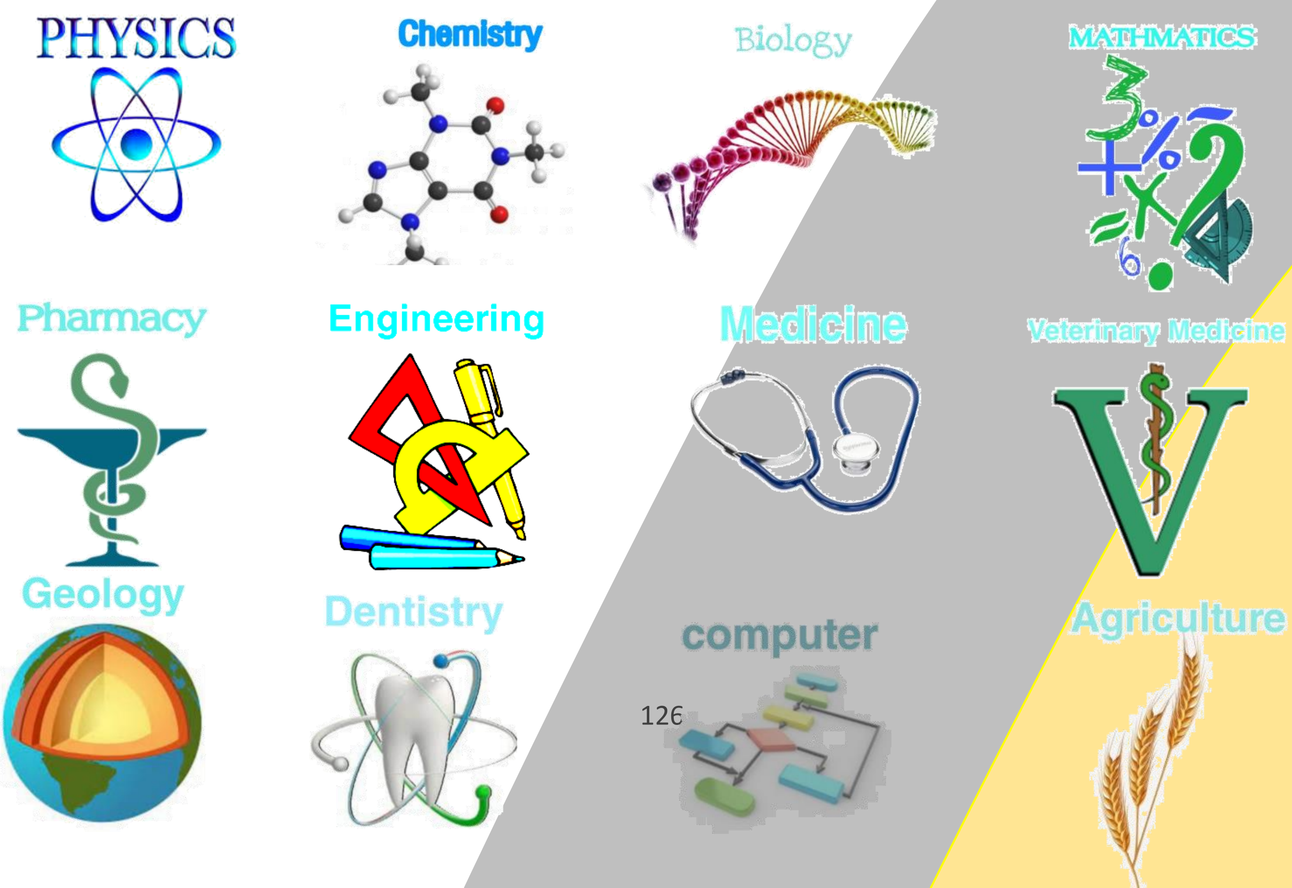
ISSN: 2958-8995. 2958-8987
Doi: 10.59799/APPP6605
No: 5 Val:2/ March/ 2024

Journal of Natural and Applied Sciences URAL

A Quarterly Multidisciplinary Scientific Journal Issued by European Academy for Development and Research / Brussels and Center of Research and Human Resources Development Ramah- Jordan



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Study of bacteremia and some factors effect on diabetic patients in Fallujah city

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ABSTRACT

This study was designed to study bacteremia and the impact of age, gender, family history, as well as the disease status on diabetic patients in Fallujah, Iraq.175 samples were collected, from (160) diabetic and (15) non-diabetic from October to November 2021. Blood culture was conducted to isolate and identify the bacteria and its antibiotic susceptibility these were evaluated for each isolate. The cultural, microscopically and biochemical results show a presence of Kocuria spp is mainly opportunistic bacteria that causes bacteremia, in our study it represents higher number compared with others. This followed by Enterobacter cloacae and Klebsiella pneumoniae. Norfloxacin and Imipenem are the most effective antibiotic from 14 antibiotics used here toward all of isolated bacteria. Female's glucose levels are higher than males, ranged from (227 to 181 mg/dL respectively). Moreover,the patients whose ages were 65-74 years show the highest glucose levels among other age groups . The study revealed significant effect of age on blood glucose levels. The mean of blood glucose in chronic disease was (277.689 ± 2) mg/dL for all age groups, while the mean of the acute(novel)disease was (238.838 ± 3) mg/dL for all age groups within 95% confidence limit. Moreover the healthy control group shows a normal blood glucose levels (122.276 ± 8) mg/dL within 95% confidence limit. The patients of diabetes mellitus with registered history of diabetes in their families show the highest blood glucose levels with more risk of disease..

Key words: Bacteremia, Diabetes, Kocuria, Enterobacter cloacae, Family history.

الملخص :

صممت هذه الدراسة للكشف عن وجود تجرثم الدم وكذلك دراسة تأثير العمر والجنس والتاريخ العائلي و الحالة المرضية على مرضى السكري في مدينة الفلوجة ,العراق. تم جمع 175 عينة , من(160) مصاب بالسكري و (15) غير مصاب بالسكري بين شهر أكتوبر إلى نوفمبر 2021. تم إجراء اختبارات و زراعة الدم لعزل وتحديد البكتيريا وحساسيتها للمضادات الحيوية . أظهرت النتائج الزرعية والمجهرية والكيميائية الحيوية التي تم تقييمها لكل عينة وجود *Kocuria* spp وهي بكتريا انتهازية تسبب تجرثم الدم وكانت الاكثر عدداً من غيرها في دراستنا الحالية تلتها البكتريا المعوية المذرقية E. cloacae وبكتريا الكلبسيلة الرئوية *K. pneumonia* . من بين 14 مضاد حيوي استخدمت هنا تجاه جميع البكتيريا المعزولة كان مضادي Norfloxacin و Imipenem الاكثر فعالية تجاه العزلات البكتيرية كما اظهرت الدراسة أن مستويات الجلوكوز لدى الإناث أعلى منها لدى الذكور، وتراوحت من (227 mg/dL إلى 181 على التوالي). علاوة على ذلك، فإن المرضى الذين تتراوح أعمارهم بين 65-74 سنة يظهرون أعلى مستويات الجلوكوز بين الفئات العمرية الأخرى. وكشفت الدراسة عن تأثير كبير للعمر على مستويات السكر في الدم. بلغ متوسط نسبة الجلوكوز في الدم في الأمراض المزمنة (277.689 mg/dL ± 2) لجميع الفئات العمرية، في حين كان متوسط نسبة السكر في الدم في حديثي الاصابة بالسكري(mg/dL 238.838 ± 3) لجميع الفئات العمرية ضمن حد ثقة 95٪. في حين أظهرت مجموعة السيطرة مستويات طبيعية لجلوكوز الدم (122.276 mg/dL ± 8) ضمن حدود الثقة 95%. وقد أظهر مرضى السكري الذين لديهم تاريخ سجل لمرض السكري في أسرهم أعلى مستويات السكر في الدم مع زيادة خطر الإصابة بالمرض.

1. INTRODUCTION

Many associated factors with diabetes especially type 2 act affect the patient life and health in general , according to the global number, type 2 diabetes patients may be at greater risk of infection (from 108 million in 1980 to 422 million in 2014) of patients over the age of 34 has only quadrupled, to 4.7% of adults over the age of 18 [1]. It is considered one of the biggest emerging threats to health in the twenty-first century**,** By 2030, according to the WHO, diabetes will be the leading cause of death [2] . There are four primary forms of Diabetes mellitus 2 (DM **II**) which primarily characterized thru high blood glucose levels (Hyper Glycaemia) resulting from defects in insulin secretion, in insulin action or both. Diabetes reports indicate that in 2025 there will be 380 million people[3].

Hyper glycaemia causes irreversible damage, dysfunction, and ravage to various organs, tissues, and cells , polydipsia, and polyphagia, caused by severe hyper glycemia may be leading to hyperosmolar syndrome and insulin, life - threatening ketoacidosis [4] . As well as in people with diabetes, there is a connection between the quantity of bacteria and active dental caries of DM. which recored high frequency of bacteria in the oral cavity of theme[5] . Iraq has gone via a painful and excruciating process to get to where it is today, multiple reasons, had a lasting impact on the health care system including wars, sectarian conflicts, politics, finances, and the security situation , This has resulted in a lack of medical apparatus, experiences , supplies and medicine [6]. The mechanisms cystopathy and micturition may predispose diabetes people to bacteremia [7]. So the infection by Enterobacteriaceae which is best described as a large, heterogeneous collection of Gram negative rods whose natural habitat is the gastrointestinal tract of both humans and animals. Enteric bacteria, also known as coliforms, have a complex antigenic structure, produce a variety of toxins, along with other virulence-enhancing factors. As an opportunistic bacteria that manifested as a nosocomial infection, *Enterobacter cloacae* gained clinical significance. pathogens in intensive care patients pathogenic, particularly those have DM type 2 patients[8] .

The aims of this study are detection of prevalence of Diabetes mellitus among females and males in different age groups, the relationship between family history and gender distributed according to disease status and, examine the association of diabetes with bacteremia, study commonly used antibiotics and identify ineffective ones among treatment protocols.

MATERIALS AND METHODS

Sample Collection

A total of one hundred and seventy-five samples were collected from Fallujah Teaching Hospital and some private laboratories in Fallujah city from patients with diabetes, among the total number of the samples; 160 samples taken from diabetic patients and 15 from non-diabetic ones from August to November in 2021. With collecting the following information: age, gender, family history (maternal, paternal or both) disease status samples were collected aseptically from (160) diabetic patients for this period of time, to study the impact of age, gender and family history along with diseases status, and the concentration of blood glucose distributed according to age and gender in addition to determination of bacteremia cases among diabetes patients

Analysis of blood and serum components:

All blood samples were collected using CBC and EDTA tubes, then analyzed with Sysmex XT-2000i system. Results included WBCs, Hb, RBCs, platelet count, MPV, and PDW with normal ranges. CRP and glucose were also tested using standard kits according to manufacturer's instructions. ESR test was conducted as described by [9].

Bacterial Strains

identification , From blood samples of diabetic patients, bacteria were detecting for isolation and f0llowing the instruction procedure [10] . Under sterile conditions, by diabetic patients 5 mL of blood were drawn , after cleansing of the skin area with an alcohol thoroughly followed by a another disinfectant. Blood loop full samples transferred to tube contain sterilized broth of brain heart infusion and incubated for 24 h. at 37℃ , then the broth had been inoculated with MacConkey agar and blood agar media used for differential diagnosis and ensure bacterial growth in the broth, the agar media incubated in the incubator for 24 hours at 37℃.

Diagnosis of bacteria

For this purpose use biochemical tests, All of the strains underwent testing using the HiMViCTM Biochemical Test Kit (HIMEDIA-KB001) and the HiAssorted Biochemical Test Kit (HIMEDIA-KB002). Add to differentiate *Klebsiella* spp from *Enterobacter* spp by using motility test which was negative for Klebsiella spp while Enterobacter spp is motility positive . And using API-20E test strips (BioMérieux, Marcy-l'Étoile, France) to identified at species level[11] . For more scertain the identity of the four bacterial isolates which obtain from diabetes blood samples, a po¬lymerase chain reaction (PCR) was performed . Utilizing the Mo Bio microbial DNA isolation kit from Mo Bio Laboratories Inc. in Solano Beach, California, and sequencing it as previously said [12]. The subsequent almost complete sequence of 1502 nucleotides the 16S rRNA gene contained. The 16S Rrna gene sequence of the isolate was subjected to BLAST sequence similarity search[13] , and EzTaxon[14] to identify the nearest taxa. The entire related 16S rRNA gene sequences were downloaded from the database (http://www.ncbi.nlm.nih.gov/) aligned using the CLUSTAL_X program[15] and the alignment corrected manually.

Susceptibility of antibiotic test

Sensitivity test against antimicrobial on Mueller-Hinton agar (Oxoid Ltd., Hampshire, United Kingdom) medium by a tablet disk diffusion method with Neo-sensitabs (Rosco Diagnostica, Taastrup, Denmark) against a panel of 14 antimicrobial drugs was done , According to the guidelines of the World Health Organization , neo-sensitabs are produced[16] .The efficacy were interpreted according to the guidelines of the National Committee for Clinical Laboratory Standards[17]. Antimicrobial agents were: Vancomycin, Rifampin, Azithromycin , Amoxicillin-clavulanic acid, Gentamicin, Ciprofloxacin, Imipenem, Cefotaxime, Ceftazidime, Cefepime, Ceftriaxone, Aztreaname, Amikacin, and Norfloxacin.

Statistical analysis

In the current study a statistical analyses were done by Microsoft Office Excel 2007, SPSS version (Statistical Package for Social Sciences) which used Chi-square, T-test, F-test, and least significant difference (LSD) [18] .

RESULTS AND DISCUSSION

The blood glucose level in this investigation was between 163-420 mg/dl in 160 blood samples of diabetic **II** , were reported from patients, whilst glucose levels range for 15 non-diabetic samples about 115±2 mg/dl . As Table (2) show eight cases of diabetic patients without bacterial infection showed high level , but all diabetic patients with bacterial infections (40 cases) showed high level ESR (25%) (75%). p=<0.001, a significant value. 26 cases of infected participants had high counts of WBCs and PLT, with significant values of p=0.0062 and 0.001, respectively. However, there was no discernible variation in the participants' hgb levels (p= 0.245).

Table (1) Infected and non-infected diabetic patients participants' frequency distribution of blood parameters

parameter	N. level	DM II with B. infection n=40		DM II without B. infection n=120		*p*-value
		No.	**%**	**No.**	**%**	
Hgb	<12	9	22.5	25	70.8	0.245
	12-14	8	19.9	38	31.7	
	16	4	10.1	5	4.12	
WBC	<4000	0	0	21	100	0.0062
	9000-13000	26	54.9	45	78.2	
ESR	Negative	0	0	75	100	<0.001
	positive	40	75	8	25	
Plt (X103/µ L)	normal	1	8.5	67	97.4	0.001
	High level	32	80	4	20	

The concentration of blood glucose distributed according to age and gender as shown in Table (2) .he elderly group (age more than 55 years) shows the highest glucose levels from the other age groups. Moreover, the patients whose ages were 65-74 years show the highest glucose levels among other age groups. Where female's glucose levels are higher than males (327 mg/dL and 281 mg/dL) respectively. So, females are higher in 59.4%, than males in our study were 40.5%.

Table (2): The distribution of blood glucose levels among study sample according to their gender and age.

age group	Renge reading mg/dl of females	Range reading mg/dl of Males
15-24	218	173
25-34	224	191
35-44	257	196
45-54	193	197
55-64	283	285
65-74	274	290
> 75	251	242
Total mean	251	224

In spite of higher glucose concentrations in females in this study than males but there is no significant elevation that indicate gender has no impact on blood glucose levels in all study groups. Otherwise, increased glucose levels of the patients with the maternal family history especially in females whose levels with acute phase of disease. In this study, a substantial increase in diabetes mellitus infections registered in the age groups over than 65 years especially those between (65-74) years with increasing of blood glucose levels, thus represent a high rate in elderly in contrast with young people. Same results obtained from the study of [19], reporting higher glucose levels in elderly but higher prevalence (64.4%) less than 55 years, this increase in elderly may be due to relation between longer durations of diabetes to further worsening of pancreatic workings, rise resistance to the insulin and an amplified risk of complications related with diabetes.[20] . In Iraq, non-communicable diseases have become a new developing problem after the 2003 war until now. Diabetes mellitus show fast growing trends, as increased prevalence of diabetes mellitus had significantly from 19.58/1000 in the year 2000 to 42.27 in 2015[21] .

Moreover, females are higher than males in our study (59.4% , 40.5%) respectively, which is convergent with [19] study who reported 54.4 % incidence in females since women identified as a more risk factor for poor control on glycemic in countries located in the MENA region, including Iraq.

