, applicable. contrary to Theorem 3.16 the grill Nano topological-space $(U, \tau_R(x) . \mathfrak{S})$ is a $\mathfrak{S}NgT_1$- space so theorem 3.15 is, applicable.

Definition 3.18 Let $(U \tau_R(x) . \mathfrak{S})$ be a grill Nano topological space, $G(NT_2 . \mathfrak{S})$ is a game defined as follows: In the m-th race, the prime round, the two players A and B compete in a race for each natural number. A shill picks $a_m \neq b_m$, whenever $a_m . b_m$. belong to U. Therefore B choose disjoin G_m *and* H_m. $\boldsymbol{i.e} G_m \cap H_m = \varphi$ belong to $\mathfrak{S}NgO(U)$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$,B get in the game whenever $\mathcal{B} = \{\{G_1 - H_1\}, \{G_2 - H_2\}, \ldots, \{G_m - H_m\}, \ldots\}$ satisfies that for all $a_m \neq b_m$ of U $\exists \{G_m . H_m\} \in \mathcal{B}$ such that $a_m \in (G_m - H_m)$, and $b_m \in (H_m - G_m)$, other hand A get .

Example 3.19 From Example 3.8

$\mathfrak{S}NgC(U) = \{U, \emptyset, \{a_1\}, \{a_2\}, \{a_1 . a_2\}, \{a_1 . a_3\}\}$

$\mathfrak{S}NgO(U) = \{\emptyset, U, \{a_{2\,.}\, a_3\}, \{a_{1\,.}\, a_3\}, \{a_3\}, \{a_2\}\}$

Then in the prime race A shall choose $a_1 \neq a_2$whenever a_1 and $a_2 \in U$ therfore B can't find two disjoin $\mathfrak{S}NgO(U)$ with $a \in G_m$, $b \in H_m$i.e., $G_m \cap H_m = \emptyset$ then A is get in the game.

Theorem 3.20 A space $(U, \tau_R(x).\mathfrak{S})$ is $\mathfrak{S}NgT_2$- space if and only if B $\hookrightarrow G(NT_2.\mathfrak{S})$.
Proof. Suppose that $(U, \tau_R(x).\mathfrak{S})$ a grill Nano topological space in the prime race A shall choose $a_1 \neq b_1$ whenever $a_1 and\, b_1 \in U$ therefore .Since$(U,\tau_R(x).\mathfrak{S})$ is a $\mathfrak{S}NgT_2$- space then B can be found G_1 and $K_1 \in \mathfrak{S}NgO(U)$ such that $a_1 \in G_1$ and $b_1 \in K_1$.$G_1 \cap K_1 = \emptyset$ in the other race A shall choose $a_2 \neq b_2$,whenever $a_2 and\, b_2 \in U$ .Therefore B choose G_2 , $K_2 \in \mathfrak{S}NgO(U)$ such that $a_2 \in G_2$ and $b_2 \in K_2$, $G_2 \cap K_2 = \emptyset$ in the m-th race A shall choose $a_m \neq b_m$ whenever a_m and $b_m \in U$, therefore B choose G_m ,$K_m \in \mathfrak{S}NgO(U)$ such that $a_m \in G_m$ and $b_m \in K_m$, $G_m \cap K_m = \emptyset$. So $\mathcal{B} = \{\{G_1 , K_1\},\{G_2 , K_2\},\dots\dots\dots.\{G_m , K_m\}\dots\dots.\}$. Is the winning strategy for B. Contrary to is Lucid.

Corollary 3.21 A space $(U,\tau_R(x).\mathfrak{S})$ is a$\mathfrak{S}NgT_2$- space if and only if A $\nrightarrow G(NT_2.\mathfrak{S})$.
Proof. From theorem3.20 the proof is lucid.

Theorem3.22 A space $(U,\tau_R(x).\mathfrak{S})$ is not a $\mathfrak{S}NgT_2$- space iff A $\hookrightarrow G(NT_2.\mathfrak{S})$.
Proof: by corollary3.21 the proof is lucid

Theorem 3.23 A space $(U,\tau_R(x),\mathfrak{S})$ is not a $\mathfrak{S}NgT_2$- space if and only if B $\nrightarrow G(NT_2.\mathfrak{S})$.
Proof. by theorem3.23 the proof is lucid

Remark 3.24 For any space $(U,\tau_R(x).\mathfrak{S})$:

- If B $\hookrightarrow G(NT_{i+1}.\mathfrak{S})$ then B $\hookrightarrow G(NT_i.\mathfrak{S})$ where $i = 0.1$
- If B $\hookrightarrow G(NT_i.\mathfrak{S})$ then B $\hookrightarrow G(NT_i,\mathfrak{S})$ where $i = 0.1$

The relationships described in the Remark 3. 34 are made clearer by **Figure 1** that follows.

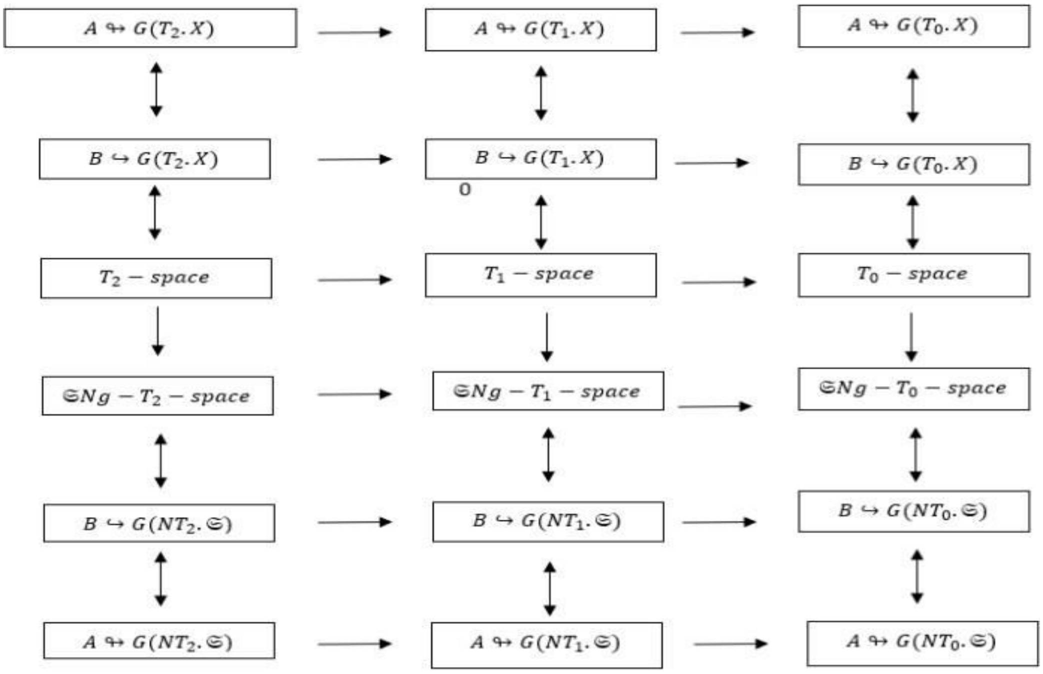


Figure 1. The relationships described in the Remark 3. 24

Any player's winning and losing tactics in in $G(T_i.X)$ and $G(T_i.\mathfrak{S})$.

Remark 3.25 For any space $(X.t.\mathfrak{S})$:

- If $A \hookrightarrow G(T_i.\mathfrak{S})$then $A \hookrightarrow G(T_{1+i}.\mathfrak{S})$, whenever $i = \{0.1\}$
- If $B \looparrowright G(T_i.\mathfrak{S})$ then $B \looparrowright G(T_{1+i}.\mathfrak{S})$, whenever $i = \{0.1\}$.
- If $A \hookrightarrow G(T_i.\mathfrak{S})$then $A \hookrightarrow G(T_i.X)$, whenever $i = \{0.1.2\}$.

The relationships described in the Remark 3. 25 are made clearer by **Figure 2** that follows.

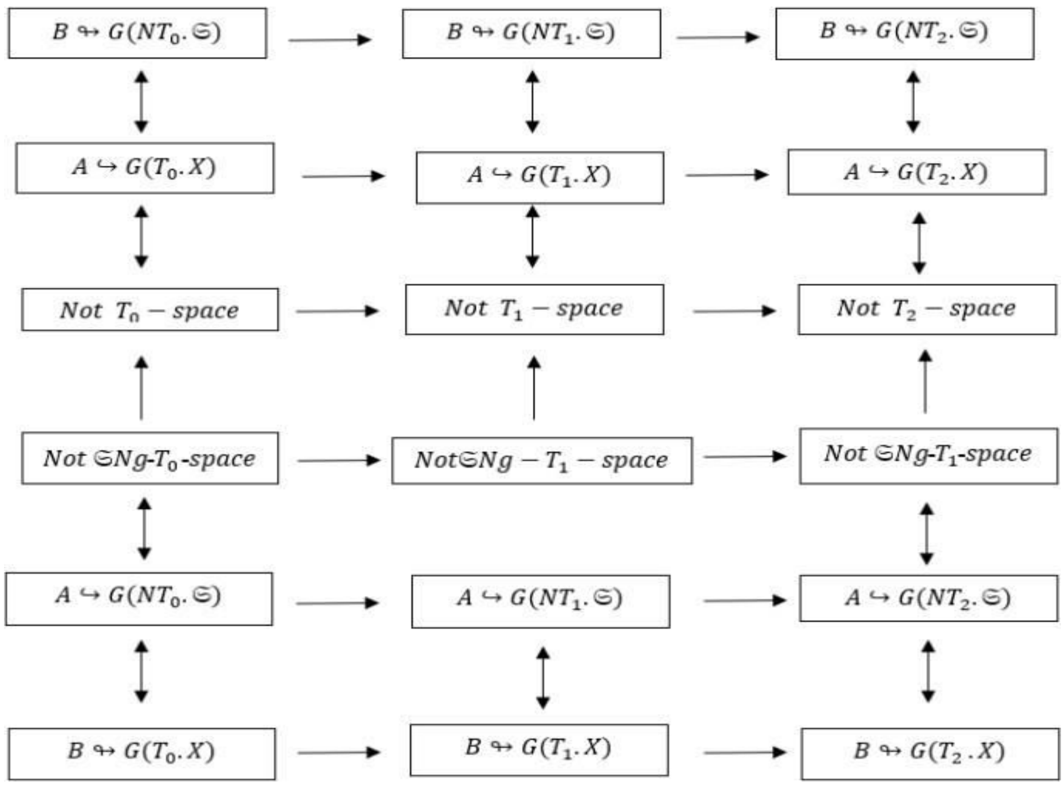


Figure 2. The relationships described in the Remark 3. 25

The winning and losing strategy whenever X is not $\mathfrak{S}Ng - T_i - space$ and not $T_i - space$

References

[1] Choquet G. Sur les notions de filter et grille. *Comptes Rendus Acad. Sci. Paris*. 1947;**(224)** :171-173

[2] Thivagar M, Richard C. On nano forms of weakly open sets. *International Journal Of Mathematics And Statistics Invention*. 2013; **1**(1): 31-37

[3] Gibbons M,Mike M. An introduction to game-theoretic modelling. Addison-Wesley Publishing Company.1991;26-34

[4] Esmaeel R, Mustafa M. New Games Via Grill-Genralized Open Sets. Al-Rafidain *Journal of Computer Sciences and Mathematics*.2021; **15** (2): 115-123

[5] Smith J. Evolutionary and theory of games. Cambridge University Press. 1976; 64(1): 41-45

[6] Mohammad R, Esmaee R. New Games via soft-I-Semi-g-Separation axioms. *Ibn AL-Haitham Journal For Pure and Applied Science*.2020; **33** (4): 122-136

[7] Rawana A, Elseidyb R. Infinite Games Via Covering Properties In Ideal Topological Spaces. *International Journal of Pure and Applied Mathematics*.2016; **106** (1):259-271

[8] Roy B, Mukherjee M. On A Typical Topology Induced By A Grill. *Soochow Journal of Mathematics*.2007; **33** (4): 771 (2007)

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

عزل وتشخيص بعض االنواع البكتيرية من نهر دجلة اثناء مروره في مدينة تكريت
وعالقتها مع بعض المتغي ارت الفيزيوكيميائية

لينا عدنان شاكر محمد الحديثي[1] و ا. م .د رغد مقداد محمود[1] [1] جامعة تكريت ،
كلية التربية للعلوم الصرفة ، الع ارق ، تكريت

عزل وتشخيص بعض االنواع البكتيرية من نهر دجلة اثناء مروره في مدينة تكريت وعالقتها مع بعض المتغي ارت الفيزيوكيميائية

لينا عدنان شاكر محمد الحديثي[1] و ا. م .د رغد مقداد محمود[1] [1] جامعة تكريت ، كلية التربية للعلوم الصرفة ، الع ارق ، تكريت

الخالصة :

أجريت الد ارسة الحالية في الفترة من شهر تموز عام 2022 لغاية شهر حزي ارن عام 2023 للتغي ارت الفصلية الحيوية والكيميائية والفيزيائية لمياه نهر دجلة وتأثير مروره في مدينة تكريت على تغيير هذه الخواص .

أظهرت النتائج وجود فروق معنوية زمانية في درجة ح اررة الماء ولجميع محطات الد ارسة كانت اقل قيمة في فصل الشتاء واعلى قيمة في فصل الصيف، كما أظهرت النتائج ان للمتطلب الحيوي لالوكسجين فروقا في القيم الزمانية والمكانية حيث كانت اقل قيمة في فصلي الصيف والربيع واعلى قيمة في فصل الشتاء ، كما بينت النتائج ان للعسرة الكلية فروقاً معنوية في القيم الزمانية والمكانية عند مستوى احتمالية $p \leq 0.05$ وكانت اقل قيمة في فصل الصيف واعلى قيمة في فصل الشتاء. وسجلت قيم ايون الكالسيوم فروقاً في القيم الزمانية إذ سجلت أعلى قيمة فصلي الشتاء والربيع وأقل قيمة فصل الصيف، أما قيم ايون المغنيسيوم فقد أظهرت فروقا في القيم الزمانية والمكانية وكانت اقل قيمة في فصل الصيف واعلى قيمة في فصل الشتاء، وسجلت نتائج ايون الصوديوم فروقاً غير معنوية في القيم الزمانية والمكانية، إذ سجلت أعلى قيمة في فصل الشتاء، وأقل قيمة في فصل الربيع، أما قيم ايون البوتاسيوم أظهرت فروقا في القيم الزمانية اذ سجلت أعلى قيمة في فصلي الشتاء والربيع و أقل قيمة في فصل الخريف، كما بينت النتائج ان للنت ارت فروقا في القيم الزمانية حيث سجلت أعلى قيمة في الصيف والخريف والشتاء، أما أقل قيمة في فصل الربيع ، كما سجلت النتائج ان للفوسفات فروقا في القيم الزمانية والمكانية حيث بلغت اعلى قيمة في فصل الصيف في المحطة الخامسة، وأقل قيمة كانت في فصل الصيف في المحطة الثالثة.

عزلت االنواع البكتيرية من المياه و لوحظ سيادة بعض االنواع البكتيرية كالبكتريا العصوية القولونية *E. coli* و *Klebsiella* spp. و *Staphylococcus* spp. و *Aeromonas sobria* و *Cirtobater amalonaticus* و *Enterobacter aerogens* و *Salmonella enterica*.

Abstract

The study was conducted from July 2022 to June 2023 to investigate the seasonal biological, chemical, and physical changes in the waters of the Tigris River and their impact on the city of Tikrit. The results showed significant temporal variations in water temperature for all study stations, with the lowest values occurring in winter and the highest in summer. The biological oxygen demand (BOD) also exhibited temporal and spatial differences, with lower values in summer and spring and higher values in winter. Total dissolved solids (TDS) exhibited significant temporal and spatial variations at a significance level of $p \leq 0.05$, with lower values in summer and higher values in winter. Calcium ion (Ca^{2+}) levels showed significant temporal variations, with the highest values in winter and spring and the lowest in summer. Magnesium ion (Mg^{2+}) levels exhibited temporal and spatial differences, with lower values in summer and higher values in winter. Sodium ion (Na^{+}) levels showed temporal variations but not significant spatial differences, with the highest values in winter and the lowest in spring. Potassium ion (K^{+}) levels exhibited temporal variations, with higher values in winter and spring and lower values in autumn. Nitrate (NO_3^-) levels showed temporal variations, with the highest values in summer, autumn, and winter, and the lowest in spring. Phosphate (PO_4^{3-}) levels exhibited temporal and spatial differences, with the highest values in summer at the fifth station and the lowest values in summer at the third station.