Now, Iraq is one of the 21 countries in the MENA region, from 537 million patients have diabetes in the world, 73 million are in the MENA Region and by 2045 this will rise to 783 million. Where the rang of diabetes in adults is 9.4% and sum cases of diabetes in adults rise to 46% [22]. In contrast with the study of [23] who showed a contrary results, and reported at higher risk of poor glycemic control in younger age groups (< 60 years). In this research the patients of diabetes mellitus with registered history of diabetes in their families show the highest blood glucose levels, where the positive history of diabetes correlate significantly (*p* value =0.000) with the appearance of diabetes and severe outcome of the disease.

In spite of higher glucose concentrations in females in our study than males but there is no significant elevation that indicate gender has no impact on blood glucose levels in all study groups (p value = 0.064), Table(3).

Table (3): The distribution of blood glucose levels according family history between females and males.

family history	Total		Males		females	
	no.	mean of glucose level	no.	mean of glucose level	no.	mean of glucose level
non	68	192.8	30	218.4	38	206.4
father	32	192	10	206	22	313.7
mother	41	397	5	235	36	258.5
both parents	19	204	0	0	19	215
total	160	246.4	45	164.9	115	248.4

It was well-known that family history could lead to state of disease risk. In spite of high impact of family history on disease presence but not all of them were infected that correlated with controlling of glycemic status such as adopting certain lifestyles or behaviors because factors such as physical activity, dietary habits and obesity therefore family history represent risk factors for the disease.

As shown in Table (4) , fmily history resulted from maternal history showed high impact on blood glucose levels in our study regardless status of the disease (chronic or acute) with higher glucose levels in contrast with those without family history.

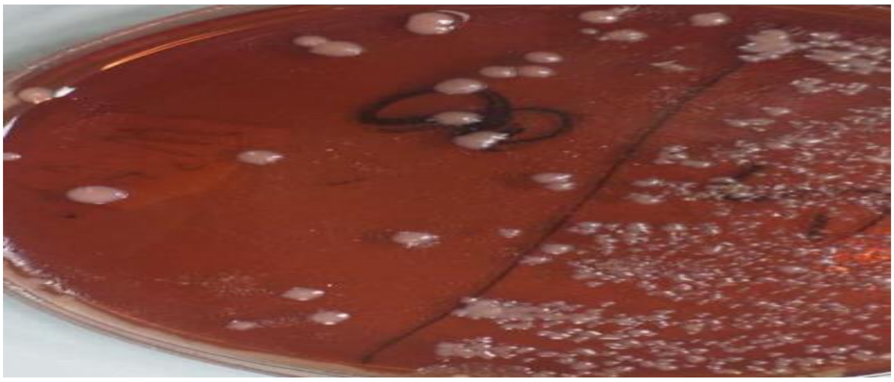
Table (4): The mean ± SD of glucose levels according to the distribution of family history for the disease by mean of gender.

Cases	Family History	Sex	Mean	Std. Deviation
Acute	N	female	197.4306	28.81380
		male	229.1073	56.01184
		Total	218.4887	50.86212
	Y	female	263.6151	95.68846
		male	237.9053	54.87994
		Total	254.2358	84.05419
	Total	female	237.4199	83.10751
		male	231.8045	55.80255
		Total	234.4385	69.99751

Chronic	N	female	249.8557	84.64259
		male	330.9647	135.19421
		Total	289.9012	119.55475
	Y	female	276.0070	101.01839
		male	252.6961	78.02915
		Total	270.1083	96.25429
	Total	female	267.7893	96.93142
		male	297.2150	120.50633
		Total	278.0920	106.70741
Non	N	female	103.7033	31.62470
		male	112.9512	4.81593
		Total	110.2923	17.90220
	Y	female	103.7033	31.62470
		male	112.9512	4.81593
		Total	110.2923	17.90220
Total	N	female	230.4522	82.38663
		male	273.4711	127.71007
		Total	254.5981	112.19028
	Y	female	273.8087	100.19847
		male	248.7362	72.84381
		Total	266.8974	94.12612
	Total	female	258.9655	96.68934
		male	264.4770	111.57480
		Total	261.2124	103.05027

The result was consistent with the study of where a poor glycemic control for patients with family history than without history registered. In study [24], diabetes frequency with a family history was more than four times higher than the incidence for whose without a family history. [25] suggest that glycaemic control is strongly influenced by family clustering and suggesting preventive strategies among them. When the affected person was kinsman most of these related reinforced were significant ,The interactions between family history of diabetes and sex with adherence to regular exercise and having a normal body composition among samples of [26] .

From 160 sample, 40 with positive blood culture bacteria, where a growth was seen on the surface of blood agar and on MacConkey agar, *Kocuria kristinae* and *Enterobacter cloacae* are the most frequently isolated in diabetes clinical specimens in (22,11) bacterial isolates respectively Table(5) . Kocuria is a Gram-positive cocci arranged in pairs, short chains, tetrads, cubical packets of eight and irregular clusters , do not produce hemolysis on blood agar, unlike most clinical isolates of Staphylococci. They usually form 2-3 mm whitish, small, round, raised, convex colonies on initial isolation and might develop non-diffusible yellowish pigmentation after prolonged incubation, as shown in Figure(1). Which is mainly opportunistic infection that causes bacteremia . Presently, by useing the 16S rRNA phylogenetic researches there are more than 18 species of Kocuria known [27]. While our study it represents higher number compared with others.



Figure(1): *Kocuria kristinae* on blood agar

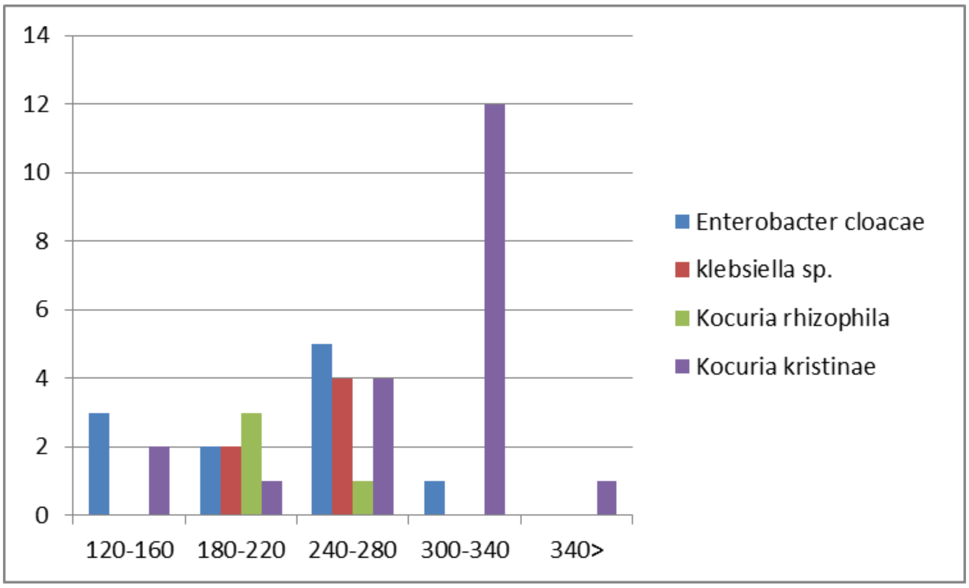
This followed by *Klebsiella pneumonia* and *Kocuria rhizophila* in (5,2)isolates respectively . compatible with systemic inflammatory response syndrome. In many healthy people, bacteremia will clear up on its own without causing illness.

Table(5): Some biochemical tests for characterization of bacterial isolates.

diagnosis	C	V	M	I	oxidase	catalase	MacConkey	blood agar	Motility	Urease	gram stain	Number of isolates
Enterobacter cloacae	+	+	-	-	-	+	growth	white colonies no haemolysis	+	-	gram negative	**11**
klebsiella pneumonia	+	+	-	-	-	+	pink growth	beta haemolysis	-	+	gram negative	**5**
Kocuria rhizophila	+	-	-	-	+	+	no growth	gamma haemolysis	-	-	gram positive	**2**
Kocuria kristinae	-	-	-	-	+	+	no growth	gamma haemolysis	-	-	gram positive	**22**

C:Citrate,V:Vogesproskauer,M:Methylred,I:Indol,(+):positiveresult(-):negative result

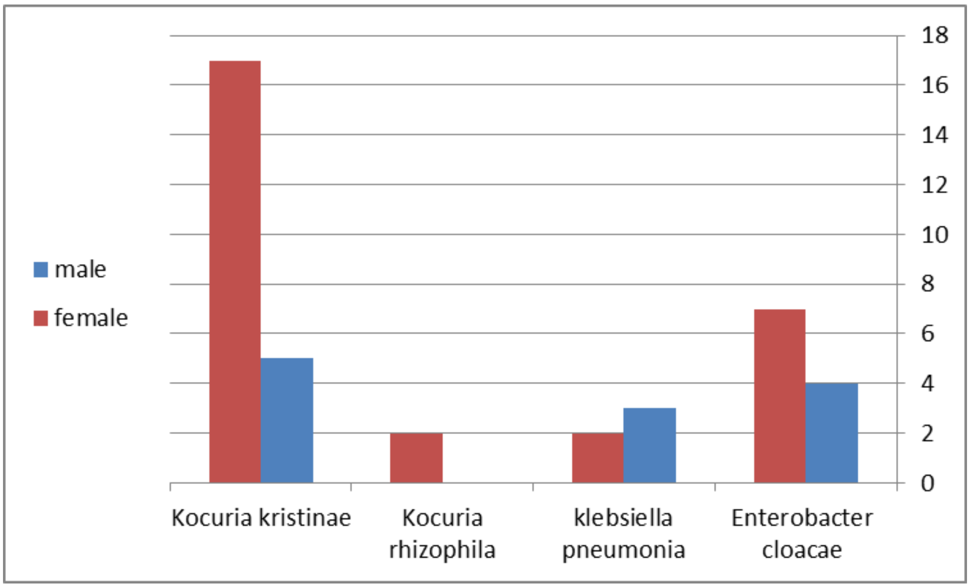
In this study the positive blood culture result represents 25% of the cultured samples that may resulted from a variety of etiologies including intestinal, urinary or catheter, etc. the incidence of bacteremia increased. While the presence of bacteria *Kocuria kristinae* increased in patients with chronic high diabetes as shown in Figure(2) .



Fig(2):Distribution of bacterial studying according to DM II level

Diabetic patients with *Klebsiella* bacteremia tend to be older than the general population that agree with our findings on ages of bacteremia positive results, [28] reported that the common underlying diseases in patients with *Klebsiella* bacteremia were DM this bacteria is proved as an infectious agent causing nosocomial infections and find *Enterobacter cloacae* represent (4.3%) in there study . Persons have diabetes mellitus could be at more hazard for contagions; in specific, which coming with the urinary tract contaminations. Earlier researches have stated a tow to four fold greater than before occurrence of bacteremia linked by DM and lately besides an raise danger of serious multiples urinary tract infection and hospitalization for pyelonephritis. Researches of the connotation with the most dangerous consequence of urinary tract infection, bacteremia due to gram negative bacilli, and enter bacteria **[29]**. Bacteremia mechanisms in diabetic patients may include be cystopathy and micturition abnormalities; glucosuria, which may motivate growth of bacteria ; and reduced function in neutrophils and macrophages caused by hyperglycemia.[30] .

As noted in this study figure (3) high levels of diabetes were accompanied by increased bacteremia in women, which was 70% while 30% of bacteremia in male. Women at high risk of type 2 diabetes are tested at their first visit to detect pre-existing (overt) diabetes [31].



Fig(3):Prevalence of bacteria in diabetic according a gender

Risk factors include of mainly due to age were the most vulnerable age group to be diagnosed with UTI , hormonal changes and pregnancy[32] Moreover; all of the forty isolated bacteria had been tested for number of antibiotics, as listed in (Table 6) to assess their resistance pattern, which appear the response of antibiotic Gram-negative and Gram-positive bacterial isolates obtained from diabetic patients blood.

Table (6) The Antibiotic sensitivity patterns with for the isolated bacteria from diabetic patients' blood.

Antibiotic types	Abbe.	No. of bacterial results			
		Kocuria kristinae	*Kocuria rhizophila*	*klebsiella pneumonia.*	*Enterobacter cloacae*
Vancomycin	VA 30 µg	(20) S (2) R	(2) R	(5) R	(5) R (6)S
Rifampin	RA 5 mcg	(20) S (2)R	(2) R	(5) R	(11) R
Azithromycin	AZM 15 µg	(18) S (4)R	(1) R (1)S	(2) R (3)S	(3) S (8)R
Norfloxacin	NOR 30 µg	(22) S	(2) S	(5) S	(11) S
Amoxicillin/calvalanic acid	AMC 20/10 µg	(12)S (10)R	(1) R (1)S	(3) R (2)S	(5) R (6)S
Ceftriaxone	CRO 30 µg	(19) R (3)S	(2) R	(5) R	(11) R
Cefotaxime	CFM 30 µg	(22)R	(2) R	(5) R	(11) R
Ceftazidime	CAZ 10 µg	(20) R (2)S	(2) R	(5) R	(11) R
Ciprofloxacin	CIP 10 µg	(21) S	(2) S	(3) R	(2) R

		(1)R		**(2)S**	**(9)S**
Amikacin	**AK 30 µg**	**(20) S** **(2) R**	**(2) S**	**(1) R** **(4)S**	**(2) R** **(9)S**
Gentamicin	**GN 10 µg**	**(20) S** **(2) R**	**(2) S**	**(5) S**	**(2) R** **(9)S**
Imipenem	**IPM 10 µg**	**(22) S**	**(2) S**	**(5) S**	**(11) S**
Cefixime	**CFM 30 µg**	**(22)R**	**(2) R**	**(5) R**	**(11) R**
Cefepime	**CPM 30 µg**	**(12)S** **(10)R**	**(2) S**	**(3) R** **(2)S**	**(2) R** **(9)S**

R:resiste , S:sensitive

Most of the isolated bacteria were resistance to Vancomycin in contrast with only it effected with *Kocuria kristinae*. Most of the isolated bacteria were resistant to Amoxicillin and Rifampin , variable bacterial response appear against Azithromycin . And all of the isolates were sensitive toward Norfloxacin representing the most effective antibiotic against the isolated bacteria . All of the isolated bacteria were resistant to Cefixime and Cefotaxime while, most of these isolates showed resistance to Cefepime . This may be as a result of the antibiotics have been in taken for too longer time and their oral route that affects their rate of absorption into the bloodstream [33].

Gentamicin group, Amikacin and Ciprofloxacin give good effect on bacterial isolates of this study which were agree in line with [34] .It had indicate by [35] to the good effect of Azithromycin on different bacterial types and biofilm forming .

Contrary to result of [36] who show that resistant of his bacterial isolates to norfloxacin was 69.0% .While These results can be consistent with the results of [37] who reported resistance in gram positive bacteria tested and isolates of *Klebsiella* to amoxicillin , So that the study of [38] from Iraq, and [39] in France agreed that *Klebsiella* has clear amoxicillin resistant by produce of βlactamases.

The dilemma is aggravated by the developing resistance of bacteria toward the common used antibiotics for prophylactic and experiential cures in those sufferer. Therefore antimicrobial cure should be done by research laboratory result sensitivity test . So, Persons with family history of diabetes especially by maternal origin advised to take attention because of the high risk of diabetes in those group, thus advised to follow a healthy with correct diet and physical activity to exclude the risk of diabetes, and patients with diabetes must take care during dealing with invasive tools such as IV catheter or others because of high risk of bacteremia origination from those sources.

Finally ,Some antibiotics lose their activity against bacteria, especially in cases of bacteremia, so it is necessary to check their effect .

CONCLUSIONS

Females are higher effected with diabetes and show high level of glucose in the blood with no significant effect of gender on disease presence or progression according to our study, elderly patients over 55 years are higher effected with diabetes mellitus in comparison with young patients and the family history of diabetes represent a high risk factor for developing of diabetes and worse progression of disease . Diabetic patients are highly effected with bacteremia caused by different organisms .While norfloxacin was the most effective antibiotic toward all of isolated bacteria.

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Modifications of Alternative Direction Implicit Method (ADI) to Find the Deflection of Elastic Thin Plates

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ABSTRACT:

The research aims to modify the Alternating Direction Implicit method for solving elastic thin plate deflection problems. The modification involves rotation between explicit and implicit method to determining mish point(nodes). In the first method, where the explicit method is used horizontally to identify nodes in the first level of mesh nodes, then the implicit method is used vertically for the second level of nodes. This repetition process continues until all retinal points (nodes) are identified. In the second method, the implicit method horizontally is used first, and then the explicit method vertically used to determine the values of the nodes in the mesh until all mesh points are complete Results from resolved cases demonstrate that the new method yields more accurate results compared to the traditional ADI method. This comparative analysis underscores the importance of selecting an appropriate numerical method based on the specific characteristics of the problem at hand, including the nature of the pressure distribution and the desired accuracy at various points within the mesh.
Keywords: Explicit method (EXP-M), Implicit method (IMP-M) , Alternating Direction Implicit (ADI).