Bacterial species were isolated from the water samples, and some predominant bacterial species were identified, including Escherichia coli, Klebsiella spp., Staphylococcus spp., Aeromonas sobria, Citrobacter amalonaticus, Enterobacter aerogenes, and Salmonella enterica.

Keywords: Filtration stations, bacteria, physical and chemical variables.

الكلمات المفتاحية : محطات التصفية ، البكتريا ، المتغي ارت الفيزيائية والكيميائية

المقدمة :

ان أهمية المياه تكمن في كونه يدخل في تركيب الخاليا الحية بنسبة 75-95% من الكتلة البروتوبالزمية لكل خلية، كما إنه يدخل في تكوين أنسجة مختلفة من جسم االنسان والحيوانات والمكونات النباتية، وال تتم أي من عمليات الهضم واالمتصاص والتمثيل الغذائي اال في وسط مائي)بريشة وشريف ، 2018 (. تعتبر مشكلة التلوث الميكروبي من اهم المشاكل بالنسبة للماء ، الن الماء يعتبر المصدر الحامل والناقل للكثير من الكائنات المجهرية ، وهو بذلك يعتبر مصد ًرأساسيا لإلصابة بالعديد من االم ارض و تعكس تأثي ارتها على عدة مجاالت كذلك أن تلوث الماء بالكائنات المرضية من بكتريا ،طفيليات ،فيروسات والتي ت نقل عن طريق المياه وتصل الى االنسان ومختلف الحيوانات االقتصادية تؤدي الى حدوث اخماج وحاالت تسمم ولها تأثي ًر كبي ًر على الصحة. لذلك فإن العديد من االم ارض أقترن وجودها بالتلوث الميكروبي حيث يقدر حوالي 500 مليون شخص في العالم يعانون سنوياً من مشاكل صحية نتيجة استخدام الماء الملوث)المجمعي ،2022(. وتعتبر المياه السطحية أفضل أنواع الماء لنمو الكائنات الدقيقة؛ ألنها تحتوي على كمية كبيرة من المواد العضوية التي تمثل غذا ًءاساسيا لمعظم الكائنات المجهرية وايضا أن درجة ح اررة المياه السطحية أكثر تالئما لنمو هذه الكائنات)Krupa and Parik, 2018(.وتعتبر اختبا ارت العدد الكلي للبكتريا مؤش ًر اعاماً للتلوث البكتيري وهو مؤشر اساسي لدرجة نقاوة الماء من الكائنات المسببة لألم ارض المنقولة)الصفاوي والطائي، 2013(.

ومن الكائنات الدقيقة المرضية التي تنتقل عن طريق المياه هي بكتريا *Salmonella typhi*)التيفوئيد(و*Vibrio cholera*)الكولي ار(و *enterobacter aerogenes* و *Shigella dysentriae*)الديزنتري(و *staphylococcus sp.* و*klebsilla sp.*، الفيروسات مثل Hepatitis virus)التهاب الكبد الفيروسي(،واالبتدائيات مثل (Giardiasis) *Giardia lambalia*(وغيرها من الكائنات الدقيقة

)Cunningham *et al*., 2007(. كما الحظت الكثير من الد ارسات ان الحيوانات هي اكبر خازن للكائنات التي تسبب لألم ارض التي تنتقل عن طريق المياه وان رمي فضالت هذه الحيوانات الى المياه او التربة يودي الى ازدياد إعداد البكتريا الممرضة مثل *E. coli*، *Salmonella spp* و *Mycobacterium spp* في هذه البيئة مما قد يؤدي الى وجود حاالت مرضية بسببها . أن التنوع الكبير لألم ارض واألوبئة المنتشرة عن طريق المياه يؤثر بصورة كبيرة على الصحة العامة حيث انه يموت سنوياً حوالي 4-2 مليون شخص أكثرهم من األطفال الذين اعمارهم اقل من خمس سنوات نتيجة هذه األم ارض ماعدا ذلك فان 2.2 مليون شخص يموتون بسبب اإلسهال الناتج عن الكولي ال والسالمونيال)Tortora *et al .,2007*(. ان من العوامل االساسية النتقال األم ارض بواسطة الماء هي فترة بقاء الممرضات في الماء حية مع االحتفاظ بقابليتها الكامنة على اإلصابة

وانتاج المرض أي ان الممرضات التي تبقى فترة طويلة في الماء تكون أكثر قابلية في نقل األم ارض حيث تستطيع بكتريا السالمونيال ان تبقى ثالثة أسابيع حية في ماء المجاري الناتج من اخر معالجة للماء في حين بكتريا *Listeria spp* لها القابلية على البقاء فترة ثمانية أسابيع دون ان تتأثر أعدادها)الساداني،2009(. وأشار))Ansama *et al*,2000 إلى إن أعداد البكتريا الممرضة تختلف في الماء اعتمادا على درجة التلوث ومصدر التلوث وخصائص الماء ذاتها مثل درجة الح ارره وتوفر المواد الغذائية ووجود المثبطات والمواد السامة فيها

ودرجة التهوية وغيرها .

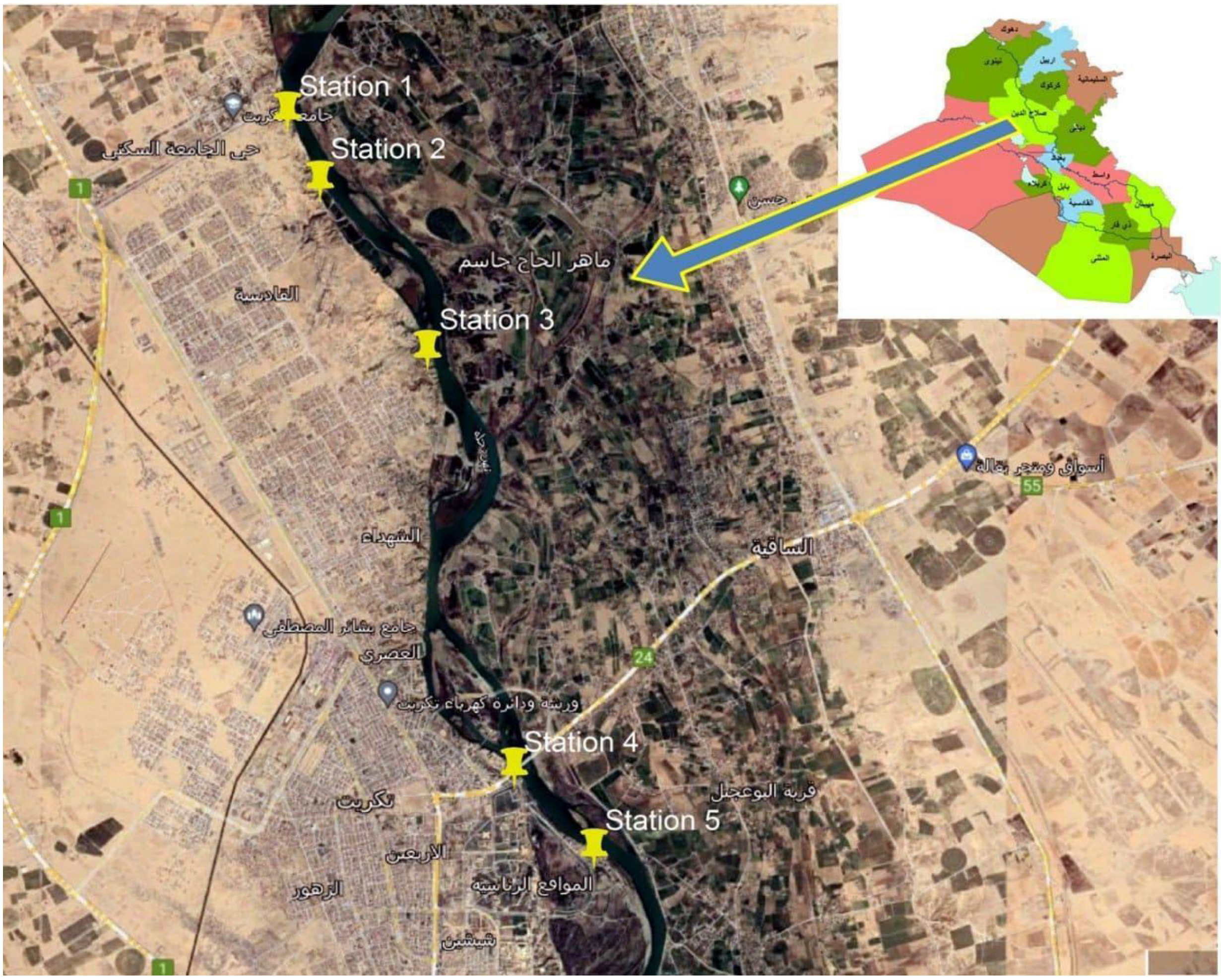
وتعتبر الخصائص الكيميائية و الفيزيائية للماء ذات اهميه كبيره في تحديد نوعية الماء من خالل اعطاء فكره لما تتكون منه من المركبات العضوية و المركبات الالعضوية و ايضا العناصر)الدوري،2014(. إذ تساعد هذه الخصائص المائية في نمو الهائمات النباتية وازدهارها مثل درجة الح ارره والضوء وتوفير المغذيات و االمالح وكمية االوكسجين المذاب وغيرها،و كذلك تساعد في حدوث تنوع في المجتمعات المائية)الزبيدي،2017(.

المواد وط ارئق العمل :

وصف عام لمنطقة الدراسة General Description of the area study

تقع منققعة الدرا~عة عمن محافالعة لعالن العدين ، واالعيعر تقعع فعي و~عو العراق ، وتنحصعر بعين عقعي عرض 35.20-33.45 شماال و عقي طول 42.30 -45.10 شرقا ،حيث يقع نهر دجلة مسافة 250 كيلو مترا تقريبا من مناطق مختلفة بقيعة تكوينها وجيومورفولوجيتها.تقع على ارتفاع 110م عن مستوى ~قح البحر.

تمتد منققة الدرا~ة الحالية ألكثر من)25(كيلومترا ً، ويعتبعر نهعر دجلعة المصعدر الرئيسعي للميعاه السعقحية فعي محافالة لالن الدين، حيث يخترق النهر هذه المدينة من الشمال الى الجنوب .



صورة)1(خارطة للمحطات الخمسة في منطقة الد ارسة

جمع العينات *samples collection*

جمعت عينات المياه من خمس مواقع بمسافات مختلفة على نهر دجلة المار في مدينة تكريت ضمن محافظة صالح الدين خالل أربعة فصول)الصيف ،الخريف ،الشتاء ،الربيع(ابتداءا من شهر تموز عام 2022 لغاية شهر حزي ارن عام 2023 ،حيث تم اخذ العينات بعمق حوالي 10cm من سطح الماء ووضعت بقناني بولي اثلين Polyethylene سعة 5 لتر، بعد أن تم غسلها مرتين بماء العينة في كل محطة . كذلك تم استخدام

قناني ونكلر Winkler سعة 250 مل لغرض قياس كمية االوكسجين المذاب في الماء، وقياس المتطلب الحيوي لالوكسجين ،حيث ملئت القناني بكامل سعتها وبدون أي فقاعة هوائية حتى ال تؤثر عملية النقل وحركة الماء على تغيير عدد من الخصائص وغلقت القناني البالستيكية جيدا وسجلت المعلومات االزمة على كل قنينة

ثم نقلت الى ثالجة بدرجة 4C ° الج ارء التحاليل الفيزيائية والكيميائية ، كما تم جمع عينات مختلفة من الهائمات النباتية باستخدام الشبكة المخصصة لجمع الهائمات النباتية والطحالب، ثم نقلت العينات الى المختبر للفحص والتشخيص. أ ّ ّ ّما بالنسبة لالختبا ارت البكتريولوجية ، فقد تم استخدام قناني زجاجية ذات فتحة ضيقة وغطاء محكم لجمع عينات المياه لغرض عزل وتشخيص البكتريا الموجودة في ماء النهر

حبث أجريت جميع التحليلات المذكورة في مختبر الد ارسات العليا قسم علوم الحياة كلية التربية للعلوم الصرفة\جامعة تكريت، وكلية الهندسة)قسم الهندسة الكيمياوية(، ودائرة ماء صالح الدين)قسم السيطرة النوعية(.

الفحوصات الفيزيائية والكيميائية:

درجة الح ارة *Temperature*

أستعمل جهاز UT3200 Mini type k/J Dual Thermometer وتام القياس بصاورة مباشرة اثناء اخاذ العينات.

المتطلب الحيوي لألوكسجين(*Biological Oxygen Demand (BOD5*) :

حسب المتطلب الحيوي لألوكسجين بواسطة الطريقة المستخدمة في قياس األوكسجين المذاب بعد وضع قناني ونكلر المعتمة لمدة خمسة أيام بدرجة ح ارة 25 درجة مئوي ثم حدد األوكسجين المذاب)DO5(وأن الفرق مع األوكسجين المذاب األولي DO0 مثلت قيمة BOD5 ملغم/لتر)APHA,2005(

$$BOD5=DO\ 0 - DO5$$

العسرة الكلية *Total Hardness*

تم اعتماد طريقة التسحيح بمحلول ثنائي أمين اإليثيلين رباعي حمض األسيتيك Na2EDTA (Ethylene Diamine Tetra Acetic Acid))APHA ، 2005(حيث تم إضافة محلول االمونيا المنظم بحجم 2 مل إلى حجم 50 مل من ماء العينة وتم التسحيح مع محلول ملح الصوديوم القياسي بتركيز) 0.02(عياري الاى ان يتحول من األحمر إلى األزرق بعد إضافة 0.2 غم من كاشف إيريوكروم األساود)ErioChrome Black –T(قبل التسحيح وٌحسبت العسرة من المعادلة التالية:

$$\text{Total Hardness (ppm) as } CaCO_3 = A \times B \times 1000 / \text{sample volume (ml)}$$

A = حجم محلول الصوديوم الملحي المسحح)مل(.

B = حجاالم CaCO3)ملغاالم(المكافئاالة لماالل واحاالد مااان محلاالول Na2EDTA الناالاتج مااان التساالحيح ماالاع محلاالول الكالسيوم القياسي.

عسرة الكالسيوم) *Calcium Hardness(Ca.H*:

كان القياس بإضافة 2 مل من محلول هيدروكسيد الصوديوم بتركيز)1(عياري إلى حجام)50(مال مان مااء العينة باستخدام)0.2(غم من بيروكسيد الهيدروجين كدليل ليتم التسحيح مع المحلول المذكور مسبقا محلول ملح الصوديوم وبالتركيز نفسه الاى ان يظهار اللاون البنفساجي باداًل عان اللاون الاوردي ، ومان خاالل اساتخدام المعادلاة التالية كان حساب عسرة الكالسيوم بالملغم / لتر كالتالي :

$$Ca^{+2}Hardness = [\ A \times B / \text{sample volume (ml)}] \times 1000$$

A= حجم Na2 EDTA المسحح)مل(.

B = حجم $CaCO_3$)ملغم(المكافئة لواحد مل من محلول ملح الصوديوم)APHA,2005(.

عسرة المغنيسيوم)*Magnesium Hard ness (mg.H* :

كان حساب تركيز ايون المغنسيوم)Mg^{+2}(من خالل المعادلة التالية ويعبر عن النتائج بوحدة ملغم / لتر .

Mg^{+2} Hardness= Total Hardness – Ca^{+2}Hardness)APHA , 2005(

أيون الصوديوم *Sodium Ion*:

استخدمت طريقة اإلنبعاث الذري اللهبي)Vogel, 1961(وذلك باستخدام جهاز Flame photometer بعد أن نعمل على معايرة الجهاز بمحلول مجهز من قبل الشركة المصنعة للمحلول)محلول قياسي للصوديوم(ويعبر عن النتائج با ppm (Part per million.

أيون البوتاسيوم *Potassium Ion*:

استخدمت طريقة اإلنبعاث الذري اللهبي ، وذلك باستخدام جهاز Flame photometer بعد أن نعمل على معايرة الجهاز بمحلول مجهز من قبل الشركة المصنعة)محلول قياسي للصوديوم(ويعبر عن النتائج باppm (Vogel, 1961).

قياس النت ارت *Nitrate (NO3)*:

استخدم جهاز Spectrophotometer في قياس النت ارت حسب)APHA, 2003(على طول موجي قدره 410 نانوميتر في خلية، اذ يتم اوالً ق ارءة Blanck؛ لتصفير الجهاز ثم تتم ق ارءة العينة وتحسب النت ارت بوحدة مايكروغ ارم / لتر.