1. INTRODUCTION

The alternative direction implicit (ADI) method is a numerical method in which contains two numerical method: Explicit method and implicit method many of research's are used this method (ADI) to solve a lot of problems like deflection problems, deformation problems, diffusion problems, and other problems in different of engineering problems, and sciences applications. The (ADI) method used to find the bending or deflection and the flexibility of elastic thin plates [1]. In 2007 Team and B; GE are used the (ADI) method to solve unsteady convention diffusion problems in two-dim [2]. Alaa and Awni and (2023) are used the (ADI) method to solve the flexibility problems [3], also in (2022) Alaa and Awni were solved Homogeneous harmonic equations using the (ADI) method [4]. Boling and Etal in (2011) are solving the fractional particle differential equation by numerical method [5]. Haneen in (2015) was solved the Bi-harmonic equation by using the alternative Implicit Direction Method [6]. Bushra in (2019) and Bushra, Awni and Firas in (2020) are used the theta equation homogenous heat diffusion problem, [8,9].

2. Theoretical Part:

2.1 ADI Method.

By combining explicit and implicit methods, the alternating path method (ADI) offers a green way to clear up elliptic partial differential equations (PDEs) numerically. It was developed with the aid of Peaceman and Rachford with a simple idea wherein the pliability equation, wave equation and biharmonic equivalent warmness equation are solved in two steps. These steps work simultaneously in two explicit and implicit methods: the primary degree searches for factors in the x-axis direction, and the second stage searches for factors inside the y-axis path. [12]. The implicit method gives more accurate results in the case of vertical pressure compared to the explicit method, which gives more accurate results in the case of lateral pressure by clamped-edges for both cases.

2.2 Modification ADI.

We have developed a new alternating implicit direction (ADI) method as follows:

1. The case type here is that the plate is clamped-edge and the pressure is vertical in both examples.
2. In the first example, the explicit method is applied horizontally at the first level, and in the second level, the implicit method is used vertically, and so on, alternating until we find all the points of the mesh.
3. In the second example, the implicit method is applied horizontally at the first level, and in the second level, the implicit method is used vertically, and so on alternately until we find all the points of the mesh. That is, the vice versa of the first example.
4. We compared the results according to the error rate after completing the results of the nodes on the network.

Note 1: By swapping the position of i and j in general formula, the **horizontally implicit method** transforms into the **vertically implicit method**.

Note 2: By swapping the position of i and j in general formula, the **horizontally explicit method** transforms into the **vertically explicit method**.

3. Particular Part:

3.1 Example: (1)

Let us have a thin elastic rectangular thin plate with dimensions (4cm*2cm) and fixed to the y and x axes at the origin points according to the following data:

h=Δx =1 cm, k=Δy=Δt=1 $secs$.

[a, b] = [0,4], [c, d] = [0,2]

Divide the two periods [a, b], [c, d] respectively depending on the values of (n) and (m) according to formula.

$h = \frac{b-a}{n} = 1$, $k = \frac{d-c}{m} = 1$, n=4, m=2, (m, n $\in z$).

In general, $xi = a + ih$, i = 0,1, 2…, n(row).

$yj = c + jk$, j = 0,1,2…, m(column).

Initial conditions, d(x,0) = 0° , t=0 on the lower side.

Boundary conditions, $d_{(0,t)} = 2^{\circ}$ on the left side, $d_{(4,t)} = 8^{\circ}$ on the right side.

$d_{(x_i,y_j)}$ the nodes on the level.

The change in time is represented by Δt.

The displacement is represented by Δx.

That is the plate is clamped-edge with vertical case.

$r = \alpha \frac{\Delta t}{h^2}$, let deflection coefficient α=2, $r = 2 \frac{1}{1\text{^}2} = 2$

Solution: - Here at the first level we find the points horizontally by the explicit method and vertically by the implicit method and so on for the second level.

First level by **explicit method horizontal.**

$$d_{i,j+1} = r\,d_{i+1,j} + (1-2r)d_{i,j} + r\,d_{i-1,j} \quad \dots\dots (1)$$

When we substitute j=0, i=1,2,3

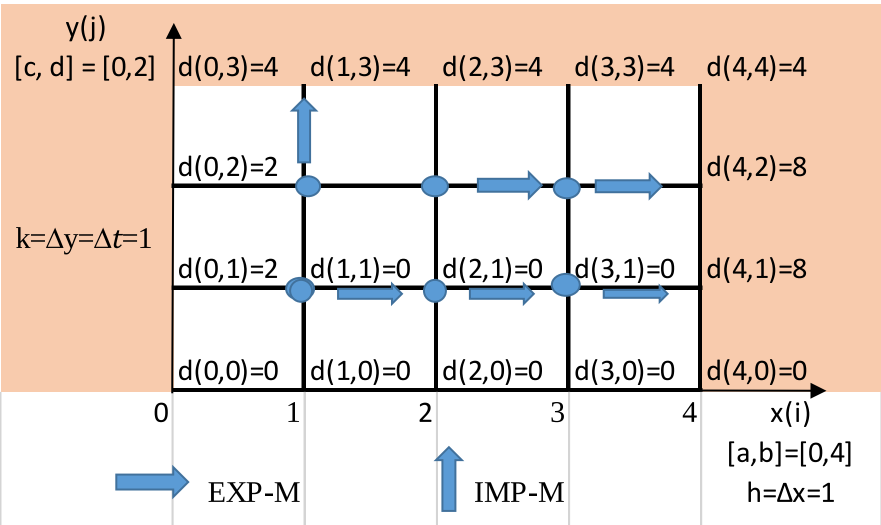
j=0, i=1, r=2 $\qquad d_{1,1} = r\,d_{2,0} + (1-2r)d_{1,0} + r\,d_{0,0}$

$$d_{1,1} = 2 * \ 0 + (1 - 2 * 2) * 0 + 2 * 0$$

$$d_{1,1} = 0$$

By the same way we find $d_{2,1} = 0,\ d_{3,1} = 0$

The following figure shows the result of first level nodes.



Fig(1): First level node (EXP-M)

Complete the solution at the second level by **implicit method vertical.**

To find $d_{1,2}$,

$$d_{i,j} = -r\,d_{i+1,j-1} + (1+2r)\,d_{i+1,j} - r\,d_{i+1,j+1} \quad \dots\dots (2)$$

When we substitute i=0, j=1

$$d_{0,1} = -r\,d_{1,0} + (1+2r)\,d_{1,1} - 2 * \ d_{1,2}$$

$$2 = -2 * \ 0 + (1 + 2 * 2) * 0 - 2 * \ d_{1,2}$$

$$2 = -2 * \ d_{1,2}$$

$$d_{1,2} = -1$$

Third level by **explicit method horizontal** to find nodes $d_{3,2}$, $d_{3,2}$

When we substitute j=1, i=2,3

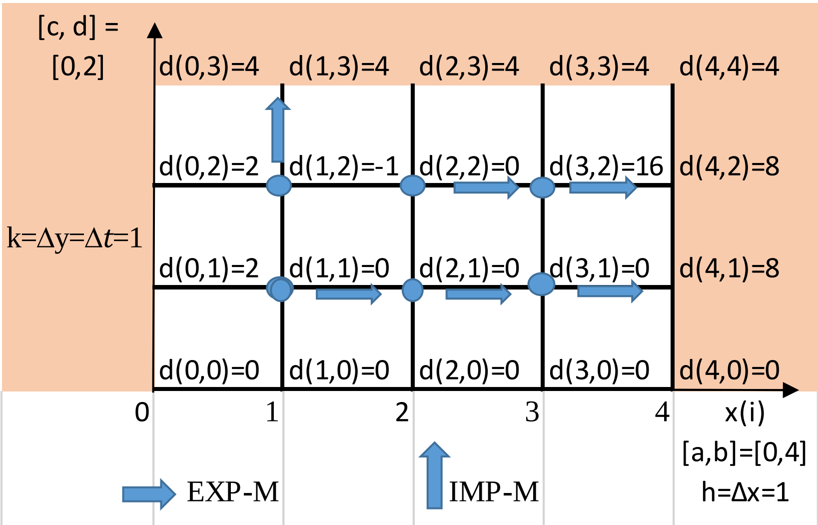
j=1, i=2, $d_{2,2} = r\, d_{3,1} + (1-2r)d_{2,1} + r\, d_{1,1}$

$d_{2,2} = 2 * 0 + (-3) * 0 + 2 * 0$

$d_{2,2} = 0$

By the same way we find $d_{3,2} = 16$

The following figure shows the result the all levels nodes.



Fig(2): All nodes on the mesh by (EXP-M) &(IMP-M).

3.2 Example: (2)

same data from example (1).

Solution: - Here at the first level we find the nodes by the implicit method horizontally and at the second level by the explicit method vertically and so on for the second level.

First level by **implicit method horizontal.**

$$d_{i,j} = -r\, d_{i-1,j+1} + (1+2r)\, d_{i,j+1} - r\, d_{i+1,j+1} \qquad \dots\dots(3)$$

When we substitute j=0, i=1,2,3, r=2

j=0, i=1, $d_{1,0} = -2 * d_{0,1} + (1+2*2)\, d_{1,1} - 2 * d_{2,1}$

$$0 = -2 * 2 + 5 * d_{1,1} - 2 * d_{2,1}$$

$$4 = 5 * d_{1,1} - 2 * d_{2,1} \qquad \dots\dots (4)$$

j=0, i=2, $d_{2,0} = -2 * d_{1,1} + (1+2*2)\, d_{2,1} - 2 * d_{3,1}$

$$0 = -2 * d_{1,1} + 5 * d_{2,1} - 2 * d_{3,1} \quad \ldots\ldots (5)$$

j=0, i=3, $d_{3,0} = -2 * d_{2,1} + (1 + 2 * 2)\, d_{3,1} - 2 * d_{4,1}$

$$0 = -2 * d_{2,1} + 5 * d_{3,1} - 2 * 8$$

$$16 = -2 * d_{2,1} + 5 * d_{3,1} \quad \ldots\ldots (6)$$

Using MATLAB for the equations (4),(5),(6).

A = Coefficient matrix.

x = Column of variables ($d_{1,1}$,$d_{2,1}$ and $d_{3,1}$).

B = Column of constants (4, 0, 16).

Here's how you can set up the matrix equation:

A = B =

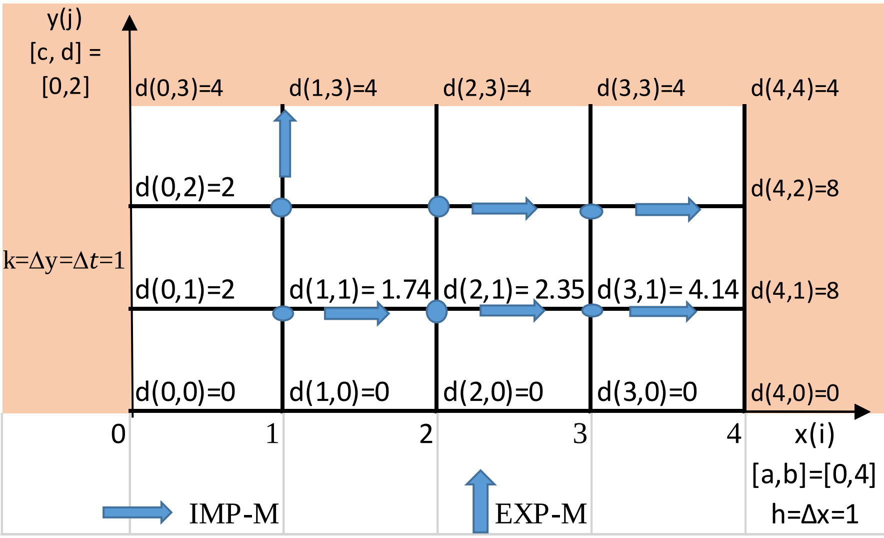
5	-2	0	4
-2	5	-2	0
0	-2	5	16

X = A inverse*B

0.2471	0.1176	0.0471	1.7412
0.1176	0.2941	0.1176	2.3529
0.0471	0.1176	0.2471	4.1412

$$d_{1,1} = 1.74 \quad , d_{2,1} = 2.35 \quad and\ d_{3,1} = 4.14$$

The following figure shows the result of first level nodes.



Fig(3): First level node (IMP-M)

Complete the solution at the second level by **explicit method vertical** to find nodes $d_{1,2}$

$$d_{i+1,j} = r\, d_{i,j+1} + (1 - 2r) d_{i,j} + r\, d_{i,j-1} \quad \ldots\ (7)$$

When we substitute i=0, j=2

$$d_{1,2} = r\, d_{0,3} + (1-2r) d_{0,2} + r\, d_{0,1}$$

$$d_{1,2} = 2 * \ 4 + (1 - 2 * 2) * 2 + 2 * 2$$

$$d_{1,2} = 8 + (-6) + 4$$

$$d_{1,2} = \ 6$$

Third level by **implicit method horizontal** to find nodes $d_{3,2}$, $d_{3,2}$

$$d_{i,j} = -r\, d_{i-1,j+1} + (1+2r)\, d_{i,j+1} - r\, d_{i+1,j+1} \qquad \text{... (3)}$$

When we substitute j=1, i=2,3, r=2

j=1, i=2, $d_{2,1} = -2 * \ d_{1,2} + (1 + 2 * 2)\, d_{2,2} - 2 * \ d_{3,2}$

$$2.35 = -2 * \ 6 + (1 + 2 * 2) * d_{2,2} - 2 * \ d_{3,2}$$

$$2.35 = (-12) + 5 * d_{2,2} - 2 * \ d_{3,2}$$

$$14.35 = 5 * d_{2,2} - 2 * \ d_{3,2} \qquad \text{… (8)}$$

j=1, i=3, $d_{3,1} = -2 * \ d_{2,2} + (1 + 2 * 2)\, d_{3,2} - 2 * \ d_{4,2}$

$$4.14 = -2 * \ d_{2,2} + 5 * \ d_{3,2} - 2 * \ 8$$

$$20.14 = -2 * \ d_{2,2} + 5 * \ d_{3,2} \qquad \text{… (9)}$$

Using MATLAB for the equations (8),(9).

A = Coefficient matrix.

x = Column of variables ($d_{2,2}$ *and* $d_{3,2}$).

B = Column of constants (14.35, 20.14).

Here's how you can set up the matrix equation:

A = B =

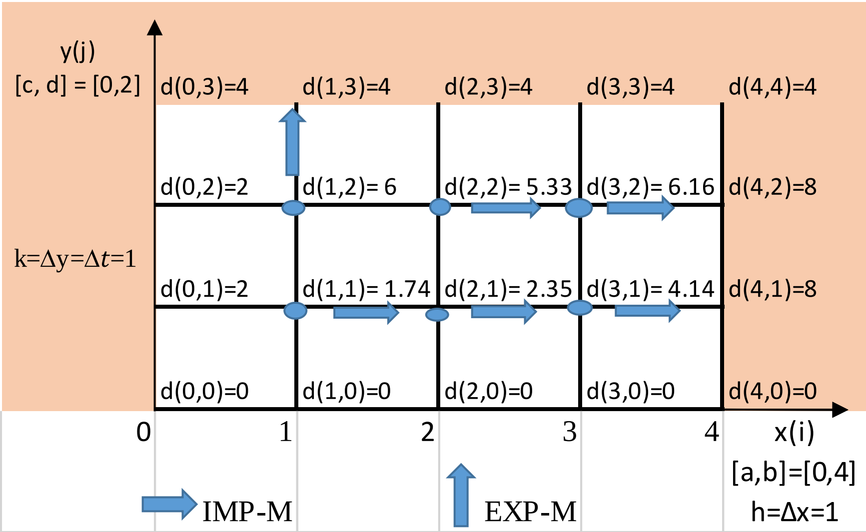
5	-2	14.35
-2	5	20.14

X = A inverse*B

0.238	0.095	5.33
0.095	0.238	6.16

$d_{2,2} = 5.33 \quad and\ d_{3,2} = 6.16$

The following figure shows the result of the all levels nodes.



Fig(4): All nodes on the mesh by (IMP-M) &(EXP-M).

Flowchart(1): Shows the results.

Table(1): shows the error rate results for Example (1) and Example (2).

d	Ex (1)	e=d1-d2	e^2	d	Ex (2)	e=d1-d2	e^2
1	0	0	0	1	0	0	0
2	0	0	0	2	0	0	0
3	0	0	0	3	0	0	0
4	0	2	4	4	0	2	4
5	2	2	4	5	2	0.26	0.0676
6	0	0	0	6	1.74	0.61	0.3721
7	0	0	0	7	2.35	1.79	3.2041
8	0	2	4	8	4.14	2.14	4.5796
9	2	3	9	9	2	4	16
10	-1	1	1	10	6	0.67	0.4489
11	0	16	256	11	5.33	0.83	0.6889
12	16			12	6.16		
	error rate		23.1667		error rate		2.4468

4. Conclusions:

A new method has been developed in this search by combining the explicit method with the implicit method by using them alternately vertically and horizontally at levels under vertical pressure. This method successfully solved every case and achieved more accurate results than the traditional alternating implicit direction (ADI) approach. The implicit method applied horizontally at the first level alternating with the explicit method vertically at the second level gave better results than the explicit method applied horizontally at the first level alternating with the implicit method vertically at the second level. The error rate in the second example is lower than the error rate in the first example, and the chart also shows the best method.

5. ACKNOWLEDGEMENT

My deep thanks and appreciation go out to my supervisor, Assistant Professor Awni M. Gaftan, for all of his assistance, direction, and advice in helping me present this knowledge in the best possible light.

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المستخلص: يهدف البحث إلى تعديل طريقة الاتجاه الضمني المتناوب (ADI) لحل مشاكل انحراف الصفائح الرقيقة المرنة. يتضمن التعديل التناوب بين الطريقة الصريحة والضمنية لتحديد نقطة (العقد) المتشابكة. في الطريقة الأولى، حيث يتم استخدام الطريقة الصريحة أفقيا لتحديد العقد في المستوى الأول من العقد الشبكية، ثم يتم استخدام الطريقة الضمنية عموديا في المستوى الثاني من العقد. تستمر عملية التكرار هذه حتى يتم تحديد جميع نقاط (عقد) الشبكية. وفي الطريقة الثانية، يتم استخدام الطريقة الضمنية أفقيا أولا، ومن ثم الطريقة الصريحة المستخدمة عموديا لتحديد قيم العقد في الشبكة حتى تكتمل جميع نقاط الشبكة، وتوضح النتائج من الحالات التي تم حلها أن الطريقة الجديدة تعطي نتائج أكثر دقة مقارنة إلى طريقة ADI

التقليدية. ويؤكد هذا التحليل المقارن على أهمية اختيار الطريقة العددية المناسبة استنادا إلى الخصائص المحددة للمشكلة المطروحة، بما في ذلك طبيعة توزيع الضغط والدقة المطلوبة في نقاط مختلفة داخل الشبكة.

الكلمات المفتاحية: طريقة الاتجاه الضمني المتناوب(ADI), الطريقة الصريحة (EXP-M), الطريقة الضمنية (IMP-M).

Mathematical Modeling Approximation Methods Accuracy

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Abstract:

Numerical approximation methods and their applications play an important role in many scientific and engineering fields. It enables us to solve mathematical equations that cannot be solved analytically, understand complex systems, and predict them. After the great development in mathematical modeling and programming, this role developed and grew. Since there are many vital applications, whether in engineering, physics, chemistry or economics, approximation methods combined with mathematical modeling have enabled us to solve complex problems that cannot be solved using only one method. Better understand and analyze complex systems, and solve complex problems. It also enables us to provide more accurate forecasts. The lenticular approximation method is based on dividing the problem into a series of small steps and then solving each step using simple analytical methods. Lenticular approximation methods can be divided into several types, including linear and nonlinear methods. In this study, this study aims to introduce the reader to numerical approximation methods in mathematical modeling. This study will discuss the types of numerical approximation methods and their applications in mathematical modeling. We studied a mathematical model and chose the appropriate method of approximation, then calculated the error rate and approximation error and verified the approximation. The results indicated that using numerical approximation methods and mathematical modeling together is a powerful tool that can be used to solve many complex problems and contributed to reducing the error rate by a very large percentage and reducing the standard deviation of the mathematical model.

Keywords: Numerical approximation, mathematical equations, mathematical modeling, complex problems programming, standard deviation.

1. Introduction

Mathematical modeling is the manner of representing a real or digital gadget with a mathematical model. This model is used to recognize, examine, and make predictions approximately the machine. In many cases, the mathematical equations describing the actual or hypothetical system can't be solved analytically. In these instances, numerical approximation strategies can be used to resolve those equations Numerical approximation techniques are a collection of methods used to approximate the answer of mathematical equations or different mathematical troubles. These techniques rely upon dividing a problem into a chain of small steps after which fixing every step using easy analytical techniques [1].

There are many varieties of lenticular approximation techniques, which include Linear approximation strategies: These techniques depend upon dividing the trouble into a sequence of small steps after which solving each step the use of linear equations. Approximation techniques the use of differential equations: These strategies depend on changing the trouble right into a differential equation and then solving this equation the use of numerical approximation methods. The significance of numerical approximation methods is because of them, They permit us to clear up mathematical equations that can't be solved analytically. It enables us to resolve mathematical equations with big or complicated numbers. It allows us to solve mathematical equations that require excessive accuracy [2].

Numerical approximation methods are widely used in many scientific and engineering fields, such as physics, chemistry, mechanical engineering, electrical engineering, and civil engineering. Here are some common applications of numerical approximation methods in mathematical modeling: Solving differential equations: numerical approximation methods are used to solve differential equations that describe Many natural phenomena, such as movement, heat, and diffusion. Solving integral equations: numerical approximation methods are used to solve integral equations that describe many natural phenomena, such as thermodynamics and electromagnetism [3].

Calculus: numerical approximation methods are used to calculate the calculus of complex mathematical functions [4,5]. Solving other mathematical problems: numerical approximation methods are used to solve many other mathematical problems, such as opti numerical approximation methods have several advantages that make them a powerful tool in mathematical modeling. However, it is important to understand the disadvantages of these methods to avoid errors [6]. Among the most common mistakes are not choosing the appropriate method for proportional variables, also not evaluating the results, relying on the human element only without resorting to computers, and despite the advantages of approximation methods, which are flexibility and accuracy [7]. solving complex problems, and the possibility of using them to solve a wide range of mathematical problems; However, it has some disadvantages, including that these methods are inaccurate if not used correctly, somewhat slow, sometimes complicated, and sometimes complex. mization problems and dynamic systems theory [8].

Studying Numerical approximation methods and mathematical modeling Some problems: A high level of mathematics is required. Studying lenticular approximation methods and mathematical modeling requires a good understanding of basic mathematics rules, such as calculus and the theory of differential equations. The complex nature of materials numerical approximation and mathematical modeling methods involve a wide range of complex methods, which can be difficult to understand and apply. The need for powerful computers: Some numerical approximation methods require the use of powerful computers to process complex data. This study aims to introduce the reader to numerical approximation methods in mathematical modeling. In this study, the types of numerical approximation methods and their applications in mathematical modeling will be discussed. This study is important because it shows how numerical approximation methods and mathematical modeling play an important role in many scientific and engineering fields. Therefore, studying these topics is important for students and researchers in these fields [9].

The methodology for studying approximation and mathematical modeling is a mixture of multiple methodologies, where the scientific methodology is used side by side with the descriptive methodology in addition to the analysis and comparison methodology, by studying a mathematical model, choosing the appropriate method for approximation, and then calculating the error rate, the approximate error, and checking the approximation.

2 .Overview of the Literature Review and Analysis

Recent trends in the study of numerical approximation literature and its application in mathematical modeling in the following areas: Development of new methods for approximate approximation of numerical, where new methods for approximate approximation of numerical are developed. Application of numerical approximation methods in new fields. Numerical approximation methods are applied in new fields, such as artificial intelligence and smart learning. Study of the effect of numerical convergence on mathematical modeling: The effect of numerical convergence on mathematical modeling is studied to understand how the effect of numerical convergence affects the modeling results.

2.1 . Historical Overview

The examine of numerical approximation dates lower back to the sixteenth century while Galileo Galilei used easy techniques to calculate distances from Earth to planets. Since then, many specific methods of numerical approximation had been advanced, which can be divided into numerous principal categories. In the seventeenth century, Newton advanced the method of calculus, which turned into an important step within the improvement of lenticular approximation techniques. The differential and vital methods allowed the development of more accurate techniques of lenticular approximation, which include linear and nonlinear approximation strategies. In the nineteenth century, several modern lenticular approximation methods have been advanced, consisting of the numerical integration technique and the technique of solving differential equations. These techniques were extra accurate and efficient than previous strategies, making them more useful in a wide variety of programs. In the twentieth century, lenticular approximation techniques continued to be advanced, with new techniques which includes the finite detail method and the mesh technique emerging. These techniques had been extra accurate and green than preceding techniques, making them extra beneficial in solving complex issues.

2.2. Related Studied

In a have a look at by way of (B Adcock, D Huybrechs Siam Review - 2019) you offer an overview of the concept of frames and the way they may be used as a framework for numerical approximation tactics. They pointed out the challenges that may be faced even as the usage of numerical approximation techniques in mathematical modeling of its various kinds: linear, non-coverage, differential, and integral, with the aid of reviewing applications of numerical approximation using superior principles of frameworks. The study affords indications of destiny guidelines and the significance of this approach in regions which includes sign processing and applied arithmetic [10].

In the have a look at of (Kumar, *At,al*, 2006), which is a presentation of an approximate approach for solving Fractional Differential Equations, which represent a sort of differential equation that consists of derivatives of an incorrect order. The proposed approach is primarily based on making use of the numerical algorithm for approximation to those equations. The proposed method turned into used to calculate approximations of the answers to a number of discretization differential equations, which lets in for managing issues in numerous fields, consisting of signal processing. The effectiveness of the proposed approach was verified by applying it to numerical examples. And examine it with other methods used in solving partition differential equations. The outcomes show that the proposed method gives a powerful and powerful answer for this magnificence of equations. The take a look at has supplied an powerful numerical tool for fixing partition differential equations, which contributes to statistics technological know-how and sign processing packages, which means that the importance of approximation strategies in solving issues and the opportunity of finding and providing effective solutions [11].

In a look at through (A. Quarteroni, *At,al*, 2019), she mentioned the use of numerical approximation methods in mathematical imaging of the human cardiovascular machine. The study used numerical approximation strategies to effectively represent the complex physical phenomena associated with the cardiovascular machine. A comprehensive overview is supplied on how to investigate and interpret medical records the usage of mathematical equipment. The results suggest that employing those mathematical fashions in various medical applications complements information and medical interplay among mathematical modeling and sensible medical programs and that numerical approximation has contributed to the medical interaction of mathematical modelling [12].

N any other observe with the aid of (Steinbach, O. 2007), the author supplied a complete review of the numerical approximation techniques used in fixing boundary cost problems related to differential equations with skewed graphs. The study centered on unique approximation techniques that are based on finite factors and boundary elements. The observe examined the mathematical concepts of boundary fee troubles. Ellipticity, after which provides details on the way to estimate those issues the use of numerical approximation techniques, with a particular awareness on methods based on finite elements and boundary elements. It provides a discussion of the advantages and demanding situations of the usage of these techniques, illustrating the applied contexts wherein those techniques can be implemented. The ebook is a precious aid for researchers and engineers working within the discipline of carried out arithmetic and finite element strategies. It offers a deep expertise of the techniques used in fixing boundary fee problems of differential equations and the significance of numerical approximation strategies in facilitating the solution of differential equations. [13]

In a look at that centered on mathematical modeling, mathematical evaluation, and numerical approximation of second-order Elliptical troubles that consist of internal elements, the author (Köppl, T. *At,al*, 2018), provided mathematical fashions for troubles with graphical deviation and analyzed them mathematically to recognize the principle houses of those equations. In addition, emphasis is placed on the way to approximate those hard equations numerically, whilst verifying the effectiveness of the strategies used. The research furnished, A comprehensive evaluation of the challenges that rise up whilst coping with skewed differential troubles and a way to reap an accurate numerical approximation to them. This paper is a valuable useful resource for researchers and those interested by applied mathematics and techniques for approximating differential equations [14].

3. The Method and Methodology

The methodology for studying approximation and mathematical modeling is a mixture of multiple methodologies, where the scientific methodology is used side by side with the descriptive methodology in addition to the analysis and comparison methodology, by studying a mathematical model, choosing the appropriate method for approximation, and then calculating the error rate, the approximate error, and checking the approximation. In this section, we will present a practical framework for the study, and the procedures that were taken, starting with data collection, passing through the formulation of the mathematical model, and ending with the results and their scientific analysis.

3.2.The Practical Framework

the applied framework of the study indicates the steps that will be followed during the study, starting from collecting data until presenting and analyzing the results. Care has been taken in choosing the appropriate approximation method to calculate the error rate in the mathematical model. See figure: 1

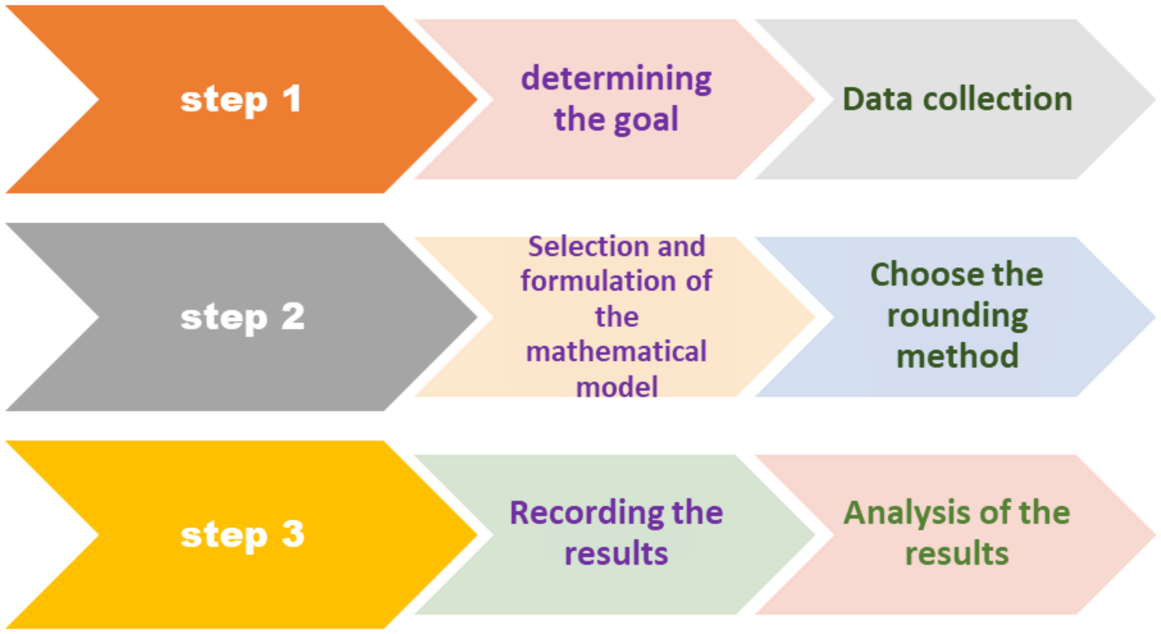

Fig 1: The Practical Framework (author)

3.3. The Procedures

The degree of the polynomial determines how accurate the polynomial approximation is in maximum mathematical modelling approximation techniques. The accuracy of the approximation increases with the degree of the polynomial. The following are the tactics:

1. Determine the purpose, that is to remedy a mathematical model problem the usage of numerical approximation techniques, the mathematical model for a laboratory test.

2. Collect information from the supervisory committee, preceding studies, and the Internet

Three. Choose the rounding technique

Most of the strategies are in the main methods that depend upon first-degree linear approximation, so we can model the values mathematically via writing a 2nd-diploma polynomial as follows:

$$P(x) = ax^2 + bx + c$$

Where:

a, b, and c are polynomial coefficients.

TTo decide the values of a, b, and c, we will use the subsequent spinoff equations: $P(x0) = y0$ $P'(x0) = y'0$

wherein: x0 is a given factor.

Y0 is the factor cost at point x0.

Y'zero is the price of the derivative at factor x0.

Four. Determining the mathematical model and the usage of numerical approximation methods to calculate the mistake rate

A third-order polynomial version may be formulated as follows:

y = ax^3 bx^2 cx d

wherein:

a, b, c, and d are the coefficients to be anticipated.

Calculate the distinction of values The difference values among the genuine values and the values anticipated from the model may be calculated as follows: diff = y_data - y_model

The third-order polynomial model can be formulated as follows:

y = ax^3 bx^2 cx d

in which:

a, b, c, and d are the coefficients to be anticipated.