الفوسفات *Orthophosphate*:

سجلت قيمة الفوسفات باالعتماد على الطريقة المنشورة من قبل)عباوي ، وحسن: 1990(وتم تحديد الفوسفات للعينات باستخدام جهاز قياس الطيف الضوئي Spectrophotometer CE 1011CECIL)(وعلى طول موجي 690 نانوميتر وعبر عن النتائج بداللة مايكرو غ ارم ذرة فسفور - فوسفات / لتر. ويتم وضع)10ج(مل من العينة في بيكر ويضاف اليه محلول 2)(مل حامض الكبريتيك مع)40(مل من المياه األيوني، ثم يبرد مزيج التفاعل، ويتم اضافة)2(مل من محلول موليبدات األلمنيوم الثنائية ويخفف المحلول الى 100، ويتم اضافة 5-7 مل قط ارت من محلول كلوريد القصديروز، و يس ّخن الى حد اتمام عملية االذابة، فيتكون لون ازرق ويعتمد شدته على تركيز الفسفور فيه ويترك المحلول لمدة)10-15(دقيقة، وبعدها يتم قياس االمتصاصية عند طول موجي)690nm(وبتحضير المحاليل القياسية تم إيجاد تركيز الفوسفات من المعادلة الخاصة لكل منحني، عبر عن النتائج بوحدة مايكرو غ ارم ذرة فسفور- فوسفيت/ لتر.)عباوي، وحسن، 1990(.

االوساط الزرعية

حضرت األوساط الزرعية حسب تعميمات الشركةالمصنعة ، ثم عقمت بالمؤصدة بدرجة ح اررة 121° م وضغط 1 جو ولمدة 15 دقيقة.

جدول)1(األوساط الزرعية المستخدمة في الد ارسة

ت	الوسط الزرعي	المنشأ
1	اكار المانيتول والملح Mannitol Salt agar	Himedia,India
2	ماكونكي اكار MacConkay agar	Himedia,India
3	االكار المغذي Nutrient agar	Himedia,India
4	المرق المغذي Nutrient broth	Himedia,India
5	اليوسين المثيل االزرق Eosin methylene blue	Himedia,India

حساب العدد الكلي للبكتريا الهوائية:

اعتمدت طريقة صب األطباق plate count Pour لغرض وتقدير العدد الكلي الحي للبكتريا، إذ تم رج نموذج عينة المياه بشدة حوالي 25 مرة، ثم حضرت سلسلة من التخافيف لغاية10-6 باستعمال المياه المقطرة المعقم وتم نقل 1 مل باستخدام ماصة نظيفة ومعقمة من كل تخفيف ومن العينة األصلية إلى أطباق بتري المعقمة، ثم صب الوسط الغذائي Nutrient agar بعد أن وصلت درجة ح اررته إلى درجة ح ارة)45-50) درجة مئوية، ثم حرك الوسط الغذائي من خالل تدوير الطبق بهدوء بطريقة دائرية مع الوسط الغذائي بصورة جيدة، ثم تركت لكي تتصلب وبعدها حضنت األطباق بصورة مقلوبة بدرجة ح ارة 37 درجة مئوية لمدة 24-48 ساعة في الحاضنة، وتم حساب عدد المستعم ارت الكلي للبكتريا الهوائية ومن ثم ضرب عدد المستعم ارت في مقلوب التخفيف وعّبر عنها CFU/100 مل)WHO,1996(،) العاني وبديوي، 1990).

حساب العدد الكلي لبكتريا القولون: ***Total count of coliform***

اتبعت طريقة العد األكثر احتمال)Most probable number (MPN في تحديد العدد الكلي لبكتريا القولون الواردة في) APH 1998(. ثم صب الوسط الغذائي MacConkey agar بعد أن وصلت ح اررته إلى درجة ح ارة)45-50(درجة مئوية على أطباق بتري المعقمة تركت لكي تتصلب، اخذت0.1 مل من كل عينة باستخدام ماصة نظيفة ومعقمة ،الى الوسط الغذائي ونشره ، وبعدها حضنت األطباق بصورة مقلوبة بدرجة ح ارة 37 درجة مئوية لمدة 24-48 ساعة في الحاضنة، حساب المستعم ارت النامية على وسط الماكونكي آكار حيث تظهر على شكل مستعم ارت صغيرة وردية.

تشخيص البكتريا : ***Identification of bacteria***

تم تشخيص البكتريا المعزولة من عينات مياه نهر دجلة عن طريق جهاز فايتك)VITIK(.

التحليل االحصائي :

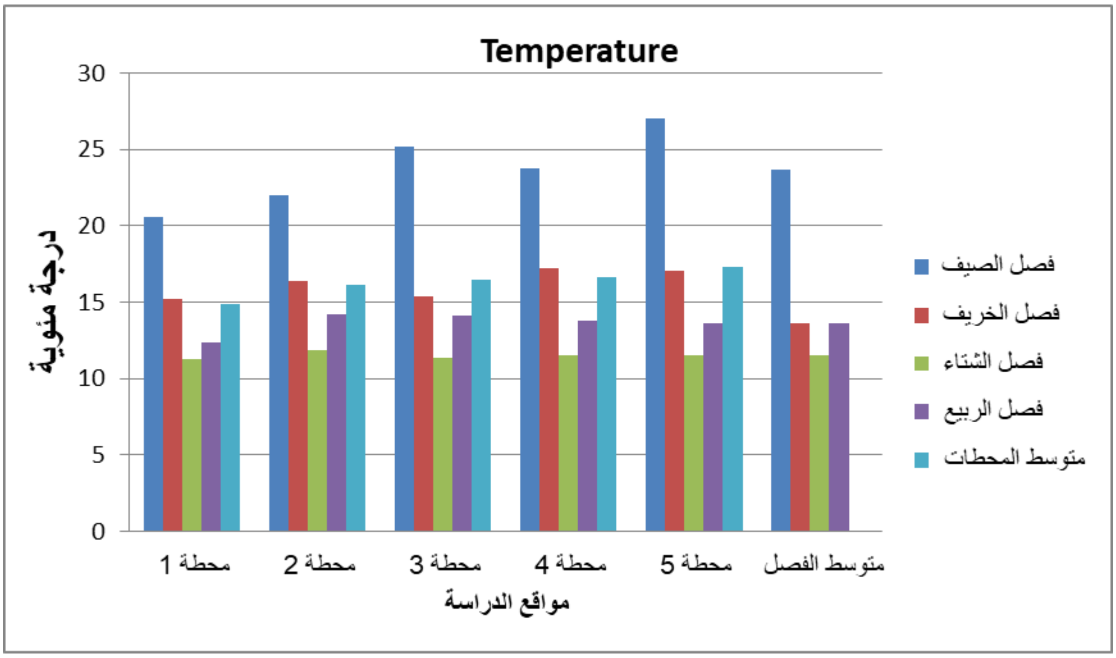
حللت النتائج احصائيا بتطبيق البرنامج االحصائي MINI TAB وطبق اختبار تحليل التباين)ANOVA(وتم مقارنة المتوسطات الحسابية باختبار دنكن متعدد الحدود بمستوى احتمالية 0.05 .

النتائج والمناقشة :

العوامل الفيزيائية والكيميائية للمياه : *Physical and Chemical Characteristics of Water*

درجة ح ارة الماء ***Water Temperature*** **:**

أظهرت نتائج الد ارسة الحالية وجود فروق معنوية زمانية في درجة ح ارة الماء ولجميع محطات الد ارسة عند مستوى احتمال p ≤ 0.05، إال أن الفروق المكانية لم تكن معنوية خالل أشهر الد ارسة وهذا ما موضح في الشكل)1(، وعموماً ت اروحت قيم درجات الح ارة للماء ما بين 11,3 -27°م، إذ إن أقل درجة ح ارة للماء سجلت 11,3 °م في فصل الشتاء عند المحطة االولى، في حين ان أعلى درجة ح ارة للماء سجلت27 °م في فصل الصيف في المحطة الخامسة.



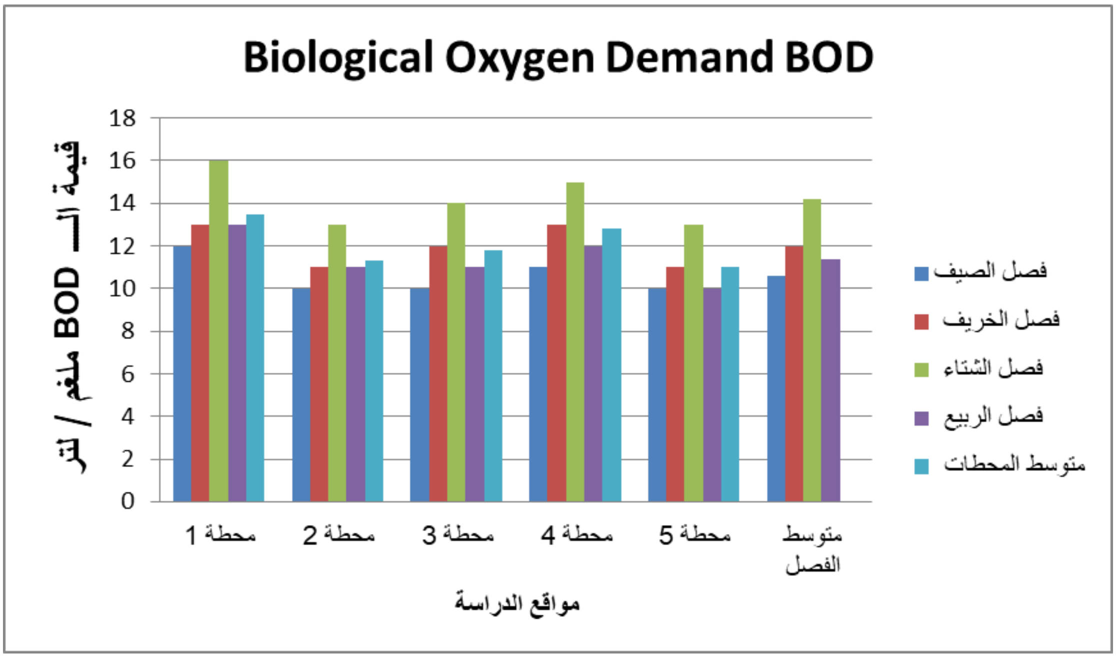
الشكل)1(: التغاير المكاني والزماني لقيم درجة ح ارة الماء في المحطات قيد الد ارسة

إن المناخ في الع ارق يتميز بإختالفات واضحة في درجات الح ارة على مدار فصول السنة. ان اإلرتفاع واإلنخفاض بدرجات ح ارة الماء قد يعود لتأثره بتغي ارت الطقس في المنطقة المدروسة وان هذا يتوافق مع نتائج

الكثير من د ارسات الباحثين على المسطحات المائية في الع ارق)اسماعيل، 2018 ؛منصور، 2019؛ محمد،2021(. إن درجات الح اررة لها تأثير مباشر على العمليات الحيوية لألحياء المائية، إذ إن في فصل الشتاء وعند انخفاض درجات الح اررة يؤدي إلى تقليل نشاط األحياء المجهرية والطفيلية نتيجة إلنخفاض نشاط العمليات األيضية واألنزيمية عند انخفاض درجة ح ارة الماء (Weiner, 2000(.

المتطلب الحيوي لألوكسجين (*Biological Oxygen Demand (BOD*:

أظهرت النتائج في الشكل)2(، إن للمتطلب الحيوي لالوكسجين فروقاً في القيم الزمانية والمكانية، إال إن الفروق الزمانية كانت معنوية عند مستوى احتمال $p \leq 0.05$، إذ سجلت أعلى قيمة بلغت 16 ملغم/لتر في فصل الشتاء عند المحطة األولى، وأقل قيمة كانت 10 ملغم/لير في كل من فصلي الصيف عند المحطة الثانية و الثالثة و الخامسة و الربيع عند المحطة الخامسة.



الشكل)2(التغاير المكاني والزماني المتطلب الحيوي لألوكسجين ملغم/ لتر في المحطات قيد الد ارسة.

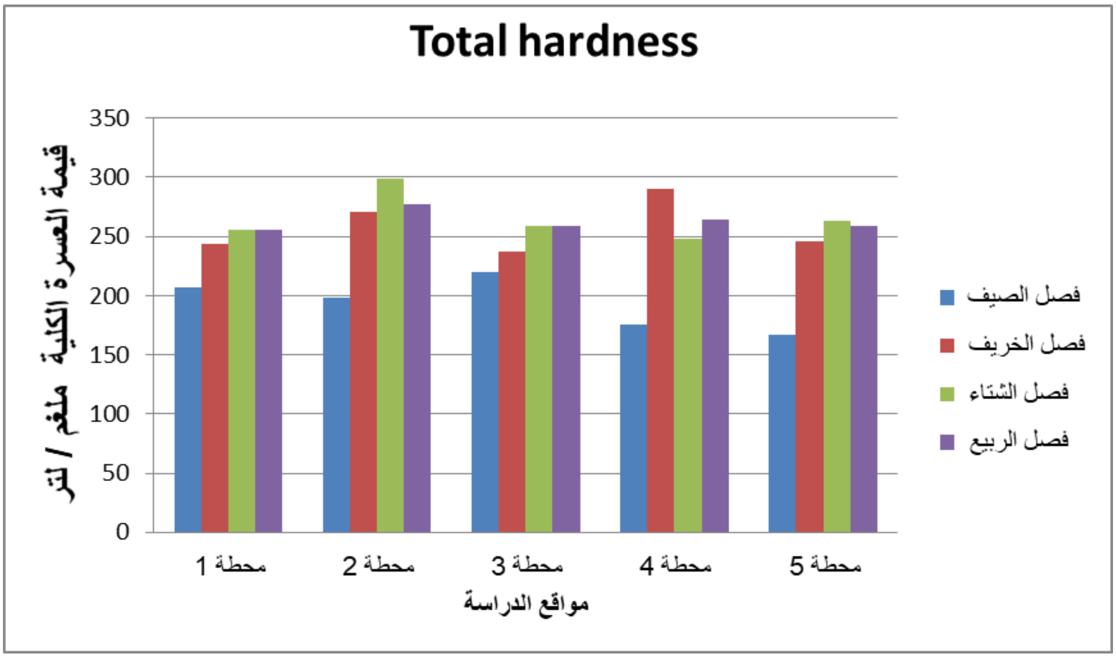
قد يعود سبب اإلنخفاض في قيم الاBOD هو لتأثره بكمية الملوثات، بينما الزيادة فيه ربما تعود لوصول بعض الملوثات البشرية الى المياه وبالتالي حدوث عملية أكسدة للمواد العضوية)Negi *et al*.،2020(. ان االرتفاع بقيم المتطلب الحيوي لألوكسجين يؤثر وينعكس سلبياً على كمية االوكسجين المذاب)Awad *et al*.،2020(، نتيجة لنشاط وفعاليات االحياء المجهرية وبالتالي تسبب زيادة بعمليات التحلل للمواد العضوية واستن ازف االوكسجين)Ibo *et al*.، 2020(، كما ان االستخدام المتعدد لمياه النهر ومنها السباحة او الز ارعة، والفضالت العضوية التي تطرحها الحيوانات المتواجدة قرب النهر، قد تسبب زيادة بقيم المتطلب الحيوي لالوكسجين، كما ان االسمدة العضوية وغير العضوية مثل النتروجين او الفسفور يسبب اث ارء غذائي للهائمات فتحتاج عند موتها نسبة عالية من االوكسجين وذلك لتحللها بواسطة االحياء المجهرية)البدري، 2012(.

أتفقت النتائج المسجلة مع محمود)2021(و العبيدي)2019(اذ سجلوا 0.2-3.9 و 0.6 -4.4 ملغم/لتر على التوالي، واقل من محمود واخرون)2018(اذ سجلت قيم ت اروحت ما بين 1.4-5.2 ملغم/لتر

وأعلى من الدوري)2020(و الدليمي وخميس)2021(إذ ت اروحت تائجهم ما بين 1-2.6 و0.1-2.9 ملغم/لتر.

العسرة الكلية *Total Hardness*)*TH*(:

بينت النتائج في الشكل)3(، إن للعسرة الكلية فروقاً معنوية في القيم الزمانية والمكانية عند مستوى احتمالية p ≥ 0.05، إذ سجلت أعلى قيمة بلغت299 ملغم/لتر في فصل الشتاء عند المحطة الثانية، وأقل قيمة كانت 167 ملغم/لتر فصل الصيف عند المحطة الخامسة.