In this situation, the model parameters had been expected using the least squares technique and the consequences have been as follows

a = -0.000002

b = 0.000001

c = -0.000001

d = 5.819000

y = -0.000002x^3 + 0.000001x^2 + -0.000001x + 5.819

by Python

Input data

x_data = np.array([8, 7, 9, 10, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52])

y_data = np.array([6.14, 6.08, 6.06, 6.04, 5.93, 5.87, 5.77, 5.52, 5.39, 5.24, 5.24, 5.24, 5.24, 5.22, 5.14, 5.14, 5.03, 5.03, 4.82, 4.69, 4.53, 4.46, 4.4, 4.31, 4.3, 4.03, 4.03, 3.99, 3.87, 3.68, 3.69, 3.61, 3.74, 3.26, 3.09, 3.05, 3.02, 2.5])

Fit a third-degree polynomial using least squares

coefficients = np.polyfit(x_data, y_data, 3)

Define the polynomial function

polynomial = np.poly1d(coefficients)

Calculate model values at the given x values

y_model = polynomial(x_data)

Calculate the difference between the observed and predicted values

diff = y_data - y_model

```
# Plot the data and model
plt.plot(x_data, y_data, 'o', label='Experimental Data')
plt.plot(x_data, y_model, '-', label='Model')
plt.plot(x_data, diff, 'r', label='Difference')
plt.legend()
plt.xlabel('x')
plt.ylabel('y')
plt.title('Third-Degree Polynomial Model')
plt.show()
coefficients
[-0.000002  0.000001 -0.000001 5.819000]
```

4. Result and Discussion

In this section, and according to the applied framework of the study, we will present the results (the fifth procedure) and we will analyze the results (the sixth procedure).

4.1. Results

The distinction of values between the proper values and the values expected from the version may be calculated as follows: Table 1

Table 1: Values between the proper values and the Values Expected

NO.	CAL.VALUES	EXP.VALUES	E.
8	6.139	6.14	0.001
7	6.079	6.08	0.001
9	6.059	6.06	0.001
10	6.039	6.04	0.001
20	5.929	5.93	0.001
21	5.869	5.87	0.001
22	5.769	5.77	0.001
23	5.519	5.52	0.001
24	5.389	5.39	0.001

25	5.239	5.24	0.001
26	5.239	5.24	0.001
27	5.239	5.24	0.001
28	5.239	5.24	0.001
29	5.229	5.22	0.009
30	5.139	5.18	0.041
31	5.139	5.14	0.001
32	5.029	5.14	0.111
33	5.029	5.03	0.001
34	4.819	5.03	0.211
35	4.689	4.82	0.131
36	4.529	4.69	0.161
37	4.459	4.53	0.071
38	4.399	4.47	0.071
39	4.309	4.4	0.091
40	4.299	4.33	0.031
41	4.029	4.3	0.271
42	4.029	4.03	0.001
43	4.029	3.99	0.039
44	3.989	3.87	0.119
45	3.869	3.68	0.189
46	3.679	3.64	0.039

47	3.689	3.61	0.079
48	3.609	3.47	0.139
49	3.739	3.26	0.479
50	3.259	3.09	0.169
51	3.08	3.05	0.03
52	2.52	2.5	0.02

The model standard deviation error was 0.001, indicating that the model provides an accurate estimate of the data:

Arithmetic mean:

¯x = (0.001 + 0.001 + ... + 0.479 + 0.02) / 30 = 0.079

Subtract the mean from each value:

x_1 - ¯x = 0.001 - 0.079 = -0.078

x_2 - ¯x = 0.001 - 0.079 = -0.078

...

x_30 - ¯x = 0.02 - 0.079 = -0.059

Square each value:

(x_1 - ¯x)^2 = (-0.078)^2 = 0.06084

(x_2 - ¯x)^2 = (-0.078)^2 = 0.06084

...

(x_30 - ¯x)^2 = (-0.059)^2 = 0.03481

σ = 0.01

figure: 2 relation of error and the squares of the difference in values and the arithmetic mean.

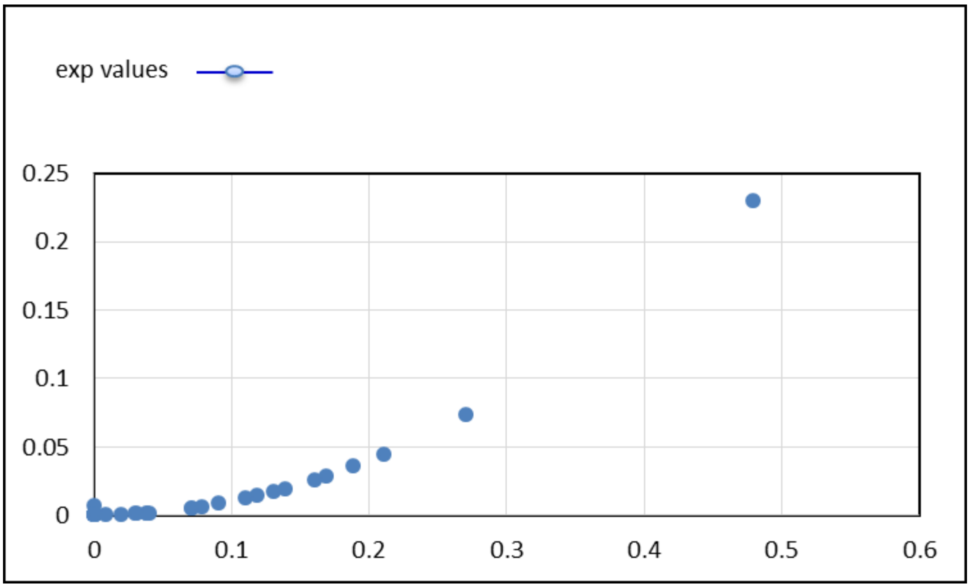

Fig 2: Relation of error and the squares of the difference in values and the arithmetic mean.

4.2. Discussion

Looking at Table (1), we discover that the virtual approximation solved the trouble of the mathematical version with such accuracy that the highest percent of mistakes became =zero.11and the lowest percent of mistakes changed into = 0.01

Looking at Figure (2), we discover that the usual deviation is a degree of the dispersion of the records across the mean. The extra the same old deviation, the extra the dispersion of the facts. In this case, the standard deviation of the values is 0.01. This way that the statistics is slightly scattered across the average.

The wellknown deviation may be used to decide the percentage error in numerical approximation methods in mathematical modeling. For instance, if we've got a mathematical feature that has an actual solution, we can use numerical approximation methods to discover an approximate solution. Then we will use the standard deviation to measure how accurate the approximate answer is. If the usual deviation is small, it method that the approximate answer may be very close to the exact answer. If the same old deviation is large, it method that the approximate solution is less accurate.

In this case, the standard deviation is small, because of this that the approximate solution is very close to the exact solution. We can expect that the approximate answer is correct enough for use in mathematical modeling

5. Conclusions

1. Numerical approximate strategies are very effective in solving mathematical modeling troubles
2. The better the diploma of approximation, the greater the accuracy [15].
3. Three. Standard deviation is a measure of the dispersion of facts across the imply.
4. The popular deviation can be used to determine the proportion error in numerical approximation strategies in mathematical modeling.
5. If the standard deviation is small, it manner that the approximate solution could be very close to the precise solution [16].
6. In the case of values for which the same old deviation is calculated, the same old deviation is small, which means that that the approximate answer is very close to the exact solution. We can anticipate that the approximate solution is accurate sufficient for use in mathematical modeling
7. To determine the share mistakes using the usual deviation:
   - Make sure the data is uniform.
   - Compare the standard deviation to the standard deviation of other data.
   - Use linear regression to determine the relationship between the exact solution and the approximate solution.

Declaration of Competing Interest

I declare that there is no competing interest with any other authors and that there is no interest or benefit that influenced the results of the research.

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θ-Differencing Hybrid Method for Solving Heat Flow Equations

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θ-Differencing Hybrid Method for Solving Heat Flow Equations

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ABSTRACT:

An equation that depends on two or more independent variables and may also contain derivatives of one or two functions is known as a partial differential equation. This type of equation can be used to describe various cases in physics, chemistry, and natural phenomena such as the movement of clouds, steam, and wind. And gases and the accompanying cases of pollution, water flow, and other cases. One of the important partial differential equations is the heat diffusion equation. In this paper, the hybrid differentiation method for solving heat diffusion equations was presented, as well as developing an algorithm for that developed method and the general formulas for that equation and what accompanies it. This formula consists of initial conditions and boundary conditions.

Finally, we compared the results of the new method with the results of the traditional method, and conducted a practical application of the new method using the MATLAB program.

Keywords: θ-Differencing method, Hybrid θ-differencing Method, Implicit method and Explicit Method, Cranck-Nicolson.

1. INTRODUCTION

The Ɵ-difference method is considered one of the modern numerical methods, as it includes three methods that are used in this method according to certain conditions to choose from these three methods, which are the explicit method, the implicit method, and the Crank-Nicolson method (C-N) [1-2].

The Ɵ differences method relies primarily on three cases of Ɵ values to apply one of the three methods (implicit, explicit, and C-N). So that each of the three methods on which the difference method depends has a different working method by dividing the area for which the heat spread is to be measured into levels, where each of these levels represents the amount of change in time, as well as different stability conditions for each of them [3].

Many researchers have presented studies related to the heat equation, including but not limited to: Many researchers have presented the heat equation by [4-8]. By [9] methods were created for the basic solution of harmonic and pheromonic BVPs. By [10] (X) the proposed TH group for the pheromonic equation. Accordingly, N. Jacob [11]. (X) Present (A) the comparative study of the 2D asymmetric diffusion problem (A) with wall convection using the theta method. Furthermore, B. J. Noye and K. J. Hayman in [12]. ADI has been used to solve two-dimensional time-dependent heat equations based on a mathematical parameter constant or degree. D. W. Peaceman and H. H. Rachford in [13]. (X) Use ADI methods to solve elliptic problems. A. Aderito et. al. [14]

1.1 Ɵ-Differences Method

The general formula for the Ɵ-difference method is as follows:

$$[\theta \frac{N_{J+1,K+1} - 2N_{J+1,K} + N_{J+1,K-1}}{(\Delta X)^2} + (1-\theta)\frac{N_{J,K+1} - 2N_{J,K} + N_{J,K-1}}{(\Delta X)^2} = \frac{N_{J+1,K} - N_{J,K}}{\Delta y} \quad (1)$$

The work of this method depends on three cases of the values of Ɵ in equation (1) to apply one of the three methods (implicit, explicit, and C-N).

$$\alpha[\frac{N_{J+1,K+1} - 2N_{J+1,K} + N_{J+1,K-1}}{(\Delta X)^2} = \frac{N_{J+1,K} + N_{J,K-1}}{\Delta y} \quad (2)$$

What is meant by α is the constant of proportionality (decreasing and increasing) of temperature, and it depends mainly on the material of which the plate is composed.
When θ=0 is used implicitly, when θ=1/2, the C-N method is used, while when θ=1, the explicit method is used.
The implicit method uses the implicit representation for each m of equations that are unknown at $N_{j+1,k}$. The general formula for this method is [15]:

$$N_{j+1,k-1} + \left(-2\frac{(\Delta x)^2}{\propto \Delta y}\right)N_{j+1,k} + N_{j+1,k+1} = \left(-\frac{(\Delta x)^2}{\propto (\Delta y)}\right)N_{j,k} \quad (3)$$

In the explicit method, the explicit method often works in the vertical direction. Thus, its general formula is when (j=0) (initial conditions in equation (3)).

Finally, the Crank-Nicolson (C-N) method: This method is one of the important numerical methods in solving heat diffusion problems, and its general formula is as follows [16]:

$$[M]\frac{\propto}{2}\left(N_{j+1,k+1} - 2\frac{N_{j+1,k-1}}{(\Delta x)^2} + N_{j,k+1} - 2\frac{N_{j,k}+N_{j,k-1}}{(\Delta x)^2}\right) = \frac{N_{j+1,k}-N_{j,k}}{\Delta y} \quad (4)$$

Since M is a matrix, the left side of the equation means the center differences of multiplying α by $\frac{\partial^2 N}{\partial y^2}$ at the points $(j+1,k),(j,k)$ which are estimates of the derivatives. Two-dimensionality at points in space C.

1.2 Hybrid Method

We will explain the hybrid method, and this is done by first dividing the region to be solved or finding its values, and after that we draw the network and set the boundary and initial conditions, and after that we find the points in the usual explicit method by substituting the values of the initial and boundary conditions in the general formula of the explicit method, and after that They are all solved. We find the middle and intended points, that is, we find the points between every two known points obtained from the previous process, by using the Crank-Nicolson formula, and after substituting the values, we obtain a set of equations that are solved directly, and then we obtain the required values. To find the values of heat diffusion.

1.3 Algorithm of Hybrid Method

1- Create the mesh or region with the boundary values and initial values given in the problem.
2- We substitute the value of (k=0 and j=1,2,3,4,5,6,7) into the general form of the Implicit method equation (3).
3- We substitute the boundary and initial values and the value of M into equation (3), and after substitution we obtain five equations resulting from substituting the values.
4- By solving the five equations simultaneously, we obtain the values of the unknowns $N_{1,}^1 N_{2,}^1 N_{3,}^1 N_{4,}^1 N_5^1, N_6^1, N_{7,}^1$ then we repeat the previous steps at the value of k=1.

5- After we obtain the solution for the explicit method for levels 1 and 2, we find the midpoints, that is, we find a middle point between every two main points, using the general formula Crank-Nicolson Hybrid.
6- We make the network again and define the points again, so the label will differ.
7- We substitute the values extracted from the implicit method into the general hybrid C_N formula and substitute the value of M. Also, after that, we will get six equations that can be solved immediately
8- By solving the six equations, we obtain the values of the unknowns $N_{1,}^{1}N_{3,}^{1}N_{5,}^{1}N_{7,}^{1}N_{9,}^{1}N_{11}^{1}N_{13,}^{1}N_{15}^{1}$, and we repeat the previous steps at k=1

2. Particular part

Let us have a rectangular plate with dimensions (4cm×2cm) and applied to the x and y axes at the origin according to the data shown:

$\Delta t = 0.2$ sec, $\Delta x = 0.5$ cm , $a = 1$

Initial conditions: $u(x,t) = 0°$ $0<x<4$ $t = 0$

Boundary conditions: $u(0,t) = 0°$, $u(4,t) = 2°$

$$M= a\frac{\Delta t}{(\Delta x)^2}$$

$$M=1\ \frac{0.2}{0.25}=0.8$$

j=1,2,3,4,5,6,7, k=0,1,2,3,4,5
Solution: We create a mesh(network) point and determined the data on it (we find two levels).

The following figure shown the distribution of boundary and initial values on mesh points

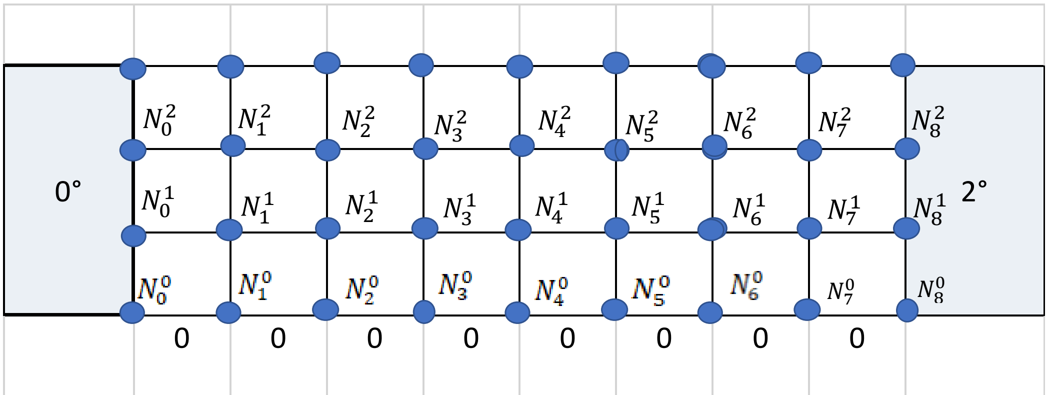


Fig. (1): Distribution of boundary and initial values on mesh points

2.1 The Explicit Method

We substitute the value of j=1,2,3,4,5,6,7 and the value of M=0.8, k=0

$$N_j^{k+1} = N_j^k + M\,(N_{j\_1}^k - 2N_j^k + N_{j+1}^k)$$

$N_1^1 = N_1^0 + M\,(N_0^0 - 2N_1^0 + N_2^0)$ =0+0.8(0-2(0) +0) = 0

$N_2^1 = N_2^0 + M\,(N_1^0 - 2N_2^0 + N_3^0)$ = 0+0.8(0-2(0) +0) =0

$N_3^1 = N_3^0 + M\,(N_2^0 - 2N_3^0 + N_4^0)$ =0+0.8(0-2(0) +0) = 0

$N_4^1 = N_4^0 + M\,(N_3^0 - 2N_4^0 + N_5^0)$ = 0+0.8(0-2(0) +0) = 0

$N_5^1 = N_5^0 + \mathrm{M}\,(N_4^0 - 2N_5^0 + N_6^0)$ =0+0.8(0-2(0) +0) =0

$N_6^1 = N_8^0 + \mathrm{M}\,(N_5^0 - 2N_6^0 + N_7^0)$ =0+0.8(0-2(0) +0) =0

$N_7^1 = N_7^0 + \mathrm{M}\,(N_6^0 - 2N_7^0 + N_8^0)$ =0+0.8(0-2(0) +2) =1

When k=1, j=1,2,3,4,5,6,7 and M=0.8 in the same way we find

$N_1^2 = 0, N_2^2 = 0,\ N_3^2 = 0,\ N_4^2 = 0,\ N_5^2 = 0,\ N_6^2 = 1.28,\ N_7^2 = 3.2$

The following figure shown distribution of values at the second and third levels

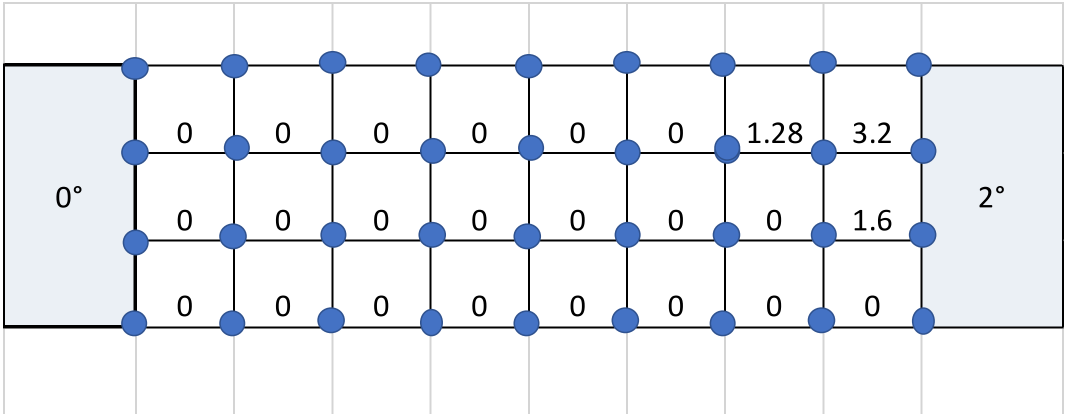


Fig. (2): Distribution of values at the second and third levels

Now we solve the previous example by implicit method

2.2 The Implicit Method

$N_j^k = -\mathrm{M}N_{j-1}^{k+1} + (1+2\mathrm{M})N_j^{k+1} - \mathrm{M}N_{j+1}^{k+1}$