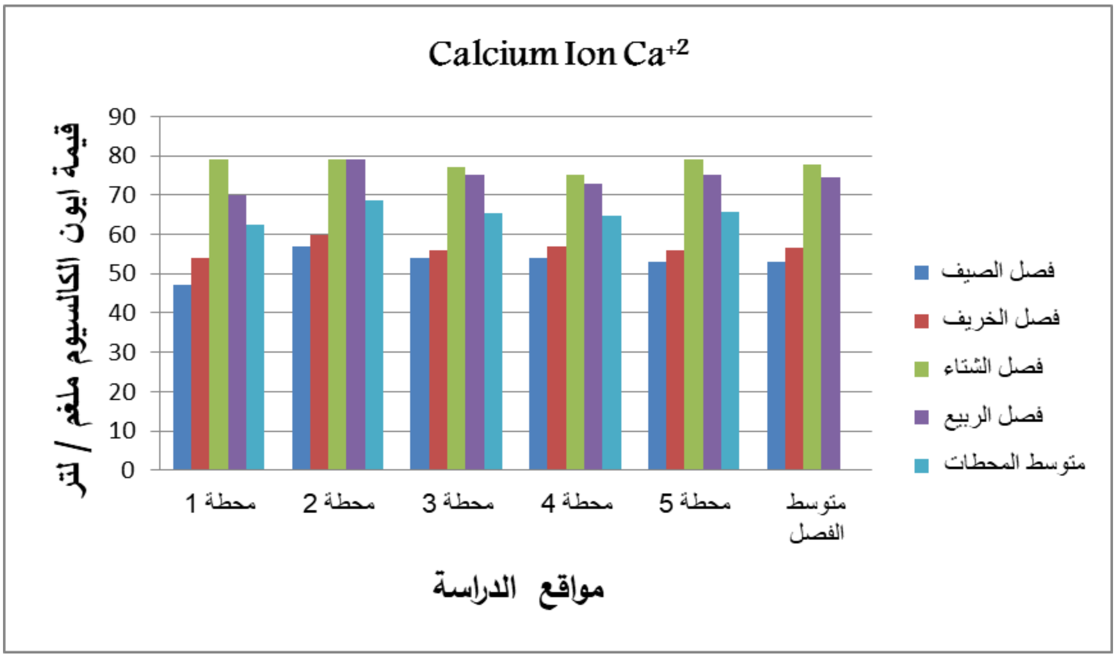
الشكل)3(: التغاير المكاني والزماني لقيم العسرة الكلية ملغم/ لتر في المحطات قيد الد ارسة.

إن سبب حدوث العسرة بالمياه قد يعود لوجود أيونات ذات شحنة موجبة ومتعددة التكافؤ مثل ايون الكالسيوم، و المغنيسيوم، فضالً عن وجود ايونات أخرى ت اركيزها قليلة جداً مثل المنغنيز و الحديد. Verma *et al.*2018) وقد تكون الزيادة بقيم العسرة ناتجة عن األرتفاع بدرجة ح ارر ة المياه و التوصيلية الكهربائية، إذ يؤدي هذا األرتفاع إلى حدوث عمليات التحلل و التجوية للصخور مما يؤدي إلى ارتفاع قيم العسرة الكلية في هذه المياه(Yaseen *et al.*2014) قد يكون االختالف بت اركيز العسرة خالل فصول الصيف والخريف ناتج بسبب عملية التبخر واالختالف في ت اركيز األمالح الذائبة بالمياه والتي تنتج عن العديد من االنشطة الطبيعية والبشرية(Pradeep *et al.*2012).

هذه القيم تتوافق مع اسماعيل)2018(إذ ت اروحت قيم نتائجها 200-300 ملغم/لتر ومع المجمعي)2022(إذ سجل قيم ت اروحت ما بين 90-290 ملغم لتر، واختلفت مع طلعت والصفاوي)2018(والسعدون)2021(إذ جاءت النتائج أقل من التي حصلوا عليها حيث ت اروحت ما بين 216-560 ملغم/لتر و273.3-1323.3 ملغم/لتر على التوالي.

ايون الكالسيوم *Ca Calcium Ion* $^{2+}$:

سجلت نتائج ايون الكالسيوم في الشكل)4(فروقاً في القيم الزمانية، إذ سجلت أعلى قيمة بلغت 79 ملغم/لتر فصلي الشتاء و الربيع عند المحطة األولى و الثانية، وأقل قيمة كانت 47 ملغم/لتر في فصل الصيف عند المحطة االولى.



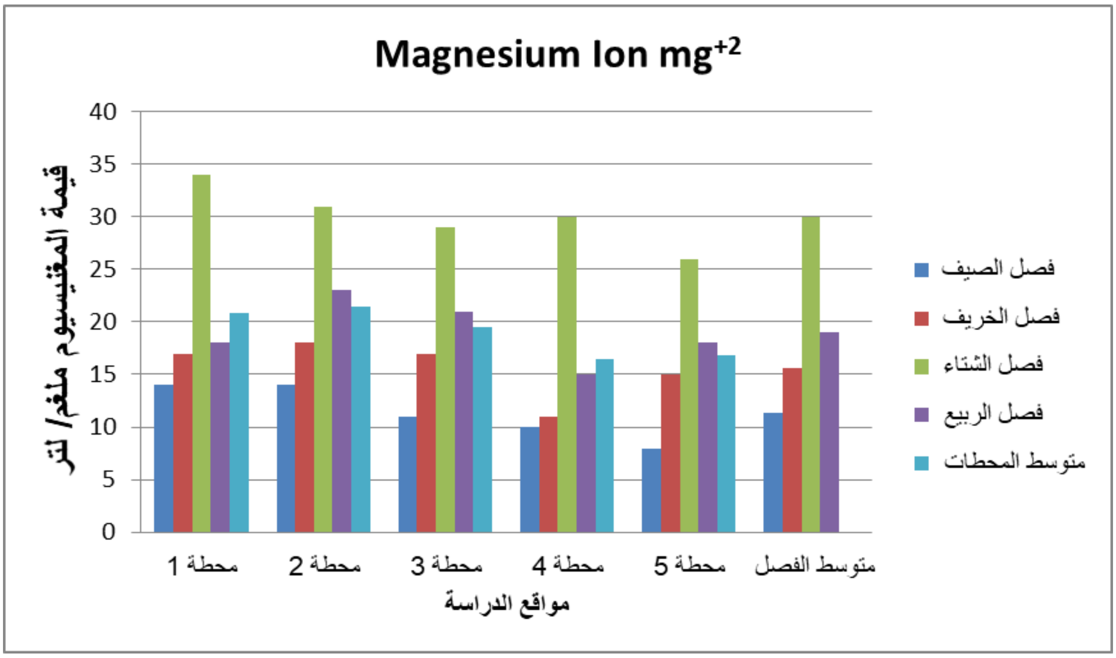
الشكل)4(: التغاير المكاني والزماني لقيم ايون الكالسيوم ملغم/ لتر في المحطات قيد الد ارسة

نالحظ من هذه النتائج أن تركيز أيون الكالسيوم كان مرتفعًا خالل فصل الشتاء ومنخفضًا خالل فصل الصيف. يمكن أن يعزى السبب و ارء ذلك إلى تكوين األرض الجيولوجي حيث تكون ت اركيز أيون الكالسيوم دائمًا أعلى من ت اركيز أيون المغنيسيوم في المياه)الشواني، 2001(.

قد يكون السبب و ارء االنخفاض في قيم الكالسيوم هو زيادة نشاط النباتات المائية في فصل الصيف، حيث يعتبر الكالسيوم من العناصر األكثر أهمية ويتم استهالكه بواسطة الطحالب والنباتات المائية األخرى، نظًار ألنه يدخل في تكوين جد ارن الخاليا النباتية، باإلضافة إلى دوره في نقل األيونات داخل وخارج الخاليا من خالل انتقائية الغشاء الخلوي)Salman *et al.* 2014(.، تتطابق هذه النتائج تقريبًا مع د ارسة القريشي)2011(التي كانت تت اروح قيمها بين 64-96 ملغم/لتر، وتكون أقل من د ارسة السعدون)2021(التي كانت تت اروح قيمها بين 61.5-231.1 ملغم/لتر، وتكون أعلى من د ارسة مطر)2021(التي كانت تت اروح قيمها بين 37-48 ملغم/لتر.

ايون المغنيسيوم ***Mg Magnesium Ion*** $^{2+}$ **:**

سجلت النتائج في الشكل)5(، فروقاً في القيم الزمانية والمكانية، إذ سجلت أعلى قيمة أليون المغنيسيوم بلغت 34 ملغم/لتر فصل الشتاء عند المحطة االولى، وأقل قيمة كانت 8 ملغم/لتر في فصل الصيف عند المحطة الخامسة.



الشكل)5(: التغاير المكاني والزماني لقيم ايون المغنيسيوم ملغم/ لتر في المحطات قيد الد ارسة.

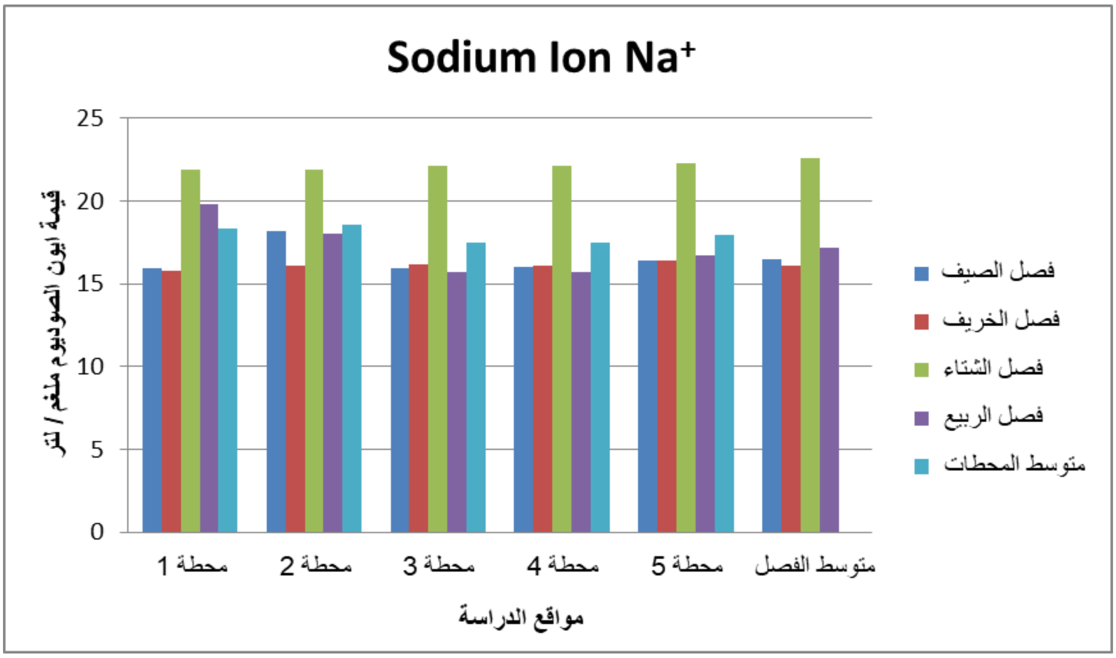
ان سبب ارتفاع المغنيسيوم بسبب الطبيعة الز ارعية للمنطقة، مما يسبب ترسب أيونات المغنيسيوم في المياه، وقد يكون بسبب التحلل للطحالب والكائنات األخرى نتيجة ارتفاع درجات الح ارر ة وبالتالي عودة أيون المغنيسيوم إلى المياه من جديد)الحمداوي، 2009(، وقد يعود السبب في تغلب ت اركيز الكالسيوم على المغنيسيوم طول مدة الد ارسة هو لوجود األ ارضي الز ارعية وإحتواء مياه الري المصرفة على بقايا من األسمدة، والتي تقوم في ترسيب

المغنيسيوم على شكل كبريتات المغنيسيوم)الفتالوي، 2005(.

جاءت النتائج أقل من د ارسة السعدون)2021(إذ ت اروحت قيم نتائجه 265.4-48.2 ملغم/لتر، و اعلى من مطر)2021(اذ ت اروحت قيم نتائجه 13-17.5 ملغم/لتر.

أيون الصوديوم)*Sodium (Na* $^{+}$ **:**

سجلت نتائج ايون الصوديوم في الشكل)6(، فروقاً غير معنوية في القيم الزمانية و المكانية، إذ سجلت أعلى قيمة بلغت 22,3 ملغم/لتر في فصل الشتاء عند المحطة الخامسة، وأقل قيمة كانت 15,7 ملغم/لتر في فصل الربيع عند كل من المحطة الثالثة و ال اربعة.



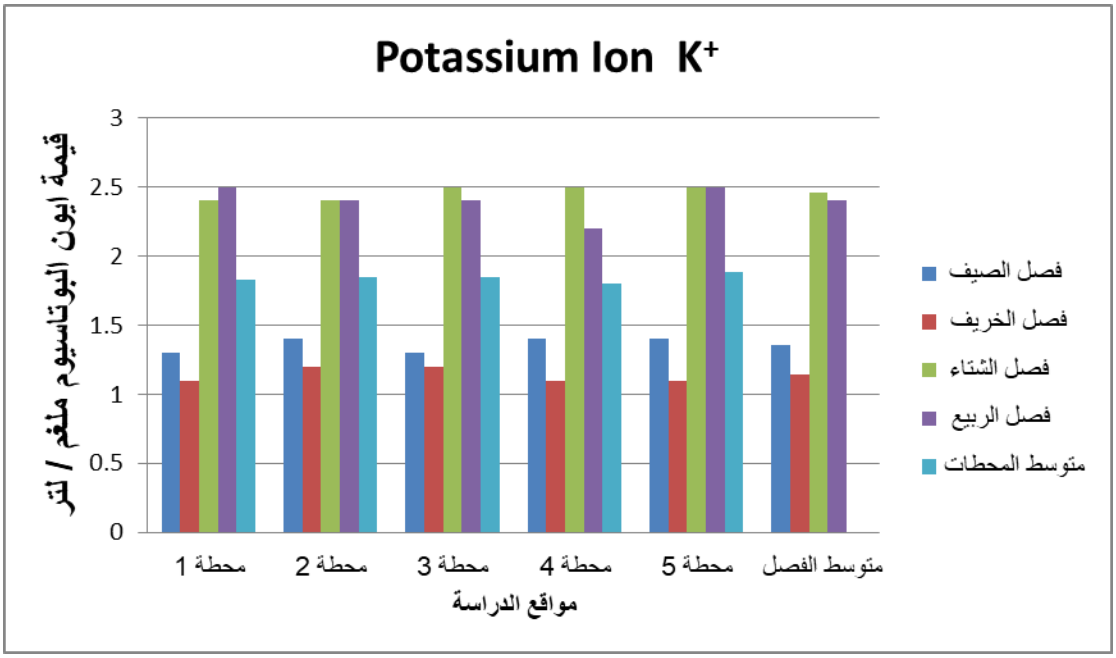
الشكل)6(: التغاير المكاني والزماني لقيم ايون الصوديوم ملغم/ لتر في المحطات قيد الد ارسة.

لقد أشا ر Fried & Sharpio)1961(الى إن أيونات الصوديوم تسود في الترية المعتدلة الملوحة وذات تفاعل قاعدي ضعيف. وقد يعزى سبب الزيادة الحاصلة في فصل الشتاء كونه ت ازمن مع سقوط األمطار، إذ ينتج عنها غسل التربة، وتجرف معها ايونات الصوديوم، كما انه يزداد عند ارتفاع درجة الح اررة والذي ربما يكون بسبب زيادة معدل التبخر)Alexander,2008(.

وهذه القيم جاءت مقاربة مع د ارسة العبيدي)2019(اذ ت اروحت قيم نتائجها 15,1-12,1 ملغم/لتر واعلى من محمد)2021(ومنصور)2019(اذ سجال نتائج د ارستهما ما بين 6-9,3 و 7,5- 8,5 ملغم/لتر على التوالي.

أيون البوتاسيوم)*Potassium (K*$^+$:

سجلت النتائج في الشكل)7(، واظهرت النتائج فروقاً في القيم الزمانية، اذ سجلت أعلى قيمة أليون البوتاسيوم بلغت 2,5 ملغم/ لتر في فصل الشتاء عند المحطة الثالثة و ال اربعة، و في فصل الربيع عند المحطة األولى و الخامسة، كما سجلت أقل قيمة وهي 1,1 ملغم/ لتر فصل الخريف عن المحطات األولى و ال اربعة و الخامسة.