When k=0, و j=1,2,3,4,5,6,7

$N_1^0 = -\mathrm{M}N_0^1 + (1+2\mathrm{M})N_1^1 - \mathrm{M}N_2^1$

$N_2^0 = -\mathrm{M}N_1^1 + (1+2\mathrm{M})N_2^1 - \mathrm{M}N_3^1$

$N_3^0 = -\mathrm{M}N_2^1 + (1+2\mathrm{M})N_3^1 - \mathrm{M}N_4^1$

$N_4^0 = -\mathrm{M}N_3^1 + (1+2\mathrm{M})N_4^1 - \mathrm{M}N_5^1$

$N_5^0 = -\mathrm{M}N_4^1 + (1+2\mathrm{M})N_5^1 - \mathrm{M}N_6^1$

$N_6^0 = -\mathrm{M}N_5^1 + (1+2\mathrm{M})N_6^1 - \mathrm{M}N_7^1$

$N_7^0 = -\mathrm{M}N_6^1 + (1+2\mathrm{M})N_7^1 - \mathrm{M}N_8^1$

We substitute the boundary and initial values and M=0.8

$2.6N_1^1 - 0.8N_2^1 = 0$ … … … (5)

$-0.8N_1^1 + 2.6N_2^1 - 0.8N_3^1 = 0$ … … … (6)

$-0.8N_2^1 + 2.6N_3^1 - 0.8N_4^1 = 0$ … … … (7)

$-0.8N_3^1 + 2.6N_4^1 - 0.8N_5^1 = 0$ … … … (8)

$-0.8N_4^1 + 2.6N_5^1 - 0.8N_6^1 = 0$ … … … (9)

$-0.8N_5^1 + 2.6N_6^1 - 0.8N_7^1 = 0$ … … … (10)

$-0.8N_6^1 + 2.6N_7^1 - 1.6 = 0 \dots \dots \dots (11)$

We solve linear equations consisting of 11 equations with 11 unknowns using MATLAB.

The coefficient matrix [A], the variable matrix [X], and the constant matrix [B] we solve in MATLAB as follows:

$$A = \begin{bmatrix} 2.6000 & -0.8000 & 0 & 0 & 0 & 0 & 0 \\ -0.8000 & 2.6000 & -0.8000 & 0 & 0 & 0 & 0 \\ 0 & -0.8000 & 2.6000 & -0.8000 & 0 & 0 & 0 \\ 0 & 0 & -0.8000 & 2.6000 & -0.8000 & 0 & 0 \\ 0 & 0 & 0 & -0.8000 & 2.6000 & -0.8000 & 0 \\ 0 & 0 & 0 & 0 & -0.8000 & 2.6000 & -0.8000 \\ 0 & 0 & 0 & 0 & 0 & -0.8000 & 2.6000 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1.6000 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0.4302 & 0.1480 & 0.0509 & 0.0175 & 0.0060 & 0.0020 & 0.0006 \\ 0.1480 & 0.4811 & 0.1656 & 0.0570 & 0.0196 & 0.0067 & 0.0020 \\ 0.0509 & 0.1656 & 0.4871 & 0.1676 & 0.0576 & 0.0196 & 0.0060 \\ 0.0175 & 0.0570 & 0.1676 & 0.4878 & 0.1676 & 0.0570 & 0.0175 \\ 0.0060 & 0.0196 & 0.0576 & 0.1676 & 0.4871 & 0.1656 & 0.0509 \\ 0.0020 & 0.0067 & 0.0196 & 0.0570 & 0.1656 & 0.4811 & 0.1480 \\ 0.0006 & 0.0020 & 0.0060 & 0.0175 & 0.0509 & 0.1480 & 0.4302 \end{bmatrix}$$

$$X = \begin{bmatrix} 0.0010 \\ 0.0033 \\ 0.0096 \\ 0.0280 \\ 0.0815 \\ 0.2369 \\ 0.6883 \end{bmatrix}$$

($N_1^1 = 0.0010$, $N_2^1 = 0.0033$, $N_3^1 = 0.0096$, $N_4^1 = 0.028$, $N_5^1 = 0.0815$, $N_6^1 = 0.2369$, $N_7^1 = 0.6883$)

Now when k=1 and j=1,2,3,4,5,6,7, we substitute the value of (M=0.8), the values of the first row of nodes in the network, and in the same way as above, we obtain the values of the second level.

$N_1^2 = 0.0043$, $N_2^2 = 0.0128$, $N_3^2 = 0.0331$, $N_4^2 = 0.0827$, $N_5^2 = 0.2008$, $N_6^2 = 0.4680$, $N_7^2 = 1.0241$

The following figure shown distribution of values at the second and third levels.

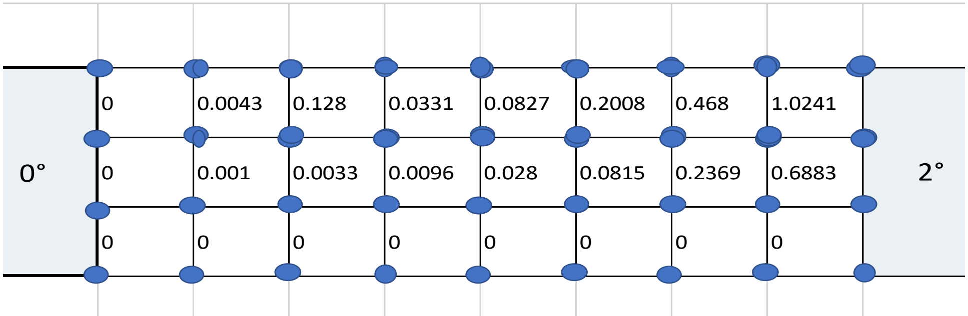
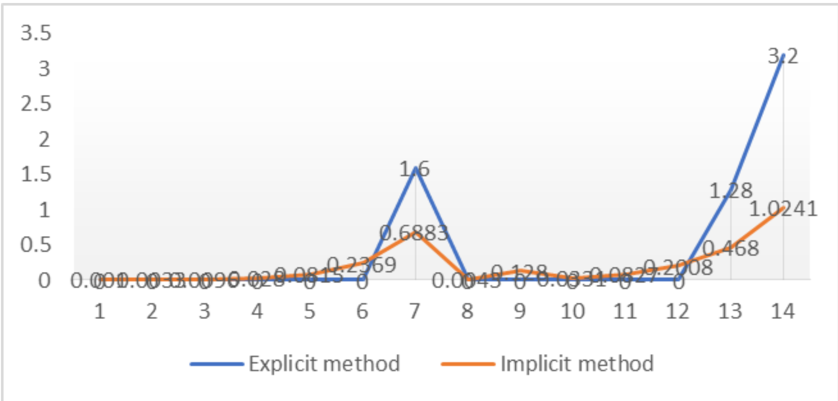


Fig. (3): Distribution of values at the second and third levels

Table (1): The table below explicit method and the implicit method

Explicit method	**0**	**0**	**0**	**0**	**0**	**0**	**1.6**	**0**	**0**	**0**	**0**	**0**	**1.28**	**3.2**
Implicit method	**0.001**	**0.003**	**0.01**	**0.028**	**0.082**	**0.237**	**0.688**	**0.004**	**0.128**	**0.033**	**0.083**	**0.201**	**0.468**	**1.0241**



Flowchart(1): The results of the previous two cases

Now we find the midpoints of the example solved by the explicit method using the hybrid Crank Nicolson.

2.3 The Explicit Hybrid Crank Nicolson Method

The following figure shown the result of the all levels nodes

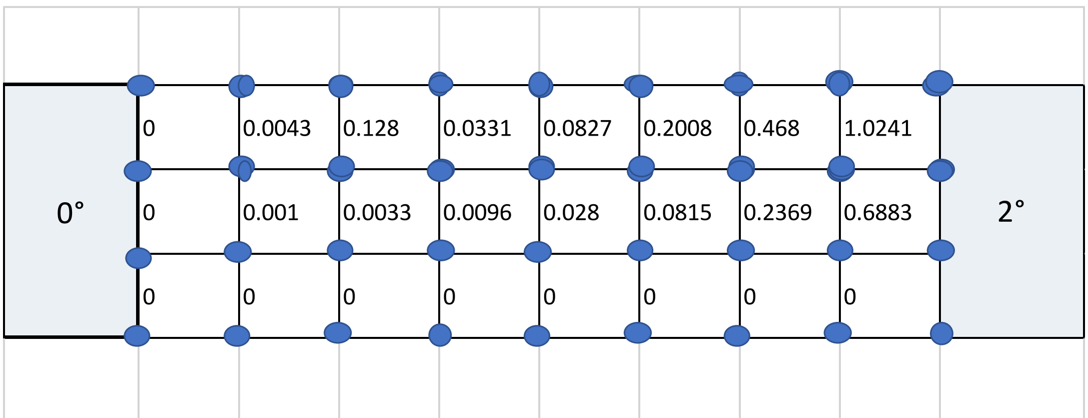


Fig. (4): The results of the mesh nodes by explicit method

$$2(1+M)N_j^{k+1} = M(N_{j-1}^{k}+N_{j+1}^{k}+N_{j-1}^{k+1}+N_{j+1}^{k+1}) + 2(1-M)N_j^{k}$$

When k=0, j= j=1,3,5,7,9,11,13,15 and M=0.8 to find $N_{1,}^1 N_{3,}^1 N_{5,}^1 N_7^1 N_{9,}^1 N_{11,}^1 N_{13,}^1 N_{15}^1$

2(1+0.8)$N_1^1 = 0.8(N_0^0+N_2^0 + N_0^1+N_2^1)+2(1-0.8)N_1^0$

3.6N_1^1=0.8(0+0+0+0) + 0

N_1^1= 0

$2(1+0.8)N_3^1 = 0.8(N_2^0+N_4^0+N_2^1+N_4^1) + 2(1-0.8)N_3^0$

3.6N_3^1= 0.8(0+0+0+0) + 0

N_3^1=0

$2(1+0.8)N_5^1 = 0.8(N_4^0+N_6^0+N_4^1+N_6^1) + 2(1-0.8)N_5^0$

3.6N_5^1=0.8(0+0+0+0) + 0

N_5^1=0

$2(1+0.8)N_7^1 = 0.8(N_6^0+N_8^0+N_6^1+N_8^1) + 2(1-0.8)N_7^0$

3.6N_7^1=0.8(0+0+0+0) + 0

N_7^1=0

$2(1+0.8)N_9^1 = 0.8(N_8^0+N_{10}^0+N_8^1+N_{10}^1) + 2(1-0.8)N_9^0$

3.6N_9^1=0.8(0+0+0+0) + 0

N_9^1=0

$2(1+0.8)N_{11}^1 = 0.8(N_{10}^0+N_{12}^0+N_{10}^1+N_{11}^1) + 2(1-0.8)N_{11}^0$

3.6N_{11}^1=0.8(0+0+0+0) + 0

N_{11}^1=0

$2(1+0.8)N_{13}^1 = 0.8(N_{12}^0+N_{14}^0+N_{12}^1+N_{14}^1) + 2(1-0.8)N_{13}^0$

3.6N_{13}^1=0.8(0+0+0+1.6) + 0

$3.6N_{13}^1$=1.28

$N_{13}^1 = 0.35$

$2(1+0.8)N_{15}^1 = 0.8(N_{14}^0+N_{16}^0+N_{14}^1+N_{16}^1) + 2(1-0.8)N_{15}^0$

3.6N_{15}^1=0.8(0+0+1.6+2) + 0

$3.6N_{15}^1$=2.88

$N_{15}^1 = 0.8$

When k=1, *j=1,3,5,7,9,11,13,15* and in the same way as above, we obtain the values of the second level. To find $N_{1,}^2 N_{3,}^2 N_{5,}^2 N_{7,}^2 N_{9,}^2 N_{11,}^2 N_{13}^2, N_{15}^2$

N_1^2=0

N_3^2=0

N_5^2=0

N_7^2=0

N_9^2=0

$N_{11}^2 = 0.28$

N_{13}^2 =1.72

N_{15}^2 =2.04

The following figure shown the result by explicit method using the hybrid C_N method

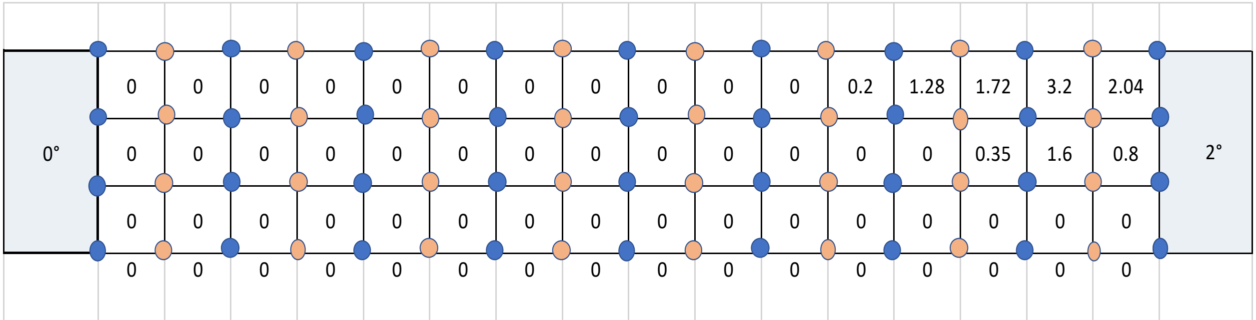


Fig. (5): Results by explicit method using the hybrid C_N method

Now we solve the previously solved example by implicit method using the hybrid C_N method to find the midpoints

2.4 The Implicit Hybrid Crank Nicolson Method

$$2(1+M)N_j^{k+1} = M(N_{j-1}^k+N_{j+1}^k+N_{j-1}^{k+1}+N_{j+1}^{k+1}) + 2(1-M)N_j^k$$

When k=0, j=1,3,5,7,9,11,13,15 to find $N_{1,}^1 N_{3,}^1 N_{5,}^1 N_7^1 N_{9,}^1 N_{11,}^1 N_{13,}^1 N_{15}^1$, M=0.8

2(1+0.8)$N_1^1 = 0.8(N_0^0+N_2^0 + N_0^1+N_2^1)+2(1-0.8)N_1^0$

3.6N_1^1=0.8(0+0+0+0.001) + 0

3.6N_1^1=0.0008

N_1^1= 0.00022

$$2(1+0.8)N_3^1 = 0.8(N_2^0+N_4^0+N_2^1+N_4^1) + 2(1-0.8)N_3^0$$

3.6N_3^1= 0.8(0+0+0.001+0.0033) + 0

3.6N_3^1= 0.00344

N_3^1=0.00095

$$2(1+0.8)N_5^1 = 0.8(N_4^0+N_6^0+N_4^1+N_6^1) + 2(1-0.2)N_5^0$$

3.6N_5^1=0.8(0+0+0.0033+0.0096) + 0

3.6N_5^1=0.01032

N_5^1=0.0028

$$2(1+0.8)N_7^1 = 0.8(N_6^0+N_8^0+N_6^1+N_8^1) + 2(1-0.8)N_7^0$$

$3.6N_7^1$=0.8(0+0+0.0096+0.028) + 0

$3.6N_7^1$=0.03

N_7^1=0.008

$2(1+0.8)N_9^1 = 0.8(N_8^0+N_{10}^0+N_8^1+N_{10}^1)+2(1-0.8)N_9^0$

$3.6N_9^1$=0.8(0+0+0.028+0.0815) + 0

$3.6N_9^1$=0.0876

N_9^1=0.024

$2(1+0.8)N_{11}^1 = 0.8(N_{10}^0+N_{12}^0+N_{10}^1+N_{11}^1)+2(1-0.8)N_{11}^0$

$3.6N_{11}^1$=0.8(0+0+0.0815+0.2369) + 0

$3.6N_{11}^1$=0.2547

N_{11}^1=0.07

$2(1+0.8)N_{13}^1 = 0.8(N_{12}^0+N_{14}^0+N_{12}^1+N_{14}^1)+2(1-0.8)N_{13}^0$

$3.6N_{13}^1$=0.8(0+0+0.2369+0.6883) + 0

$3.6N_{13}^1$=0.74

$N_{13}^1 = 0.20$

$2(1+0.8)N_{15}^1 = 0.8(N_{14}^0+N_{16}^0+N_{14}^1+N_{16}^1)+2(1-0.8)N_{15}^0$

$3.6N_{15}^1$=0.8(0+0+0.6883+2) + 0

$3.6N_{15}^1$=2.88

$N_{15}^1 = 0.597$

When k=1, *j=1,3,5,7,9,11,13,15* and in the same way as above, we obtain the values of the second level.