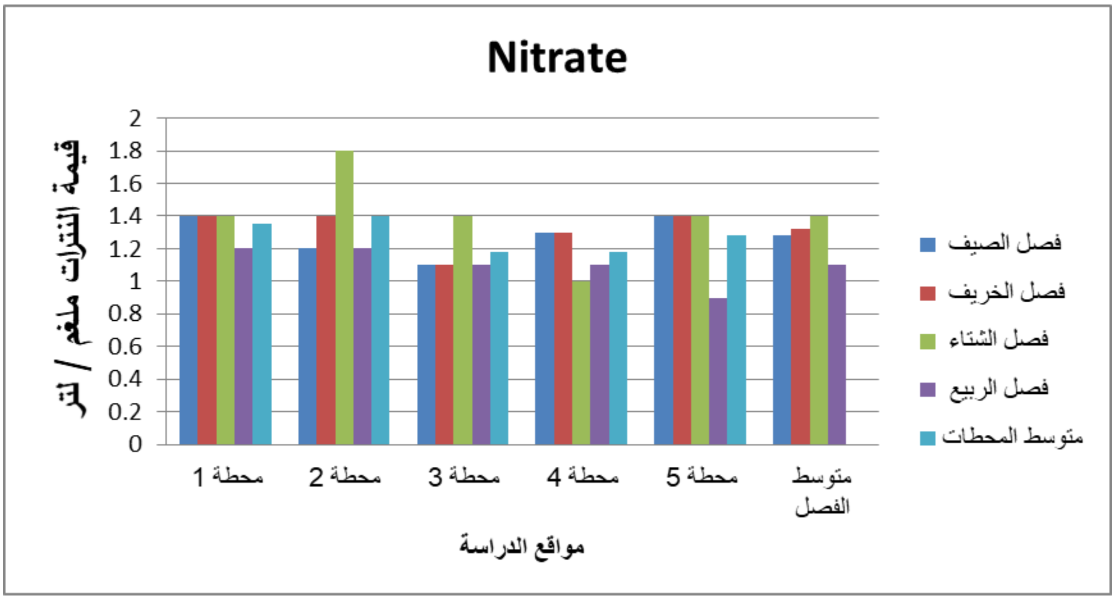
الشكل)7(: التغاير المكاني والزماني لقيم ايون البوتاسيوم ملغم/ لتر في المحطات قيد الد ارسة.

أن تركيز أيون البوتاسيوم في مياه النهر سجل أقل تركيز من تركيز أيون الصوديوم، وان هذا قد يعزى لعدم وجود النباتات المتحللة، والتي بدورها تعطي البوتاسيوم للماء، وكذلك قلة األنشطة الز ارعية في المناطق المجاورة)الفتالوي،2007(. إذ يتميز البوتاسيوم بمقاومته العالية للتجوية مقارن ًةبالصوديوم، فضالً عن قابليته على االمت ازز من قبل التربة ضمن طبقات األرض لذلك يقل تركيزه (Nofal & Ibrahim)2020)

وهذه القيم تتوافق مع د ارسة محمد)2021(اذ ت اروحت قيم نتائجها 2.0-4.1 ملغم/لتر ومع د ارسة منصور)2019(اذ ت اروحت قيم نتائجه 1.5-1.8 ملغم/لتر، واقل من اسماعيل)2018(اذ ت اروحت نتائجها ما بين 2-5.5 ملغم/ لتر.

النت ارت *Nitrate* :

اظهرت النتائج في الشكل)8(، انه كان للنت ارت فروقاً في القيم الزمانية، إذ سجلت أعلى قيمة بلغت 1,8 ملغم/لتر في كل من فصل الصيف و الخريف و الشتاء و في اغلب المحطات، أما أقل قيمة فكانت 0,9 ملغم/ لتر في فصل الربيع عند المحطة الخامسة.

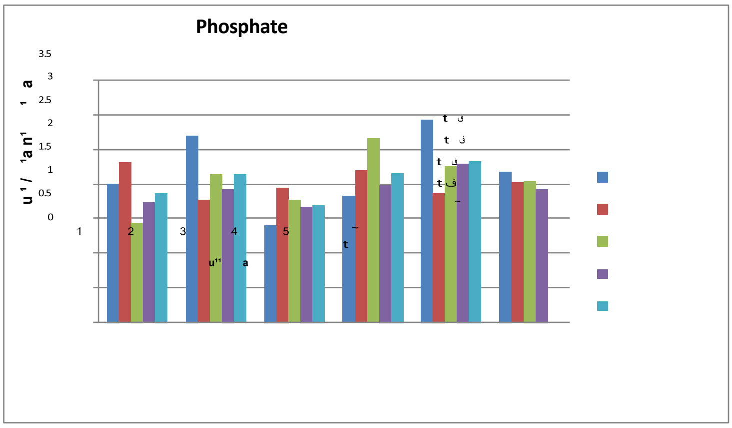


الشكل)8(التغاير المكاني والزماني لقيم النت ارت ملغم/ لتر في المحطات قيد الد ارسة

توجد النت ارت في حالتها الطبيعية وذائبة في التربة ، وتخترق التربة والمياه الجوفية كما تصرف في المجاري المائية)2013 Ghazali and Zaid،(. فقد ظهر في الفترة ألاخيرة اهتماما كبي ار بمشكلة تواجد النت ارت في المياه السطحية و الجوفية على حد سواء، و ذلك بعد ان اثبتت ألابحاث الطبية مضار النت ارت على الصحة و خاصة ألاطفال الرضع، حيث أنها تسبب اختناقا نتيجة نقص االوكسجين في الدم بسبب تحول النت ارت الى نتريت داخل الجهاز الهضمي)الحايك، 1989(. ان الزيادة في تركيز النت ارت مع مرور السنوات يرجع الى وجودها في المناطق الز ارعية، حيث يقوم الم ازرعون برش ألاسمدة الغير عضوية والحيوانية وبعد الري يمكن أن يرشح النتروجين، فضال عن استخدام سكان الريف كثي ار للحفر الفنية للتخلص من مياه صرفهم التي يمكن أن تكون مصدر للنت ارت التي تصل الى المياه الجوفية)السعدي، 2006(.

الفوسفات *Phosphate* :

بينت النتائج في الشكل)9(، كان للفوسفات فروقاً في القيم الزمانية و المكانية، إذ سجلت أعلى قيمة بلغت 2,92932 ملغم/لتر في فصل الصيف عند المحطة الخامسة، وأقل قيمة كانت 1,39993 ملغم/ لتر في فصل الصيف عند المحطة الثالثة.



الشكل)9(: التغاير المكاني والزماني لقيم الفوسفات ملغم/ لتر في المحطات قيد الد ارسة

توجد المركبات الفوسفاتية في الصخور الرسوبية والبركانية والترسبات الحاوية على العظام الحيوانية وصخور الا Apatite وعند تماسها مع الماء تذوب وتزيد من تركيزها في الماء فضال عن مخلفات المياه البشرية والحيوانية التي تحوي على ت اركيز من مركبات الفوسفات)الصفاوي و آخرون، 2009(.

وجاءت النتائج اعلى لما حصل عليها)إب ارهيم، 2010(، في د ارسته لنوعية المياه الجوفية لمناطق مختارة من محافظة نينوى اذ ت اروحت قيمة الفوسفات ما بين 0.001-0.19 ملغم. لتر-1، وكذلك الحال مع ما توصل اليه)الصفاوي، 2007(، حيث بلغت اعلى قيمة له 0.14 ملغم/لتر.

ان االنخفاض في قيمة الفوسفات ربما يعزى الى قابلية ترسيب الفوسفات بشكل فوسفات الكالسيوم اضافة الى امت ازره من قبل اسطح دقائق الطين مما يقلل انتقاله الى البيئة المائية، كما يعد االستعمال الكثير لألسمدة الفوسفاتية والمنظفات المصدر الرئيسي للفوسفات في المياه الجوفية فضلا عن تصريف المياه الثقيلة الى الفضالت المدنية .)Manahan, 2004(.

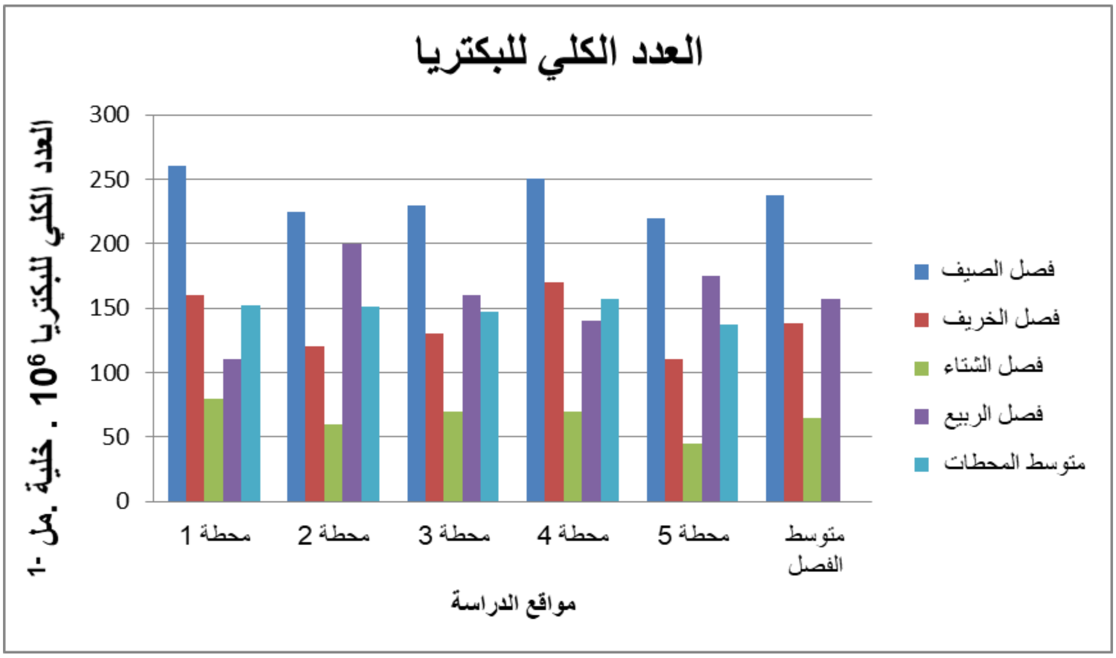
أنواع البكتريا المعزولة من المحطات المدروسة:

أظهرت النتائج في الشكل)10(توضيحا للتغي ارت الفصلية للعدد الكلي للبكتيريا)بالماليين(في المياه خالل فترة الد ارسة، اذ سجلت المحطة االولى في فصل الصيف، 260 مليون خلية بكتيرية في المِّل1. أما في فصل الخريف، انخفض هذا العدد إلى 160 مليون خلية بكتيرية. في فصل الشتاء، انخفض العدد أكثر إلى 80 مليون خلية بكتيرية. أما في فصل الربيع، ف ازد العدد قليالً ليصل إلى 110 مليون خلية بكتيرية.

في المحطة الثانية، كان هناك 225 مليون خلية بكتيرية في المِّل1 في فصل الصيف. في فصل الخريف، انخفض هذا العدد إلى 120 مليون خلية بكتيرية. في فصل الشتاء، انخفض العدد أكثر إلى 60 مليون خلية بكتيرية. أما في فصل الربيع، فارتفع العدد بشكل كبير ليصل إلى 200 مليون خلية بكتيرية.

في المحطة الثالثة، كان هناك 230 مليون خلية بكتيرية في المّل1 في فصل الصيف. انخفض هذا العدد إلى 130 مليون خلية بكتيرية في فصل الخريف. في فصل الشتاء، انخفض العدد قليالً إلى 70 مليون خلية بكتيرية. أما في فصل الربيع، فارتفع العدد ليصل إلى 160 مليون خلية بكتيرية.

في المحطة ال اربعة، كان هناك 250 مليون خلية بكتيرية في المّل1 في فصل الصيف. في فصل الخريف، انخفض هذا العدد إلى 170 مليون خلية بكتيرية. في فصل الخريف انخفض العدد الى 70 مليون خلية بكتيرية، اما في فصل الربيع فقد ارتفع العدد الى 140 خلية بكتيرية. اما في المحطة الخامسة و األخيرة فقد بلغ العد الكلي للبكتريا الى 220 مليون خلية بكتيرية في فصل الصيف، و 110 في فصل الخريف، و انخفض الى 45 في فصل الشتاء، ثم ارتفع الى 175 مليون خلية بكتيرية في فصل الربيع.



الشكل)10(: يبين التغي ارت الفصلية والموقعية للعدد الكلي للبكتريا)10^6.خلية.مل$^{-1}$(في المياه خالل مدة الد ارسة

تعود الزيادة في قيم العدد اإلجمالي للبكتيريا إلى زيادة المغذيات من المواد العضوية وغير العضوية واألمالح بكميات كبيرة بحيث تكون بيئة ووسيلة مناسبة لنمو البكتيريا ، فضالً عن زيادة في تعكر المياه ودرجة ح اررة مناسبة لنمو ونشاط الكائنات الحية الدقيقة)الحمداني ، 2018(.واختبا ارت العدد اإلجمالي للبكتيريا هي مؤشر عام للتلوث الجرثومي، وهو معيار مهم لدرجة نقاء الماء ومدى خلوه من مسببات األم ارض المنقولة)الصفاوي وآخرون، 2012 (.

وتتفق الد ارسة الحالية مع ما توصل إليه كل من)AL-Hashimi وآخرون 2017(و AL-Azawi) وآخرون 2018(، وأعلى من النتائج التي حصل عليها)اللهيبي،2021(عند د ارسته لتقييم كفاءة محطتي اسالة ماء الشرقاط القديم والموحد في قضاء الشرقاط ومدى كفاءتهما في تصفية ماء الشرب التي ت اروحت فيها قيم العدد الكلي للبكتريا بين(51× 10^{-3}) إلى(160× 10^{-3}CFU/مل(.

الجدول)١(: مجموعة ألأنواع البكتيرية المعزولة من المحطات المدروسة حسب فصول السنة

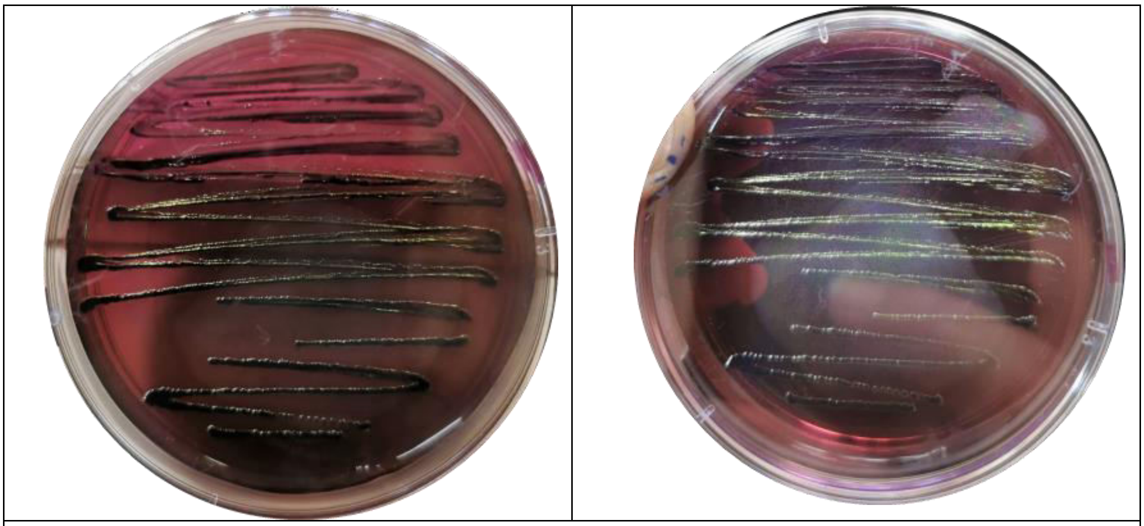
فصل الربيع	فصل الشتاء	فصل الخريف	فصل الصيف	المحطات
E.coli , Klebsiella pneumoniae Enterobacter aerogens	*E.coli , Klebsiella pneumoniae*	*Aeromonas sobria*	*Klebsiella oxytoca Staphylococcus aureus*	**محطة 1**
E.coli	*E.coli*	*Aeromonas sobria*	*E.coli Staphylococcus epidermidis Staphylococcus saprophyticus*	**محطة 2**
E.coli Klebsiella pneumoniae	*E.coli Klebsiella pneumonia Aeromonas sobria*	*Staphylococcus lentus Aeromonas veronii*	*E.coli Salmonella enterica Staphylococcus epidermidis Staphylococcus saprophyticus Staphylococcus aureus*	**محطة 3**
E.coli	*E.coli Citrobacter amalonaticus*	*Klebsiella pneumoniae*	*Klebsiella oxyoca E.coli Staphylococcus saprophyticus Staphylococcus aureus*	**محطة 4**
E.coli Aeromonas sobria	*E.coli*	*Citrobacter amalonaticus Ralstonia mannitolilytica*	*Enterobacter aerogens, Staphylococcus saprophyticus Staphylococcus aureus*	**محطة 5**

واظهرت النتائج في الجدول)1(األنواع البكتيرية المعزولة من المحطات المدروسة و حسب فصول السنة. تواجدت بكتريا *Salmonella sp* الذي يسبب داء الحمى المعوية)التيفوئيد والبا ارتيفوئيد(في فصل الصيف، كما يعتمد تواجد هذا الجنس البكتيري في الطبيعة على وجود الحيوانات ومن أهم العوامل التي تساعد على تواجد هذا الجنس البكتيري الطيور الداجنة والماشية والقوارض والقطط ويمكن أ ّنيستفيد من االنسان كعائل.