To find $N_{1,}^2 N_{3,}^2 N_{5,}^2 N_{7,}^2 N_{9,}^2 N_{11,}^2 N_{13}^2, N_{15}^2$

N_1^2=0.0014

N_3^2=0.03

N_5^2=0.0389

N_7^2=0.0349

N_9^2=0.084

$N_{11}^2 = 0.227$

N_{13}^2 =0.425

N_{15}^2 =1.335

The following figure shown the result by explicit method using the hybrid C_N method

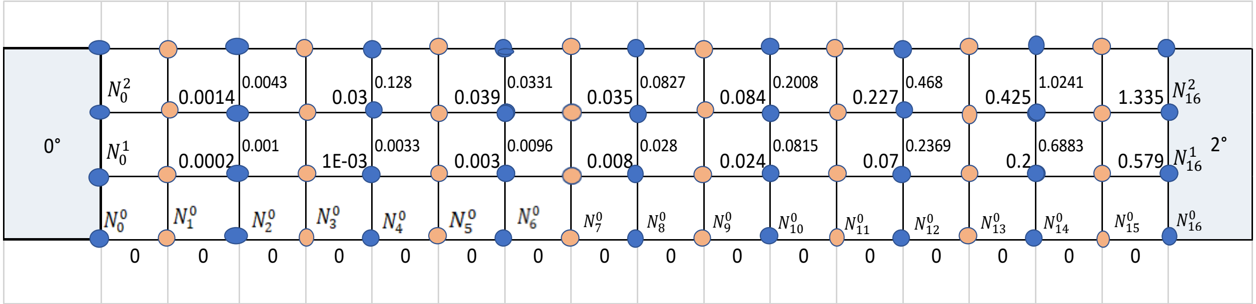
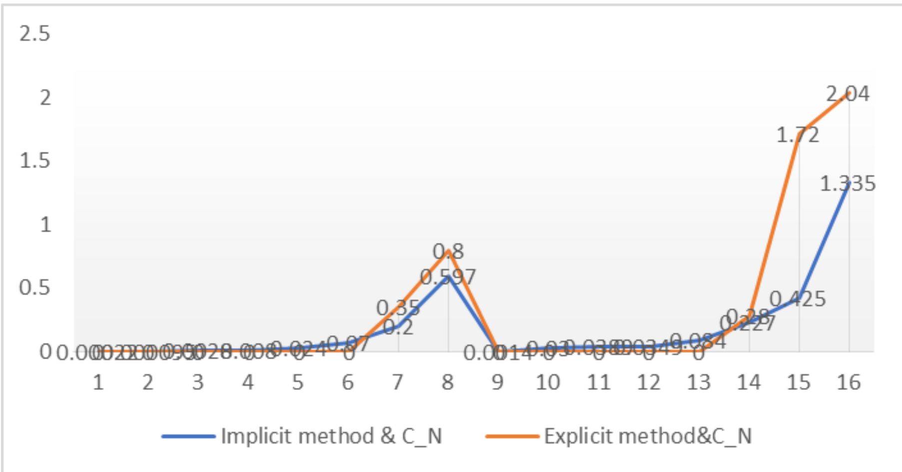


Fig. (6): Results by implicit method using the hybrid C_N method

Table (2): The table of results two cases above

Implicit method C_N &	0.0002	9.5E-05	0.003	0.008	0.024	0.07	0.2	0.597	0.001	0.03	0.039	0.035	0.084	0.227	0.425	1.335
Explicit method&C_N	0	0	0	0	0	0	0.35	0.8	0	0	0	0	0	0.28	1.72	2.04



Flowchart(2): The chart showing the results of the previous two cases

3. Conclusions

In this study, two hybrid methods were studied, and the results were divided in terms of speed, accuracy of the solution, and difficulty of the solution. The results of the first hybrid method were faster in reaching the solution, but more difficult in application compared to the second hybrid method, which was the opposite of the first method, as it was slower in reaching the solution. The solution is easier to implement and takes more time, especially in the first levels. However, there is an agreement in most of the results in the last levels.

4. ACKNOWLEDGEMENT

I extend my sincere thanks and gratitude to my supervisor, Assistant Professor, Awni M. Gaftan, for the help, guidance and advice he has shown in order to display this knowledge in its best light.

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المستخلص:تعرف المعادلة التي تعتمد على متغيرين مستقلين أو أكثر، وقد تحتوي أيضا على مشتقات لواحدة أو اثنتين، بالمعادلة التفاضلية الجزئية. يمكن استخدام هذا النوع من المعادلات لوصف حالات مختلفة في الفيزياء والكيمياء والظواهر الطبيعية مثل حركة السحب والبخار والرياح. والغازات وما يصاحبها من حالات التلوث وتدفق المياه وغيرها من الحالات. إحدى المعادلات التفاضلية الجزئية المهمة هي معادلة انتشار الحرارة. تم في هذا البحث عرض طريقة التمايز الهجين لحل معادلات انتشار الحرارة، وكذلك تطوير خوارزمية لتلك الطريقة المطورة والصيغ العامة لتلك المعادلة وما يصاحبها. تتكون هذه الصيغة من الشروط الأولية والشروط الحدودية.

وأخيرا قمنا بمقارنة نتائج الطريقة الجديدة مع نتائج الطريقة التقليدية، وقمنا بالتطبيق العملي للطريقة الجديدة باستخدام برنامج MATLAB

الكلمات المفتاحية: طريقة التفريق (θ)، طريقة التفريق (θ) الهجينة، الطريقة الضمنية، الطريقة الصريحة، كرانك-نيكولسون.

A New Class of Generalized Contra ω-Continuity and Some Their Applications

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Abstract:

Generalized ω-open sets are now distinguished concepts in topological spaces and their applications. Our article is devoted to study a original structure of generalized contra ω-continuity called, contra $\omega_{\delta\text{-}ß}$-continuous mappings which is a weak form of contra continuity via a novel notion of extended δ-ß-open and ω-open sets called, $\omega_{\delta\text{-}ß}$-open sets, also this manuscript is dealing with the applications of notion $\omega_{\delta\text{-}ß}$-open sets. Diverse essential properties and basic characterizations related to this class of generalization continuous maps have been discussed. As well, we investigate the relations between this kind of extended contra ω-continuous maps and other distinguished structures of generalized contra ω-continuity in topological spaces. Additionally, some clarifying examples to highlight the superiority of our outcomes have been provided.

Mathematics Subject Classification 2010: 54C05,54C08,54C10.

Keywords: $\omega_{\delta\text{-}ß}$-open sets, $\omega_{\delta\text{-}ß}$-continuity, $\omega_{\delta\text{-}ß}$-connectedness, contra $\omega_{\delta\text{-}ß}$-continuity.

1. Introduction

Continuity and of course generalized contra continuity stand among the most vital concepts in the whole of mathematics and related sciences, therefore various forms generalized contra ω-continuous maps have been offered over the last years, and their variations have been extensively studied". ω-open and ω-closed sets have been described and studied via H. Z. Hdeib in [1],and offered the idea of ω-continuous map as extension of continuity in [2]. A novel class of continuity called contra continuous map has been introduced via J. Dontchev, in[3]. AL-Zoubi [4] proved that the family of all ω-open sets in $\mathcal{X}$ form topology which finer than $\mathcal{T}$. Afterward, two new weaker forms of contra continuity are offered, the first type called, contra ß-continuity which is presented by M. Caldas in[5], and the second type namely, contra ω-continuity which is studied by A. Al-Omari in[6]. Later, regular generalized ω-closed sets and ω-continuous map have been supplied as new classes of generalized closed sets via A. Al-Omari in [7]. "A new class of generalized open sets, called δ-ß-open sets was presented in a general topology along with δ-ß-continuity via E. Hatir & T. Noiri in [8]". In [9] A.Al-Omari et al, presented other strong and weak structures of ω-Continuity. H. Aljarrah [10] defined new kinds of generalized mappings namely, ωß-irresolute, ωß-open and ωß-closed maps by utilizing $\omega_{ß}$-open sets. Later than generalized ω_{β}-closed sets have been offered by H. Aljarrah et al, [11]. Newly, S. Al Ghou, in [12] presented the concept of ω_{s}-irresoluteness as strong form of ω_{s}-continuity and established that ω_{s}-irresoluteness is independent of both continuity and irresoluteness. As well, B. J. Waqas [13] presented another structure of continuity called, *contra ω_{pre}-continuous* maps, by the notion of ω_{pre}-open sets. Also, P. Sasmaz [14], defined δ_{ω}-open sets and offered some sorts of continuity. In addition refer the reader to see [15-19]. The main goal of our manuscript is to present and investigate some interesting properties and essential characterizations concerning of new class of generalized contra ω-continuity namely, contra $\omega_{\delta\text{-ß}}$-continuous maps by new notion of generalized *δ*-*ß*-open and *ω*-open sets called, $\omega_{\delta\text{-ß}}$-open sets . Additionally, investigate the relation between this new notion of extended continuity and other recognized of extended contra ω-continuity.

2. Prerequisites

In this manuscript, $(\mathcal{X},\mathcal{T})\&(\mathcal{Y},\mathcal{T}^*)$(briefly, $\mathcal{X}\&\mathcal{Y}$)mean topological spaces (briefly. Top-Sp). We recall the following definitions and the significant concepts that play vital role in our study .

Definition 2.1: [1] Let $(\mathcal{X},\mathcal{T})$ be a Top-Sp and $\mathcal{K}\subseteq\mathcal{X}$ then :

$(\boldsymbol{a})$ $\mathcal{p}\in\mathcal{X}$ *is* condensation point of $\mathcal{K}$ if $\forall\ \mathcal{N}\in\mathcal{T}$ and $\mathcal{p}\in\mathcal{N}$, $\mathcal{N}\cap\mathcal{K}$ is Uncountable.

$(\boldsymbol{b})$ $\mathcal{K}\subseteq\mathcal{X}$ *is* *ω*-closed if it contains each its condensation points. The complement of ω-closed is called ω-open . It is well-known a subset $\mathcal{K}$ of $\mathcal{X}$ is *ω*-open *if and only if* $\forall\ \mathcal{p}\in\mathcal{K}\ \exists\ \mathcal{N}\in\mathcal{T}$(s. t)$\mathcal{p}\in\mathcal{N}\&\mathcal{N}-\mathcal{K}$ is countable set.

$(\boldsymbol{c})$ *ω*-interior with ω-closure of $\mathcal{K}$ indicated $Int_{\omega}(\mathcal{K})$ and $Cl_{\omega}(\mathcal{K})$ in that order and are described as:

$(\mathbf{1})$ $Int_{\omega}(\mathcal{K})=\bigcup\{\mathcal{B}\subseteq\mathcal{X}:\mathcal{B}\ is\ \omega-\text{open}\ \&\ \mathcal{B}\subseteq\mathcal{K}\}$.

$(\mathbf{2})$ $Cl_{\omega}(\mathcal{K})=\bigcap\{\mathcal{A}\subseteq\mathcal{X}:\mathcal{A}\ is\ \omega-\text{closed}\ \&\ \mathcal{K}\subseteq\mathcal{A}\}$.

Remark.2.2: Family of all *ω*-open subsets of $\mathcal{X}$ defined $\mathcal{T}_{\omega}$ or $\omega\Sigma(\mathcal{X},\mathcal{T})$,forms a topology finer than $\mathcal{T}$.

Definition 2.3: [20] A subset $\mathcal{K}$ of $\mathcal{X}$ is *δ*-open if $\forall\ x\in\mathcal{K}\ \exists$ open $\mathcal{N}$ (s. t), $x\in\mathcal{N}\subseteq Int(Cl(\mathcal{N}))\subseteq\mathcal{K}$. Collection of all *δ*-open sets in $\mathcal{X}$ is indicated via $\delta\Sigma(\mathcal{X},\ \mathcal{T})$.

Definition2.4: [8] A subset $\mathcal{K}$ of $\mathcal{X}$ is said to *δ*-*ß*-open if $\mathcal{K}\subseteq Cl(Int(Cl_{\delta}(\mathcal{K})))$,intersection of each *δ*-*ß*-closed sets containing $\mathcal{K}$ is *δ*-*ß*-closure of $\mathcal{K}$ and is indicated by $Cl_{\delta-ß}(\mathcal{K})$,union of each *δ*-*ß*-open sets of $\mathcal{X}$ contained in $\mathcal{K}$ is *δ*-*ß*-interior of $\mathcal{K}$ and is indicated by $Int_{\delta-ß}(\mathcal{K})$.

Remark.2.5: Collection of each *δ*-*ß*-open and *δ*-*ß*-closed subsets containing $\mathcal{p}\in\mathcal{X}$ is defined as $\delta-ß\Sigma(\mathcal{X},\mathcal{p})\ (resp.\ \delta-\beta C(\mathcal{X},\mathcal{p})$. Family of all *δ*-*ß*-open and *δ*-*ß*-closed are defined via$\delta-ß\Sigma(\mathcal{X},\mathcal{T})(resp.\ \delta-ßC(\mathcal{X},\mathcal{T}))$.

Proposition2.6: [8] Following statements hold for $\mathcal{X}$:

$(\boldsymbol{a})$ Arbitrary union of any family of *δ*-*ß*-open sets in $\mathcal{X}$, is *δ*-*ß*-open .

$(\boldsymbol{b})$ Arbitrary intersection of any family of *δ*-*ß*-closed sets in $\mathcal{X}$, is an *δ*-*ß*-closed .

Lemma2.7: [8] Assume $\mathcal{X}$ be a Top-sp and $\mathcal{K},\mathcal{D}\subseteq\mathcal{X}$. *If* $\mathcal{K}$ is open set & $\mathcal{D}$ is *δ*-*ß*-open so, $\mathcal{K}\cap\mathcal{D}$ is *δ*-*ß*-open.

Definition 2.8: [21] A subset $\mathcal{N}$ of $(\mathcal{X},\mathcal{T})$ is said to be *δ*-*ß*-Neighborhood of $\mathcal{p}\in\mathcal{X}$ if $\exists$ *δ*-*ß*-open set $\mathcal{K}$ of $\mathcal{X}$ (s. t) $\mathcal{p}\in\mathcal{K}\subseteq\mathcal{N}$.

Definition.2.9: [3] $\mathfrak{F}:(\mathcal{X},\mathcal{T})\longrightarrow(\mathcal{Y},\mathcal{T}^*)$ is said to be contra-continuous if $\mathfrak{F}^{-1}(\mathcal{N})$ is closed in $\mathcal{X}\ \forall$ open set $\mathcal{N}\ of\ \mathcal{Y}$.

Definition 2.10: [22] Let $(\mathcal{X},\mathcal{T})$ be a Top-sp and $\mathcal{H}\subseteq\mathcal{X}$. The set $\bigcap\{\mathcal{N}\in\mathcal{T}:\mathcal{H}\subseteq\mathcal{N}\}$ is called the kernel of $\mathcal{H}$ and denoted by $ker(\mathcal{H})$.