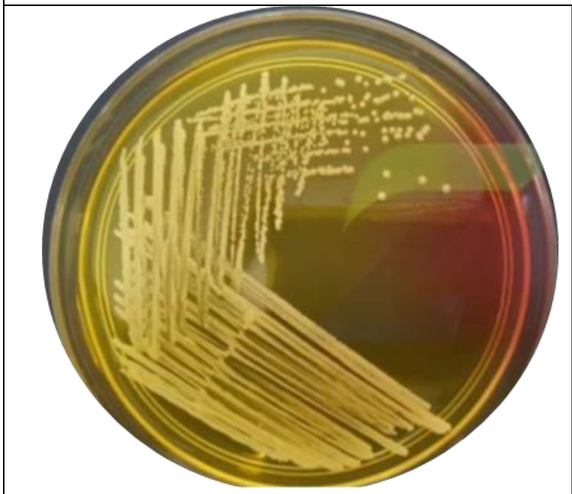
اظهرت العزله *Klebsiella pneumoiae* قدره على النمو في درجات ح ارة متفاوتة تت اروح ما بين)12- 43(مْ. يتواجد في الجهاز التنفسي والب ارز لحوالي 5% من االشخاص االصحاء ويسبب االلتهابات المزمنة منها إلتهاب الرئة ونتيجة تحملها لدرجات الح ارة المتفاوتة فأنها تكون متواجدة في أشهر الصيف والشتاء)االمام وآخرون، 2013 (.

قد تم عزل بكتريا القولون *Escherichia coli* بكثرة خالل أشهر الدراسة ولجميع العينات المدروسة وت ُعدالشريكية القولونية المؤشر الحقيقي الوحيد للتلوث الب ارزي الحديث، وهي تعود للعائلة المعوية التي تشابه بمفعولها بكتريا الكولي ڤورتنمو بدرجة ح ارة 37 مْ فضالً عن نموها بدرجة 44 م ْ.

وال يحدد موسم معين لزيادة أعاداد البكتريا القولونية بل ترتبط أعداد الزيادة والنقصان بحسب الوساط التي تعيش به ووفرة المغذيات المالءمة لنموهاا.ً مع ارتفاع الملوحاة والعناصار الثقيلة . إن تلوث الماء ببكتريا القولون يعاد مؤشا ار خطي ار حيث يجب أن يخلو ماء الشرب من أية خلية لبكتريا القولون في 100 مل)عبد الر ازق، 2017 (. إن لعملية مرور المياه خالل انابيب االسالة تأثير واضح في أعداد ومحتوى المياه من الملوثات العالقة أوالذائبة العضوية وغير العضوية وأن أهم هذه الملوثات هي تلك الناتجة من الفعاليات اليومية لألنسان والتي تشمل المخلفات المنزلية و مخلفات المستشفيات والمدارس والمحالت التجارية التي تختلط بمياه المجاري وتتسرب بقايا تلك الفضالت المتفسخة بصورة مباشرة أو غير مباشرة الى أنابيب االسالة مسّببة تلوث تلك المياه)عبد الر ازق، 2017(، وفي عملية تقدير العدد الكلي للبكتريا الهوائية من غير الممكن توفير ظروف مالئمة وتهيئة أوساط زرعية مناسبة لجميع انواع بكتريا المياه لذلك فأن أعداد البكتريا التي تنمو في االوساط الزرعية التعكس العدد الحقيقي وهي أقل منه بكثير)عبد الر ازق، 2017 (. ان وجود بكتريا القولون في المياه يعتبر داللة على عدم صالحيتها لإلستهالك البشري، وقد لوحظ تسجيل أعلى النتائج ولعدد من المواقع خالل فصل الشتاء في حين سجلت النتائج المنخفضة والنتائج التي تخلو من تواجد بكتريا القولون الكلية في فصل الربيع مع ارتفاع درجة الح ارة، إذ وجد إن هنالك عالقة عكسية بين أعداد بكتريا القولون الكلية ودرجة الح ارة كما في د ارسة)شرتوح وآخرون، 2014(التي أجريت بهدف د ارسة بعض الخصائص الفيزيائية والكيميائية والبكتريويوجية في قناة مجمع الجادرية الجامعي في بغداد أذ سجل أعلى معدل لبكتريا القولون الكلية للمحطة خالل المدة الشتوية وبلغت معدالتها 248±MPN17.1 / 100 مل و 12±1.1 MPN/ 100 مل خالل المدة الربيعية، أن أرتفاع درجة الح ارة وزيادة تأثير أشعة الشمس يثبط نمو األحياء المجهرية خالل فصل الربيع والصيف فضآل عن زيادة نشاط بعض الهائمات النباتية التي تتغذى على الكائنات المجهرية وباألخص البكتريا خالل هذه المدة فيما تتعكس هذه الظروف خالل مدة الشتاء مما يجعل العد الحي لألحياء المجهرية يزداد خالل هذه المدة)Sabae & Yehia، 2011(.



Escherichia coli



Staphylococcus aureus

Klebsiella pneumoiae

كل)6(: أن ع ك عز ن ه د ~

- **االستنتاجات :**

1. جميع الخواص الكيميائية والفيزيائية كانت ضمن الحدود المسموح بها للمواصفات القياسية والع ارقية .

2. المتغي ارت الشهرية خاصة فيما يتعلق بسقوط االمطار وارتفاع منسوب المياه لها تأثير كبير على الخصائص الفيزيائية والكيميائية والبيولوجية

لمياه مشروع تنقية مياه الشرب في منطقة الد ارسة . 3. ظهرت

الفحوصات البكتريولوجية ان العدد االجمالي للبكتريا كان موجودا عند مستويات عالية وتتجاوز المحددات الع ارقية ومحددات منظمة

الصحة العالمية .

المصادر :

بريشة، جابر ازيد؛ شريف، محمد احمد.)2018(.ملوثات المياه ومصادرها وطرق معالجتها ، دار النشر للجامعات، مصر.

المجمعي، غازي شمس صالح)٢٠٢٢(. تقييم بعض الخصائص الفيزيائية والكيميائية والبكتريولوجية لنهر االسحاقي في محافظة صالح الدين. رسالة ماجستير، كلية العلوم - جامعة تكريت. **الصفاوي،**

عبد العزيز يونس طليع؛ الطائي، نور ضياء .)2013(د ارسة بيئية وبكتريولوجية للفضالت السائلة من مستشفيات مدينة الموصل. مجلة تكريت للعلوم الصرفة.18)4(، 97-86.

Tortora, G. J. Funke, B. R and Case, C. L.(2007). Microbiology an introduction. Benjamin Cummings, San Francisco.

الساداني، إب ارهيم احمد حسين حسن.)2009(. د ارسة بيئية وبكتريولوجية لنهر دجلة ضمن محافظة صالح الدين، رسالة ماجستير، كلية العلوم، جامعة تكريت.

Ansama,M.,Saeed,G. and Mona,T.(2000). Bacterial contamination of drinking water supplies in Albo-saff village Mousal city first national sentific conference in environmental pollution and means of protection .Baghdad,5-6 Nov.

الدوري ،ايمان شاكر محمود.)2014(.مستويات بعض المركبات الهايدروكاربونية العطرية متعدد الحلقات)PAHs(في نهر دجله ضمن محافظة صالح الدين ،رسالة ماجستير ،كلية العلوم ،جامعة تكريت.

الزبيدي، ختام عباس مرهون.)2017(. تأثير التعاقب في اوقات الليل والنهار في مجتمعي الهائمات النباتية والحيوانية في نهر الديوانية، رسالة دكتو اره، كلية التربية، جامعة القادسية.

APHA, American Public Health Association. (2003). Standard Methods for the Examination of water and wastewater, (20thed).A.P.H.A.1015 Fifteenth Street, NW. Washington. DC, USA

عباوي، سعاد عبد، وحسين، محمد سليمان)1990(. الهندسة العلمية للبيئة. فحوصات المياه دار الحكمة للطباعة والنشر، جامعة الموصل، الموصل/ الع ارق

WHO (World Health Organization). (1996). Guideline for Drinking Water Quality Health Criteria and Other Supporting Information 2 nd. Ed. Vol. 21.Geneva.

العاني، فائز عزيز، وأمين سليمان بدوي)1990(. مبادئ األحياء المجهرية، و ازرة التعليم العالي والبحث العلمي، جامعة الموصل.

APHA. American public health Association. (1998). Standard methods for examination of water and wastewater. 20th ed .A.P.H.A., 101 S fifteenth street. New York.

اسماعيل، هال محمود)2018(. تأثير الملوثات البيئية على الطفيليات الخارجية في ثالثة أنواع من أسماك نهر دجلة. رسالة ماجستير، كلية التربية للبنات / جامعة تكريت.

منصور، معد عبد هلا)2019(. تأثير بعض الجوانب البيئية إلصابة ثالث أنواع من أسماك نهر ال ازب الصغير بالطفيليات الخارجية في مدينة كركوك. رسالة ماجستير. كلية التربية للبنات- جامعة تكريت.

محمد، ايناس حمدون)2021(. تأثير الظروف البيئية لمياه نهر ال ازب الصغير ضمن قضاء التون كوبري على اصابة ثالثة انواع من االسماك بالطفيليات الخارجية. رسالة ماجستير/ كلية التربية للبنات جامعة تكريت. الع ارق.

Weiner, E.R. (2000). Application of Environment Chemistry . Baco Raton .London .U.K. Lewis publisher CRC press LLC.273 pp.

Negi, P.; Mor, S. & Ravindra, K. (2020). Impact of landfill leachate on the groundwater quality in three cities of North India and health risk assessment. Environment, Development and Sustainability, 22(2):1455-1474.

Awad, A., Eldeeb, H. & El-Rawy, M. (2020). Assessment of surface and groundwater interaction using field measurements: A case study of Dairut City, Assuit, Egypt. Journal of Engineering Science and Technology, 15(1), 406-425.

Ibo, E. M.; Orji, M. U. & Umeh, O. R. (2020). Seasonal Evaluation of the Physicochemical Properties of Some Boreholes Water Samples in Mile 50, Abakaliki Ebonyi State. South Asian Journal of Researc.h in Microbiology,6(1):1-15.

البدري، أف ارح طعمة خلف مطر)2012(. د ارسة تأثير مياه مجاري سام ارء على نوعية المياه في نهر دجلة . رسالة ماجستير . كلية التربية للبنات / جامعة تكريت .

العبيدي، مروه محمد مديد)2019(. د ارسة مقارنة تشخيصية وبيئية للطفيليات الخارجية والمعوية لبعض أسماك نهر الزآب الصغير وأسماك األحواض االصطناعية في مدينة كركوك . رسالة ماجستير/ كلية التربية للبنات جامعة تكريت. الع ارق. **محمود، ايمان شاكر، عبد الجبار، رياض عباس وخرنوب، حسين حسن)٢٠١٨(.** الصفات الفيزيائية والكيميائية لمياه نهر دجلة ضمن محافظة صالح الدين/ الع ارق. مجلة جامعة تكريت للعلوم الز ارعية، 18)2(: 120-128.

الدددليمي، هبددة حمددد محمدددد وخمددديس، حميددد سددلمان.)2021(. د ارساة تشخيصااية لابعض انااواع الطحالاب المتواجادة فاي نهاار دجلاة المااار فااي مديناة الضاالوعية، صالح الدين.Samarra Journal of pure Applied Science, 3(3):95-107.

Ghazali, D. and Zaid, A. (2013). Etude de la qualité physico-chimique et bactériologique des eaux de la source Ain Salama-Jerri (Région de Meknès-Maroc). LARHYSS Journal ISSN 1112-3680 (12).

الحايك، نصر) 1989(. تلوث المياه و تنقيتها)ديوان المطبوعات الجامعية الجازئر(.

السعدي، حسن علي ناصر)2006(. أساسيات علم البيئة المائية.

الصفاوي، عبد العزيز يونس والبرواري، مشير رشيد احمد وخدر، نوزت خلف)2009(. دارسة الخصائص الطبيعية والكيمياوية والبيولوجية لمياه وادي دهوك. مجلة تكريت للعلوم الصرفة، 14)2(: 60-54. **الصفاوي،**

عبد العزيز يونس طليع)2007(. د ارسة صالحية المياه الجوفية لمنطقة الكونسية، ناحية حميدات لألغ ارض ال ازرعية. مجلة التربية و العلم،10)20(:191-204.

اب ارهيم، أحمد خليل)2010(. د ارسة نوعية المياه الجوفية لمناطق مختارة من محافظة نينوى. رسالة ماجستير/ كلية الهندسة/ قسم الهندسة المدنية/ جامعة تكريت/ تكريت/ الع ارق

Manahan, S.E. (2004). Environmental chemistry, CRC Press, 8th. Ed., Washington DC USA. 763.

الحمداني، أحمد شهاب أحمد)2018(. لتقييم WQI تطبيق معامل نوعية المياه. مياه بعض أحياء مدينة الموصل. رسالة ماجستير. كلية التربية للعلوم الصرفة - علوم الحياة.

الصفاوي، عبد العزيز يونس طليع؛ الصائغ، خالد سعيد؛ و الفاضلي، فائزة عزيز محمود)2012(. االمطار الحامضية الساقطة على مدينة الموصل. مجلة التربية و العلوم للعلوم الصرفة، المجلد)25(العدد)25(: 129.

Al-Hashimi, M. A.; Al-Bakri, S. A. and Okab,Ayah A.(2017). Assessment of WQI and Microbial pollution for plants in Baghdad city. Journal of University 235.

Alazawii, L. H.; Nashaat, M.R and Muftin, F.S (2018). ASSES (2018). Assessing the the Quality of Tigris Effects of Al- Rasheed Electrical Power Plant on the Quality River, Southern of Baghdad by Canadian Water Quality Indian WQI). Iraqi Journal of Science.59(3A): 1162-1168.

اللهيبي، عبدهللا محمود عجيل)2021(. تقييم كفاءة محطتي إسالة ماء الشرقاط القديم والموحد في قضاء الشرقاط ومدى كفاءتهما في تصفية ماء الشرب، رسالة ماجستير، كلية العلوم /جامعة تكريت.

االمام، محمد محمد وابو زويده، عبد الباسط رمضان)2013(. اساسيات التشخيص البكتريولوجي المعملي السريري. مركز بحوث التقنيات الحيوية. ط اربلس: 1- 160.

عبدالر ازق، هالة عبد الحافظ)2017(. التحري عن نوعية مياه الشرب للدور السكنية في منطقة حي المستنصرية/بغداد بداللة البكتريا المخاطية خالل أشهر الصيف لسنة 2016 . مجلة المستنصرية للعلوم . المجلد)1(28.

شرتوح، سفيان محمد وعبد المجيد، أحمد عالء الدين و الع ازوي، محمد نافع علي و محمد، أحمد جاسم و هادي، محمد أحمد عبد األمير)2014(. د ارسة بعض الخصائص الفيزيائية والكيميائية والبكتريولوجية في قناة مجمع الجادرية الجامعي، بغداد-الع ارق مجلة جامعة بابل . المجلد)22(8 .

Yehia, H. M., & Sabae, S. Z. (2011). Microbial pollution of water in El- Salam canal, Egypt. American-Eurasian Journal of Agricultrue & Environm ental Science, 11, 305–309.

Verma, D. K., Bhunia, G. S., Shit, P. K. and Tiwari, A. K. (2018). Assessment of groundwater quality of the central Gangetic plain area of India using geospatial and WQI techniques.Journal of the Geological Society of India,92(6):743-752.

Yaseen, B.R. ; Al-Asaady, K.A.; Kazem, A.A. & Chaichan, M.T. (2014). Environmental impacts of salt tide in Shatt Al-Arab Basra/Iraq. J. Environ. Sci. Toxicol. Food Technol. 10, 35–4.

Pradeep, V.; Deepika, C.; Urvi, G. & Hitesh, S.(2012). Water Quality Analysis of an Organically Polluted Lake by Investigating Different Physical and Chemical Parameters. Int. J. Res. Chem. Environ. 2, 105–111.

اسماعيل، هال محمود)2018(. تأثير الملوثات البيئية على الطفيليات الخارجية في ثالثة أنواع من أسماك نهر دجلة. رسالة ماجستير، كلية التربية للبنات / جامعة تكريت.

المجمعي، غازي شمس صالح)٢٠٢٢(. تقييم بعض الخصائص الفيزيائية والكيميائية والبكتريولوجية لنهر االسحاقي في محافظة صالح الدين. رسالة ماجستير، كلية العلوم - جامعة تكريت.

السعدون، ضياء وسام كامل)2021(. د ارسة بيئية عن الطحالب الطينية في ارفد ديالى / الع ارق. كلية التربية /جامعة سام ارء.

الشواني، طاووس محمد كامل احمد)٢٠٠١(. د ارسة بيئية و مايكروبولوجية لنهر ال ازب االسفل من منطقة التون كبري الى الحويجة محافظة التأميم . رسالة ماجستير ، كلية التربية للبنات . جامعة تكريت.

Salman, J. M; Hassan, F. M; Hadi, S. J. & Motar, A. A. (2014). An ecological study of epiphytic algae on two aquatic macrophytes in lotic ecosystem. Asian Journal of Natural and Applied Science; 3(3): 37-51.

مطر، عبدهللا احمد شويش)2021(. تأثير بعض العوامل البيئية على مياه نهر ال ازب األسفل وتقييم كفاءة محطة تصفية مياه الشرب في قضاء الحويجة. رسالة ماجستير، كلية التربية للعلوم الصرفة-جامعة تكريت.

الحمداوي، علي عبيد شعواط)2009(. االنتاجية األولية في نهر الدغارة . رسالة ماجستير. كلية التربية- جامعة القادسية .

الفتالوي، حسن جميل عواد)2005(. د ارسة بيئية لنهر الف ارت بين سدة الهندية وناحية الكفل-الع ارق. رسالة ماجستير- جامعة بابل.

Sharpio, R.F. & Fried, J.M. (1961). A practical hand book of seawater analysis . 2nd ed.Bulletin Fisheries Research Board of Canda .311PP.

العبيدي، مروه محمد مديد)2019(. د ارسة مقارنة تشخيصية وبيئية للطفيليات الخارجية والمعوية لبعض أسماك نهر الزآب الصغير وأسماك األحواض االصطناعية في مدينة كركوك . رسالة ماجستير/ كلية التربية للبنات جامعة تكريت. الع ارق.

محمد، ايناس حمدون)2021(. تأثير الظروف البيئية لمياه نهر ال ازب الصغير ضمن قضاء التون كوبري على اصابة ثالثة انواع من االسماك بالطفيليات الخارجية. رسالة ماجستير/ كلية التربية للبنات جامعة تكريت. الع ارق.

منصور، معد عبد هللا)2019(. تأثير بعض الجوانب البيئية إلصابة ثالث أنواع من أسماك نهر ال ازب الصغير بالطفيليات الخارجية في مدينة كركوك. رسالة ماجستير. كلية التربية للبنات- جامعة تكريت.

اسماعيل، هال محمود)2018(. تأثير الملوثات البيئية على الطفيليات الخارجية في ثالثة أنواع من أسماك نهر دجلة. رسالة ماجستير، كلية التربية للبنات / جامعة تكريت.

ISSN (Print): 2958-8995
ISSN (Online): 2958-8987

Doi: 10.59799 /APPP6605

Best one-sided algebraic approximation by average modulus

Raheam A. Al-Saphory[1,*], Abdullah A. Al-Hayani[2] and **Alaa A. Auad[3]

[1, 2] Department of Mathematics; College of Education for Pure Sciences;
Tikrit University, Salahaddin; IRAQ.
[3]Department of Mathematic; College of Education for Pure Sciences University of Anbar
; Ramadi; IRAQ.

*E-mail: *saphory@tu.edu.iq ** alaa.adnan.auad@uoanbar.edu.iq

Best one-sided algebraic approximation by average modulus

Raheam A. Al-Saphory[1,*], Abdullah A. Al-Hayani[2] and **Alaa A. Auad[3]

[1, 2] Department of Mathematics; College of Education for Pure Sciences; Tikrit University, Salahaddin; IRAQ.
[3]Department of Mathematic; College of Education for Pure Sciences University of Anbar; Ramadi; IRAQ.

*E-mail: *saphory@tu.edu.iq ** alaa.adnan.auad@uoanbar.edu.iq

Abstract

The aim of this work is to introduced the concept of the best one-sided approximation of unbounded functions in weighted space by using algebraic operators in terms the average modulus of smoothness. We also show an estimate of the degree of best one-sided approximation of unbounded functions in the terms of average modulus of smoothness.

Keywords: weighted space, algebraic polynomial unbounded function, and average modulus of smoothness.

1. INTRODUCTION

Ronald [1] in [1968] studied the problem of approximation in a normed linear space has associated with it a dual problem of maximizing functionals. Thus, Doronin and Ligun [2] discussed the problem of the one-sided approximation of functions by n-dimensional subspaces and find the exact value of the best one-sided approximation of the class $W^r L_1$. So, Nurnberger [3] studied Unity in one-sided L1-approximation and quadrature formulae. Babenko and Glushko [4] studied the problem of the uniqueness of elements of the best approximations in the space $L_1[a,b]$ and expressed the problem of the best approximation and the best (α,β)-approximation of continuous functions and the problem of the best one-sided approximation of continuously differentiable functions. Thus, Yang [5] presented one sided L_p norm and best approximation in one sided L_p norm. So, Motornyi and Sedunova [6] found the best one-sided approximations of the class W_∞^1 of differentiable functions by algebraic polynomials in L_1 space. Thus, Bustamante et al. [7] studied polynomials of the best one-sided approximation to a step function on [-1,1] and they proved that polynomials are obtained by Hermite interpolation at the zeros of some quasi orthogonal Jacobi polynomial. So, Alexander [8] in 2016 found the Best One-Sided Approximation of Some Classes of Functions of Several Variables by Haar Polynomials defined using modulus of continuity $\omega(f,t)$ and $\omega_{\rho i}(f,\delta)$ and in the same year, Alaa and Mousa [9] studied some positive factors for the one-sided approximation of the infinite functions in the weighted space $L_{p,\alpha}(X)$ and give an estimate of the degree of the best one-sided approximation in terms of the mean continuity coefficient Thus, Sedunova [10] studied the best one-sided approximation for the class of differentiable functions by algebraic polynomials in the mean. Also, Jianbo and Jialin [11] presened the Strong and Weak Convergence Rates of a Spatial Approximation for Stochastic Partial Differential Equation with One-sided Lipschitz Coefficient. Furthermore, Raad et al. [12] found the best one-sided multiplier approximation of unbounded functions by trigonometric polynomials in term of averaged modulus.

2. PRILIMINARIES

Continuing our previous investigations on polynomial operators for one-sided approximation to unbounded functions in weighted space (see [5]), it is the aim of this paper to develop a notion of direct estimation polynomial approximation with constructs which fits, to gather with results (see [8] and [9]) for unbounded function approximation processes.

Let $X = [0,1]$, we denoted by $L_{p,\beta}(X), 1 \leq p < \infty$ be the space of all unbounded functions that defined on X, with every function in this space has the norm given by

$$\|\rho\|_{p,\beta} = \left(\int_X |\rho(x)|^p\right)^{\frac{1}{p}} < \infty. \quad (1)$$

Let W be the suitable set of all weight functions on X, such that $|\rho(x)| \leq M\beta(x)$, where M is positive real number and

$\beta : X \to \mathbb{R}$ weight function, which are equipped with the following norm

$$\|\rho\|_{p,\beta} = \left(\int_X \left|\frac{\rho(x)}{\beta(x)}\right|^p\right)^{\frac{1}{p}} < \infty. \quad (2)$$

The local modulus of continuity of a function $\rho : [0,1] \to \mathbb{R}$ in the pint x is denoted by

$$\omega_k(\rho, x, \delta)_{p,\beta} = \sup\{\left|\Delta_r^k \rho(x)\right| : x, x + rk \in [x - \frac{\delta k}{2}, x + \frac{\delta k}{2}] \quad (3)$$

such that

$$\Delta_r^k(\rho, x) = \sum_{r=0}^{k} (-1)^{r+k} \binom{k}{r} \rho(x + r\delta) \ if \ x, x + r\delta \in X \quad (4)$$

The average modulus of smoothness we defined by

$$\tau_k(\xi, \delta)_{p,\beta} = \|\omega_k(\rho, ., \delta)\|_{p,\beta}. \quad (5)$$

Let $\mathbb{N}$ be the set of natural numbers and $\mathbb{H}_k$ the set of all algebraic polynomials of degree less than or equal to $k \in \mathbb{N}$.

The degree of best one-sided approximation in $L_{p,\beta}(X), 1 \leq p < \infty$, of unbounded function as ρ by operators $\mathcal{P}_k$ & q_k are denoted by:

$$\tilde{\mathcal{E}}_k(\rho, x)_{p,\beta} = inf\{\|\rho - \mathcal{P}_k\|_{p,\beta} \ : \mathcal{P}_k \in \mathbb{H}_k \} \quad (6)$$

$$\tilde{\mathcal{E}}_k(\rho,x)_{p,\beta} = inf\{\|q_k - \mathcal{P}_k\|_{p,\beta} : \mathcal{P}_k, q_k \in \mathbb{H}_k \& \mathcal{P}_k(x) \le \rho(x) \le q_k(x)\}. \quad (7)$$

It easy to verify that there are not linear operators for one-sided approximation in X. Some non-linear construction have been proposed in [3] and [6].

Let us consider the step function

$$\Phi(x) = \begin{cases} 0, & if -1 \le x \le 0, \\ 1, & if \ \ 0 < x \le 1, \end{cases} \quad (8)$$

determine two collections of functions $\{\mathcal{P}_k\}$ and $\{q_k\}, \mathcal{P}_k,\ q_k \in \mathbb{H}_k$ such that

$$\mathcal{P}_k(x) \le \Phi(x) \le q_k(x),\ x \in [-1,1] \quad (9)$$

and

$$C_k = \|\rho - \mathcal{P}_k\|_{p,\beta} \to 0,\ p = 1 \quad (10)$$

For the first one we work in space $L_{p,\beta}(X)$. For $1 \le p < \infty$, we construct two different sequences of operators, for $x \in X$, $k \in \mathbb{N}$ and $\rho \in L_{p,\beta}(X)$ define

$$\mathcal{M}_k(\rho,x) = \rho(0) + \int_0^1 \mathcal{P}_k(t-x)\,\rho'_+(t)dt - \int_0^1 q_k(t-x)\,\rho'_-(t)dt \quad (11)$$

and

$$\mathcal{N}_k(\rho,x) = \rho(0) + \int_0^1 q_k(t-x)\,\rho'_+(t)dt - \int_0^1 \mathcal{P}_k(t-x)\,\rho'_-(t)dt \quad (12)$$

it is clear $\mathcal{M}_k(\rho), \mathcal{N}_k(\rho) \in \mathbb{H}_k$, we will prove that

$\mathcal{M}_k(\rho,x) \le \rho(x) \le \mathcal{N}_k(\rho,x), x \in X$ and both

$\|\rho - \mathcal{M}_k(\rho)\|_{p,\beta} \le C_k\|\rho'\|_{p,\beta}$ and

$\|\rho - \mathcal{N}_k(\rho)\|_{p,\beta} \le C_k\|\rho'\|_{p,\beta}$, where C_k be as in (10).

In the second part, for function $L_{p,\beta}(X)$, we construct operators

$$G_y(\rho,x) = \int_0^1[\rho(1-y)x + yt) - \omega(\rho,(1-y)x + yt, y)]dt \quad (13)$$

and

$$H_y(\rho,x) = \int_0^1[\rho(1-y)x + yt) + \omega(\rho,(1-y)x + yt, y)]dt. \quad (14)$$

It is clear $G_k(\rho), H_k(\rho) \in \mathbb{H}_k$ and so we can define

$$\mathcal{L}_{k,y}(\rho,x) = \mathcal{M}_k(G_y(\rho),x) \quad (15)$$

$$\mathcal{J}_{k,y}(\rho, x) = \mathcal{N}_k\big(H_y(\rho), x\big), \qquad (16)$$

Where $\mathcal{M}_k(\rho)$ and $\mathcal{N}_k(\rho)$ by as equations (11) and (12) in that order.

We will prove that

$$\mathcal{L}_{k,y}(\rho, x) \leq \rho(x) \leq \mathcal{J}_{k,y}(\rho, x), \ x \in \mathrm{X} \text{ and}$$

present the degree of best one-sided approximation of unbounded functions by operators $\mathcal{L}_{k,y}(\rho, x)$ and $\mathcal{J}_{k,y}(\rho, x), x \in \mathrm{X}$ in terms averaged modulus of continuity.

In the last years there has been interest in studying open problems related to one-sided approximations (see [1] , [2]).

We point out that other operators for one-sided approximations have constructed in [7].

In particular, the operators presented in [6] yield the non-optimal rate $O(\tau\left(\rho, \frac{1}{\sqrt{k}}\right))$ where is ones consider in [4] give the optimal rate, but without an explicit constant. The paper is organized as follows. In section (3) we calculate the degree of best one-sided approximation of unbounded functions by mean of the operators define (13) and (14).

Finally in the some section, we consider the degree of the best onesided approximation by mean of the operators defined in (15) and (16)

3. AUXILIARY LEMMAS

Lemma 3.1:

Let $\rho \in L_{p,\beta}[0,1], \ y \in (0,1)$, and the operators $G_y(\rho) \& H_y(\rho)$ are defined through equations (13) and (14) in that order.

Then, $G_y(\rho, x) \leq \rho(x) \leq H_y(\rho, x), \ x \in X = [0,1]$ and

$$Max\{\|G_y'\|_{p,\beta}, \|H_y'\|_{p,\beta}\} \leq \frac{3}{k}\tau_k(\rho, y)_{p,\beta}.$$

Lemma 3.2:

Let $\Phi(x)$ be given in equation (8). Every $x \in [-1,1]$ we known

$$\mathcal{P}_k(\rho) = T_k^-(arc\ cosx) \text{ and}$$

$$q_k(\rho) = T_k^+(arc\ cosx).$$

Then, $\mathcal{P}_k, q_k \in \mathbb{H}_k, \mathcal{P}_k(\rho) \leq \rho(x) \leq q_k(\rho), \ x \in [-1,1]$ and

$$\|q_k(\rho) - \mathcal{P}_k(\rho)\|_{p,\beta} \leq \frac{4\pi^2}{k+2}.$$

Lemma 3.3:

Let $\rho \in L_{p,\beta}(X)$,$1 \leq p < \infty$, $k \in \mathbb{N}$, and $k \geq 2$. Let $\mathcal{M}_k(\rho)$ and $\mathcal{N}_k(\rho)$ by as equations (11) and (12) in that order. Then,

$$\mathcal{M}_k(\rho), \mathcal{N}_k(\rho) \in \mathbb{H}_k \text{ and}$$

$$\mathcal{M}_k(\rho, x) \leq \rho(x) \leq \mathcal{N}_k(\rho, x), x \in X = [0,1].$$

Proof:

From equations (9), (10), (11) and (12), it is clear that

$\mathcal{M}_k(\rho), \mathcal{N}_k(\rho) \in \mathbb{H}_k$. We have

$$\mathcal{M}_k(\rho, x) = \rho(0) + \int_0^1 \mathcal{P}_k(t - x)\, \rho'_+(t)dt - \int_0^1 q_k(t - x)\, \rho'_-(t)dt$$

where $\mathcal{P}_k, q_k \in \mathbb{H}_k$, such that $\mathcal{P}_k(\rho) \leq \rho(x) \leq q_k(\rho),\ x \in X$

and $\|\mathcal{P}_k - q_k\|_{p,\beta} \to 0$

since, $\mathcal{P}_k(\rho) \leq \rho(x) \leq q_k(\rho), x \in [0,1]$,

thus

$$\mathcal{M}_k(\rho, x) \leq \rho(0) + \int_0^1 \Phi(t - x)\, \rho'_+(t)dt - \int_0^1 \Phi(t - x)\, \rho'_-(t)dt$$

$$= \rho(0) + \int_0^1 \Phi(t - x)\rho'(t)dt$$

$$= \rho(0) + \rho(x) - \rho(0)$$

$$= \rho(x).$$

So,

$$\rho(x) = \rho(0) + \rho(x) - \rho(0)$$

$$= \rho(0) + \int_0^1 \rho'(t)dt$$

$$= \rho(0) + \int_0^1 \Phi(t - x)\, \rho'(t)dt$$

$$= \rho(0) + \int_0^1 \Phi(t - x)\, \rho'_+(t)dt - \int_0^1 \Phi(t - x)\, \rho'_-(t)dt$$

$$\leq \rho(0) + \int_0^1 q_k(t - x)\, \rho'_+(t)dt - \int_0^1 \mathcal{P}_k(t - x)\, \rho'_-(t)dt$$

$$= \mathcal{N}_k(\rho, x).$$

Lemma 3.4:

For $\rho \in L_{p,\beta}(X)$,$1 \leq p < \infty$, $k \in \mathbb{N}$, and $k \geq 2$. Let $\mathcal{M}_k(\rho)$ and $\mathcal{N}_k(\rho)$ by as equations (11) and (12) in that order. Then,

$$Max\{\|\rho-\mathcal{M}_k(\rho)\|_{p,\beta},\|\rho-\mathcal{N}_k(\rho)\|_{p,\beta}\}\le C_k\|\rho'\|_{p,\beta}.$$

Proof:

Since

$$|\rho(x)-\mathcal{M}_k(\rho,x)|\le\int_{-x}^{1-x}(q_k(h)-\mathcal{P}_k(h))|\rho'(x+h)|dh,$$

We put $\alpha_k(h)=q_k(h)-\mathcal{P}_k(h)$ and from Holder's inequity, we get

$$(\|\rho-\mathcal{M}_k(\rho)\|_{p,\beta})^p\le\int_0^1\left|\frac{\int_{-x}^{1-x}\alpha_k(h)|\rho'(x+h)|dh}{\beta(x)}\right|^p dx$$

$$\le\int_0^1(\left|\int_{-x}^{1-x}\alpha_k(h)dh\right|^{p-1})(\left|\frac{\int_{-x}^{1-x}\alpha_k(h)|\rho'(x+h)|^p dh}{\beta(x)}\right|)\,dx$$

$$\le(\int_0^1|\alpha_k(u)|^{p-1}du)(\int_0^1\left|\frac{\rho'(v)}{\beta(v)}\right|^p(\int_{v-1}^{v}\frac{\alpha_k(h)}{\beta(h)}dh)\,dv)$$

$$\le(\int_0^1|\alpha_k(u)|^p du)(\int_0^1\left|\frac{\rho'(v)}{\beta(v)}\right|^p dv)$$

Thus

$$\|\rho-\mathcal{M}_k(\rho)\|_{p,\beta}\le(\int_0^1|\alpha_k(u)|^p du)^{1/p}(\int_0^1\left|\frac{\rho'(v)}{\beta(v)}\right|^p dv)^{1/p},$$

Hence

$$\|\rho-\mathcal{M}_k(\rho)\|_{p,\beta}\le\|\alpha_k\|_p\|\rho'\|_{p,\beta}=C_k\|\rho'\|_{p,\beta}\,.$$

Likewise, we show that,

$$\|\rho-\mathcal{N}_k(\rho)\|_{p,\beta}\le C_k\|\rho'\|_{p,\beta}.$$

4. MAIN RESULTS

Theorem 4.1:

Let $\rho\in L_{p,\beta}(\mathrm{X}), 1\le p<\infty,\ k\in\mathbb{N}$, and $k\ge 2.$ Let $G_y(\rho)\&\,H_y(\rho)$ are defined through equations (13) and (14) in that order.

Then,

$$Max\{\|\rho-G_y(\rho)\|_{p,\beta},\|\rho-H_y(\rho)\|_{p,\beta}\}\le C_1(y,p)\tau_k(\rho,y)_{p,\beta}$$

and

$$\tilde{\mathcal{E}}_k(\rho)_{p,\beta}\le C_k(y,p)\tau_k(\rho,y)_{p,\beta}.$$

Proof:

It is used to, take q such that $1/p+1/q=1$, from equations (13), (14) and Holder's inequality, we get

$$(y\|\rho - G_y(\rho)\|_{p,\beta})^p = y^p \int_0^1 \left|\frac{\rho(x)-G_y(\rho,x)}{\beta(x)}\right|^p dx$$

$$\leq y^p \int_0^1 \left|\frac{H_y(\rho,x)-G_y(\rho,x)}{\beta(x)}\right|^p dx$$

$$\leq 2^p y^p \int_0^1 \int_0^y \left|\frac{\omega(\rho,(1-y)x+yt,y)}{\beta((1-y)x)}\right|^p dtdx.$$

Put $h = (1-y)x$ implies $dh = (1-y)dx$

$$(y\|\rho - G_y(\rho)\|_{p,\beta})^p \leq \frac{2^p y^p}{1-y} \int_0^y \int_t^{1-y+t} \left|\frac{\omega(\rho,h,y)}{\beta(h)}\right|^p dhdt$$

$$\leq \frac{2^p y^{p/q}}{1-y} \int_0^y \int_0^1 \left|\frac{\omega(\rho,h,y)}{\beta(h)}\right|^p dhdt$$

$$\leq \frac{2^p y^{\frac{p}{q}+1}}{1-y} \int_0^1 \left|\frac{\omega(\rho,h,y)}{\beta(h)}\right|^p dh$$

Thus,

$$\left\|\rho - G_y(\rho)\right\|_{p,\beta} \leq \frac{2}{(1-y)^{1/p}} \left(\int_0^1 \left|\frac{\omega(\rho,h,y)}{\beta(h)}\right|^p dh\right)^{1/p}$$

$$\leq \frac{2}{(1-y)^{1/p}} \|\omega(\rho,.,y)\|_{p,\beta}$$

$$= \frac{2}{(1-y)^{1/p}} \tau_k(\rho,y)_{p,\beta}$$

We have $\frac{2}{(1-y)^{1/p}}$ constant depending on y and p, then

$$\left\|\rho - G_y(\rho)\right\|_{p,\beta} \leq C_1(y,p)\tau_k(\rho,y)_{p,\beta}.$$

Analogously, we can show $\left\|\rho - H_y(\rho)\right\|_{p,\beta} \leq C_1(y,p)\tau_k(\rho,y)_{p,\beta}$.

We go to the following inequality:

$$\tilde{\mathcal{E}}_k(\rho)_{p,\beta} \leq \left\|H_y(\rho) - G_y(\rho)\right\|_{p,\beta}$$

$$\leq \left\|\rho - H_y(\rho)\right\|_{p,\beta} + \left\|\rho - G_y(\rho)\right\|_{p,\beta}$$

$$\leq C_k(y,p)\tau_k(\rho,y)_{p,\beta}.$$

Theorem 4.2:

Let $\rho \in L_{p,\beta}(X), 1 \leq p < \infty,\ k \in \mathbb{N}$. Let $\mathcal{L}_{k,y}(\rho)$ and $\mathcal{J}_{k,y}(\rho)$ are defined through equations (13) and (14) in that order.

Then,

$$\mathcal{L}_{k,y}(\rho,x) \leq \rho(x) \leq \mathcal{J}_{k,y}(\rho,x),\ x \in X$$

$$Max\,\{\|\rho - \mathcal{L}_{k,y}(\rho)\|_{p,\beta}, \|\rho - \mathcal{J}_{k,y}(\rho)\|_{p,\beta}\} \leq (C_1(y,p) + \frac{3C_k}{y})\tau_k(\rho,y)_{p,\beta}$$

and

$$\tilde{\mathcal{E}}_k(\rho)_{p,\beta} \leq (C_1(y,p) + \frac{6C_k}{y})\tau_k(\rho,y)_{p,\beta}.$$

Proof:

Let $G_y(\rho)$ and $H_y(\rho)$ are defined through equations (13) and (14) in that order.

So, from equations (15) and (16), it is clear $\mathcal{L}_{k,y}(\rho), \mathcal{J}_{k,y}(\rho) \in \mathbb{H}_k$.

Also, from equations (15), (16), Theorem 4.1, Lemma 3.3, Lemma 3.4 and Lemma 3.1, since

$$\mathcal{L}_{k,y}(\rho,x) = \mathcal{M}_k\left(G_y(\rho),x)\right) \leq G_y(\rho,x) \leq \rho(x)$$

$$\leq H_y(\rho,x) \leq \mathcal{N}_k\left(H_y(\rho),x)\right) = \mathcal{J}_{k,y}(\rho,x),\ x \in [0,1].$$

Moreover,

$$\begin{aligned}
\|\rho - \mathcal{L}_{k,y}(\rho)\|_{p,\beta} &\leq \|\rho - G_y(\rho)\|_{p,\beta} + \|G_y(\rho) - \mathcal{L}_{k,y}(\rho)\|_{p,\beta} \\
&\leq C_1(y,p)\tau_k(\rho,y)_{p,\beta} + \left\|\rho - \mathcal{M}_k\left(G_y(\rho,x)\right)\right\|_{p,\beta} \\
&\leq C_1(y,p)\tau_k(\rho,y)_{p,\beta} + C_k\|G'_y(\rho)\|_{p,\beta} \\
&= C_1(y,p)\tau_k(\rho,y)_{p,\beta} + C_k\left\|\frac{G'_y(\rho,.)}{\beta(.)}\right\|_p \\
&\leq C_1(y,p)\tau_k(\rho,y)_{p,\beta} + \frac{3C_k}{y}\tau_k(\frac{\rho}{\beta},y)_p \\
&= C_1(y,p)\tau_k(\rho,y)_{p,\beta} + \frac{3C_k}{y}\tau_k(\rho,y)_{p,\beta} \\
&= (C_1(y,p) + \frac{3C_k}{y})\tau_k(\rho,y)_{p,\beta}.
\end{aligned}$$

The approximation for $\|\rho - \mathcal{J}_{k,y}(\rho,x)\|_{p,\beta}$ follows similarly.

Thus,

$$\begin{aligned}
\tilde{\mathcal{E}}_k(\rho)_{p,\beta} &\leq \|\mathcal{J}_{k,y}(\rho,x) - \mathcal{L}_{k,y}(\rho)\|_{p,\beta} \\
&\leq \|\rho - \mathcal{L}_{k,y}(\rho)\|_{p,\beta} + \|\rho - \mathcal{J}_{k,y}(\rho,x)\|_{p,\beta}
\end{aligned}$$

$$\leq 2(C_1(y,p)+\frac{3C_k}{y})\tau_k(\rho,y)_{p,\beta}$$

$$\leq (C_1(y,p)+\frac{6C_k}{y})\tau_k(\rho,y)_{p,\beta}.$$

Theorem 4.3:

Let $\rho \in L_{p,\beta}(\mathrm{X}), 1 \leq p < \infty,\ k \in \mathbb{N}$, and $k \geq 2$. Let $\mathcal{P}_k(\rho)$ and $q_k(\rho)$ be the sequence of polynomials constructed as in (9), set

$$A_k(\rho) = \mathcal{L}_{k,\frac{1}{k}}(\rho) \text{ and } B_k(\rho) = \mathcal{J}_{k,\frac{1}{k}}(\rho),$$

where

$\mathcal{L}_{k,\frac{1}{k}}(\rho)$ and $\mathcal{J}_{k,\frac{1}{k}}(\rho)$ are given as (15) and (16) respectively.

Then

$$A_k(\rho,x) \leq \rho(x) \leq B_k(\rho,x),\ x \in \mathrm{X},$$

$$Max\,\{\|\rho - A_k(\rho)\|_{p,\beta}, \|\rho - B_k(\rho)\|_{p,\beta}\} \leq (C_1(y,p)+\frac{3C_k}{y})\tau_k(\rho,\frac{1}{k})_{p,\beta}$$

and

$$\tilde{\mathcal{E}}_k(\rho)_{p,\beta} \leq 2(C_k(y,p)+\frac{12k\pi^2}{k+2})\tau_k(\rho,\frac{1}{k})_{p,\beta}.$$

Proof:

Using equations (15) and (16) with $y=\frac{1}{k}$ and $k \geq 2$, we get

$$\mathcal{L}_{k,\frac{1}{k}}(\rho,x) = \mathcal{M}_k\left(G_{\frac{1}{k}}(\rho,x)\right) \text{ and } \mathcal{J}_{k,\frac{1}{k}}(\rho,x) = \mathcal{N}_k\left(H_{\frac{1}{k}}(\rho,x)\right)$$

where

$G_{\frac{1}{k}}(\rho), H_{\frac{1}{k}}(\rho) \in \mathbb{H}_k$. Also

$$\mathcal{M}_k\left(G_{\frac{1}{k}}(\rho)\right), \mathcal{N}_k\left(H_{\frac{1}{k}}(\rho)\right) \in \mathbb{H}_k$$

Using Lemma 3.3, since $\mathcal{M}_k(\rho,x) \leq \rho(x) \leq \mathcal{N}_k(\rho,x),\ x \in \mathrm{X}$.

Hence, $A_k(\rho,x) \leq \rho(x) \leq B_k(\rho,x),\ x \in \mathrm{X}$.

We need an approximate for $\|\rho - A_k(\rho)\|_{p,\beta}$ one has:

Using (15), Lemma 3.2 and Theorem 4.2 we obtain

$$\|\rho - A_k(\rho)\|_{p,\beta} = \left\|\rho - \mathcal{L}_{k,\frac{1}{k}}(\rho)\right\|_{p,\beta} \leq (C_k(y,p) + \frac{3C_k}{\frac{1}{k}})\tau_k(\rho,\frac{1}{k})_{p,\beta}$$

$$\leq (C_k(y,p) + \frac{12k\pi^2}{k+2})\tau_k(\rho,\frac{1}{k})_{p,\beta}.$$

Analogously, we can show

$$\|\rho - B_k(\rho)\|_{p,\beta} \leq (C_k(y,p) + \frac{12k\pi^2}{k+2})\tau_k(\rho,\frac{1}{k})_{p,\beta}.$$

Thus,

$$\tilde{\mathcal{E}}_k(\rho)_{p,\beta} \leq \|B_k(\rho) - A_k(\rho)\|_{p,\beta}$$

$$\leq \|B_k(\rho) - \rho\|_{p,\beta} + \|\rho - A_k(\rho)\|_{p,\beta}$$

$$\leq 2(C_k(y,p) + \frac{12k\pi^2}{k+2})\tau_k(\rho,\frac{1}{k})_{p,\beta}.$$

6. REFERENCES

[1] R. Devore, One-Sided Approximation of Functions, JOURNAL OF APPROXIMATION THEORY 1, p.p. 11-25, (1968).

[2] V. G. Doronin and A. A. Ligun, Upper bounds for the best one-sided approximation by splines of the classes $\mathbf{W^r L_1}$, Mathematical notes of the Academy of Sciences of the USSR, volume 19, p.p. 7–10, (1976).

[3] G. Nurnberger, Unicity in one-sided L_1-approximation and quadrature formulae, Journal of Approximation Theory, Volume 45, Issue 3, (1985), p.p. 271-279.

[3] V. F. Babenko and V. N. Glushko, On the uniqueness of elements of the best approximation and the best one-sided approximation in the space L_1, Ukrainian Mathematical Journal, volume 46, p.p. 503–513 (1994).

[4] C. Yang, one sided L_p norm and best approximation in one sided L_p norm, The Rocky Mountain Journal of Mathematics, **41**(5), (2011), p.p. 1725-1740.

[5] P. Motornyi and V. Sedunova, best one-sided approximations of the class of differentiable functions by algebraic polynomials in L_1 space, Vol. 20, p.p. , (2012).

[6] J. Bustamante, R. Martinez-Cruz and J. M. Quesada, Quasi orthogonal Jacobi polynomials and best one-sided approximation to step functions, Journal of Approximation Theory, Volume 198, (2015), p.p. 10-23

[7] Alexander N. Shchitov, Best One-Sided Approximation of Some Classes of Functions of SeveralVariables by Haar Polynomials, International Journal of Advanced Research in Mathematics, Vol. 6, p.p. 42-50, (2016).

[8] A. A. Auad and M. M. Khrajan, Direct Estimation for One-Sided Approximation By Polynomial Operators, MJPS, **3** (2), p.p. 54-59, (2016).

[9] V. Sedunova, Best One-Sided Approximation for the Class of Differentiable Functions by Algebraic Polynomials in the Mean, Ukrainian Mathematical Journal, **69**(2), p.p. 1-14, (2017).

[10] J. Cui and J. Hong, Strong and Weak Convergence Rates of a Spatial Approximation for Stochastic Partial Differential Equation with One-sided Lipschitz Coefficient, SIAM Journal on Numerical Analysis, Vol. 57, Iss. 4, p.p. 1545-2042, (2019).

[11] Raad F. Hassan, Saheb K. Al-Saidy and Naseif J. Al-Jawari, Best One-Sided Multiplier Approximation of Unbounded Functions by Trigonometric Polynomials, Al-Nahrain Journal of Science, **24** (4), (2021), p.p. 40-45.