Lemma 2.11: [23] Let $\mathcal{H}$ and $\mathcal{K}$ be subsets of $\mathcal{X}$, so the next statements satisfied:

(i) $p \in Ker(\mathcal{H})$ *iff* $\mathcal{H} \cap \mathcal{M} \neq \emptyset$ for at all closed set $\mathcal{M}$ in $(\mathcal{X}, \mathcal{T})$ containing p.

(ii) If $\mathcal{H} \in Ker(\mathcal{H})$ & $\mathcal{H}$ is open in $\mathcal{X}$, so $\mathcal{H} = Ker(\mathcal{H})$.

(iii) If $\mathcal{H} \subseteq \mathcal{K}$, so $Ker(\mathcal{H}) \subseteq Ker(\mathcal{K})$.

3- Various Characterizations of Contra $\omega_{\delta\text{-}\beta}$-Continuous Mappings

This segment, devoted to present diverse characterizations and essential properties related to contra $\omega_{\delta\text{-}\beta}$-continuous (briefly. C-$\omega_{\delta\text{-}\beta}$-Cont.) maps in topological spaces.

Definition 3.1: A subset $\mathcal{K}$ of $(\mathcal{X}, \mathcal{T})$ is said to be $\omega_{\delta\text{-}\beta}$-open if $\forall\, p \in \mathcal{K}, \exists$ δ-β-open subset $\mathcal{N}_p \subseteq \mathcal{X}$ containing p (s. t) $\mathcal{N}_p - \mathcal{K}$ is countable .

Remark 3.2: Complement of $\omega_{\delta\text{-}\beta}$-open subset is $\omega_{\delta\text{-}\beta}$-closed, as well the collection of each $\omega_{\delta\text{-}\beta}$-open(*resp*. $\omega_{\delta\text{-}\beta}$-open) subsets of $(\mathcal{X}, \mathcal{T})$ indicated by $\omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})\left(resp.\, \omega_{\delta-\beta}C(\mathcal{X}, \mathcal{T})\right)$.

Definition 3.3: $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is called C-$\omega_{\delta\text{-}\beta}$-Cont. if $\mathfrak{F}^{-1}(\mathcal{W})$ is $\omega_{\delta\text{-}\beta}$-closed in $(\mathcal{X}, \mathcal{T})$ for each open $\mathcal{W}$ of $(\mathcal{Y}, \mathcal{T}^*)$

Remark 3.4: For a space $\mathcal{X}$ apparent that:

(a) If $\mathcal{X}$ is a C-set, so each map $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ C-$\omega_{\delta\text{-}\beta}$-Cont.

(b) Every C-Cont map is C-$\omega_{\delta\text{-}\beta}$-Cont., but the converse not necessary to be true in general as shown in the next example:

Example 3.5: Assume $\mathcal{X} = \mathbb{R}$ with the topology $\mathcal{T} = \mathcal{T}_{\mathcal{U}}$ and $\mathcal{Y} = \{r, s\}$ with the topology $\mathcal{T}^* = \{\emptyset, \mathcal{Y}, \{s\}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is a map described by:

$$\mathfrak{F}(p) = \begin{cases} s & p \in \mathbb{Q} \\ r & p \in \mathbb{R} - \mathbb{Q} \end{cases}$$

So therefore $\mathfrak{F}$ is contra $\omega_{\delta\text{-}\beta}$-continuous but not C-Cont.

Lemma 3.6: The following statements hold for $\mathcal{X}$:

(a)Union of any family of $\omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$ sets is $\omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$.

(b)Intersection of an $\omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$ set and ω-open is $\omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$.

Definition.3.7: Let $\mathcal{K} \subseteq \mathcal{X}$, then, $\omega_{\delta\text{-}\beta}$-interior and $\omega_{\delta\text{-}\beta}$-closure of $\mathcal{K}$ are defined through $Int_{\omega_{\delta-\beta}}(\mathcal{K})$ & $Cl_{\omega_{\delta-\beta}}(\mathcal{K})$ in that order and are indicated as:

(a) $Int_{\omega_{\delta-\beta}}(\mathcal{K}) = \bigcup\{\mathcal{B} \subseteq \mathcal{X}: \mathcal{B} \text{ is } \omega_{\delta-\beta} - \text{open} \,\&\, \mathcal{B} \subseteq \mathcal{K}\}$.

(b) $Cl_{\omega_{\delta-\beta}}(\mathcal{K}) = \bigcap\{\mathcal{A} \subseteq \mathcal{X}: \mathcal{A} \text{ is } \omega_{\delta-\beta} - \text{closed} \,\&\, \mathcal{K} \subseteq \mathcal{A}\}$.

Theorem.3.8: Following statements are equivalent for $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$:

(i) $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont.

(ii) $\mathfrak{F}^{-1}(\mathcal{M})$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$ for each closed $\mathcal{M}$ of $(\mathcal{Y}, \mathcal{T}^*)$.

(iii) For each $p \in \mathcal{X}$ and all closed $\mathcal{M}$ in $(\mathcal{Y}, \mathcal{T}^*)$containing $\mathfrak{F}(p)$, $\exists$ $\omega_{\delta\text{-}\beta}$-open $\mathcal{N}$ in $(\mathcal{X}, \mathcal{T})$ containing p (s. t) $\mathfrak{F}(\mathcal{N}) \in \mathcal{M}$.

(iv) $\mathfrak{F}\left(Cl_{\omega_{\delta-\beta}}(\mathcal{H})\right) \subseteq Ker(\mathfrak{F}(\mathcal{H}))$ for each subset $\mathcal{H}$ of $(\mathcal{X}, \mathcal{T})$.

(v) $Cl_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(\mathcal{K})) \subseteq \mathfrak{F}^{-1}(Ker(\mathcal{K}))\forall$ subset $\mathcal{K}$ of $(\mathcal{Y}, \mathcal{T}^*)$.

proof: Statements$(i) \Leftrightarrow (ii)$ &$(ii) \Rightarrow (iii)$ are apparent.

$(iii) \Rightarrow (ii)$: Suppose $\mathcal{M}$ be arbitrary closed set of $\mathcal{Y}$ & $p \in \mathfrak{F}^{-1}(\mathcal{M})$. So $\mathfrak{F}(p) \in \mathcal{M}$ and $\exists\, \mathcal{N}_p \in \omega_{\delta-\beta}\Sigma(\mathcal{X}, p)$ (s. t) $\mathfrak{F}(\mathcal{N}_p) \subseteq \mathcal{M}$. Consequently, obtain $\mathfrak{F}^{-1}(\mathcal{N}) = \cup\{\mathcal{N}_p | p \in \mathfrak{F}^{-1}(\mathcal{M})\} \in \omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$.

$(ii) \Rightarrow (iv)$: Assume $\mathcal{H}$ be any subset of $\mathcal{X}$, and $q \notin ker(\mathfrak{F}(\mathcal{H}))$. So by Lemma 2.11 $\mathfrak{F}(\mathcal{H}) \cap \mathcal{M} = \emptyset$. Thus, $\mathcal{H} \cap \mathfrak{F}^{-1}(\mathcal{M}) = \emptyset$ & $Cl_{\omega_{\delta-\beta}}(\mathcal{H}) \cap \mathfrak{F}^{-1}(\mathcal{M}) = \emptyset$. Therefore, obtain $\mathfrak{F}(Cl_{\omega_{\delta-\beta}}(\mathcal{H})) \cap \mathcal{M} = \emptyset$ & $q \notin \mathfrak{F}(Cl_{\omega_{\delta-\beta}}(\mathcal{H}))$. Thus $\mathfrak{F}(Cl(\mathcal{H})) \subseteq ker(\mathfrak{F}(\mathcal{H}))$.

$(iv) \Rightarrow (v)$: Suppose $\mathcal{K}$ be arbitrary subset of $\mathcal{Y}$. Via (4) and Lemma 2.11, obtain $\mathfrak{F}(Cl_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(\mathcal{K})))) \subset ker(\mathfrak{F}(\mathfrak{F}^{-1}(\mathcal{K}) \subset ker(\mathcal{K})$ with $Cl_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(\mathcal{K})) \subset \mathfrak{F}^{-1}(ker(\mathcal{K}))$.

$(v) \Rightarrow (i)$: Assume $\mathcal{W}$ is arbitrary open set of $\mathcal{Y}$. So, via Lemma 2.11, obtain $Cl_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(\mathcal{W})) \subset \mathfrak{F}^{-1}(ker(\mathcal{W})) = \mathfrak{F}^{-1}(\mathcal{W})$& $Cl_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(\mathcal{W})) = \mathfrak{F}^{-1}(\mathcal{W})$. This shows $\mathfrak{F}^{-1}(\mathcal{W})$ is $\omega_{\delta\text{-}\beta}$-closed.

Definition 3.9: [6] $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is called C-ω-Cont if $\mathcal{F}^{-1}(\mathcal{W})$ is ω-closed in $\mathcal{X}$ for all open set $\mathcal{W}$ in $\mathcal{Y}$.

Remark 3.10: Since each ω-open set is $\omega_{\delta\text{-}\beta}$-open, then every contra ω-continuous mapping is contra $\omega_{\delta\text{-}\beta}$-continuous, but the converse not necessary to be true in general as shown in the following example :

Example 3.11: Assume $\mathcal{X} = \mathbb{R}$ with the topology $\mathcal{T} = \mathcal{T}_u$ & $\mathcal{Y} = \{r, s\}$ with $\mathcal{T}^* = \{\emptyset, \mathcal{Y}, \{r\}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ defined by:

$$\mathfrak{F}(p) = \begin{cases} s & p \in \mathbb{Q} \\ r & p \in \mathbb{R} - \mathbb{Q} \end{cases}$$

So therefore, $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, but not C-$\omega$-Cont.

Definition3.12: [24] $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is called C-δ-β-cont, if $\mathfrak{F}^{-1}(\mathcal{W}) \in \delta - \beta C(\mathcal{X}, \mathcal{T})$ for each open set $\mathcal{W}$ in $\mathcal{Y}$.

Remark 3.13: Since every δ-β-open set is $\omega_{\delta\text{-}\beta}$-open, then every contra δ-β-continuous mapping is contra $\omega_{\delta\text{-}\beta}$-continuous, but the converse not necessary to be true in general as shown in the following example :

Example 3.14: Assume $\mathcal{X} = \{r, s, t\}$, $\mathcal{T} = \{\emptyset, \mathcal{X}, \{s\}, \{t\}, \{s, t\}\}$ & $\mathcal{T}^* = \{\emptyset, \mathcal{X}, \{r\}, \{s\}, \{r, s\}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ mapping defined by:

$$\mathfrak{F}(p) = \begin{cases} r & p = s \\ t & p = r \end{cases}$$

So therefore, $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, but not C-$\delta$-$\beta$-Continuous.

Definition3.15: $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \longrightarrow (\mathcal{Y}, \mathcal{T}^*)$ is called $\omega_{\delta\text{-}\beta}$-Cont. at $p \in \mathcal{X}$ if $\forall$ open $\mathcal{W}$ of $(\mathcal{Y}, \mathcal{T}^*)$ containing, $\mathfrak{F}(p)\ \exists\ \omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$-open set $\mathcal{N}$ of $\mathcal{X}$ including p (s. t) $\mathfrak{F}(\mathcal{N}) \subseteq \mathcal{W}$. *If* $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-Cont. at every $p \in \mathcal{X}$, then it is called $\omega_{\delta\text{-}\beta}$-Cont..

Remark3.16: The concepts of $\omega_{\delta\text{-}\beta}$-continuity and contra $\omega_{\delta\text{-}\beta}$-continuity are independent of each other as shown in the following examples :

Example3.17: Assume $\mathcal{X} = \mathbb{R}$, $\mathcal{T} = \mathcal{T}_{coc}$ & $\mathcal{Y} = \{r, s\}$ with $\mathcal{T}^* = \{\emptyset, \mathcal{Y}, \{r\}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ defined by:

$$\mathfrak{F}(p) = \begin{cases} s & p \in \mathbb{Q} \\ r & p \in \mathbb{R} - \mathbb{Q} \end{cases}$$

So therefore, $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-continuous, but not C-$\omega_{\delta\text{-}\beta}$-Cont.

Example3.18: Presume $\mathcal{X} = \mathbb{R}$, $\mathcal{T} = \mathcal{T}_{coc}$ & $\mathcal{Y} = \{r, s\}$ with $\mathcal{T}^* = \{\emptyset, \mathcal{Y}, \{r\}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ defined by:

$$\mathfrak{F}(p) = \begin{cases} r & p \in \mathbb{Q} \\ s & p \in \mathbb{R} - \mathbb{Q} \end{cases}$$

So therefore, $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, but not $\omega_{\delta\text{-}\beta}$-continuous.

Proposition3.19: Suppose $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is C-$\omega_{\delta\text{-}\beta}$-Cont. Then, $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-Cont if one of the next conditions holds:

$(\boldsymbol{i})$ $(\mathcal{Y}, \mathcal{T}^*)$ *is regular*.

$(\boldsymbol{ii})$ $Int_{\omega_{\delta-\beta}}\left(\mathfrak{F}^{-1}(Cl(\mathcal{W}))\right) \subseteq \mathfrak{F}^{-1}(\mathcal{W})$ $\forall$ open $\mathcal{W}$ in $(\mathcal{Y}, \mathcal{T}^*)$.

Proof: $(\boldsymbol{i})$ Suppose $p \in \mathcal{X}$ & $\mathcal{W}$ be open of $(\mathcal{Y}, \mathcal{T}^*)$ including $\mathfrak{F}(p)$. Since $(\mathcal{Y}, \mathcal{T}^*)$ is regular, $\exists$ open set $\mathcal{W}$ in $(\mathcal{Y}, \mathcal{T}^*)$ including $\mathfrak{F}(p)$ (s. t) $Cl(\mathcal{D}) \subseteq \mathcal{W}$. Since $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, so by theorem 3.8, there exists $\omega_{\delta\text{-}\beta}$-open $\mathcal{N}$ in $(\mathcal{X}, \mathcal{T})$ including p (s. t) $\mathfrak{F}(\mathcal{N}) \subseteq Cl(\mathcal{D})$; thus $\mathfrak{F}(\mathcal{N}) \subseteq \mathcal{W}$. Consequently $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-Cont.

$(\boldsymbol{ii})$ Presume $\mathcal{W}$ be any open of $(\mathcal{Y}, \mathcal{T}^*)$. Since $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont and $Cl(\mathcal{W})$ is closed, by theorem 3.8, obtain $\mathfrak{F}^{-1}(Cl(\mathcal{W}))$ is $\omega_{\delta\text{-}\beta}$-open in $\mathcal{X}$ and through $(\boldsymbol{ii})$, $\mathfrak{F}^{-1}(Cl(\mathcal{W})) \subseteq Int_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(Cl(\mathcal{W}))) \subseteq \mathfrak{F}^{-1}(\mathcal{W})$. Consequently, $\mathfrak{F}^{-1}(\mathcal{W}) = Int_{\omega_{\delta-\beta}}(\mathfrak{F}^{-1}(Cl(\mathcal{W})))$ and therefore $\mathfrak{F}^{-1}(\mathcal{W})$ is $\omega_{\delta\text{-}\beta}$-open in$(\mathcal{X}, \mathcal{T})$. As a result $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-Cont map.

Definition 3.20: A space $(\mathcal{X}, \mathcal{T})$ is said to be:

$(\boldsymbol{i})$ A $\omega_{\delta\text{-}\beta}$-space if each $\omega_{\delta\text{-}\beta}$-open is open in $(\mathcal{X}, \mathcal{T})$.

$(\boldsymbol{ii})$ Locally $\omega_{\delta\text{-}\beta}$-indiscrete if each $\omega_{\delta\text{-}\beta}$-open is closed in $(\mathcal{X}, \mathcal{T})$.

Proposition3.21: For a C-$\omega_{\delta\text{-}\beta}$-Cont map $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$, the next statements hold:

$(\boldsymbol{i})$ If $(\mathcal{X}, \mathcal{T})$ is a $\omega_{\delta\text{-}\beta}$-sp, so $\mathfrak{F}$ is C-Cont, C-ω-Cont & C-$\omega_{\delta\text{-}\beta}$-Cont.

$(\boldsymbol{ii})$ If $(\mathcal{X}, \mathcal{T})$ is locally $\omega_{\delta\text{-}\beta}$-indiscrete, then $\mathfrak{F}$ is Cont.

$(\boldsymbol{iii})$ If $(\mathcal{X}, \mathcal{T})$ is a $\omega_{\delta\text{-}\beta}$-sp with $\mathfrak{F}$ is closed surjection, so $(\mathcal{Y}, \mathcal{T}^*)$is locally indiscrete .

Proof: $(\boldsymbol{i})$ & $(\boldsymbol{ii})$ follows immediately from their definitions .

$(\boldsymbol{iii})$ Assume $\mathcal{W}$ is open in $(\mathcal{Y}, \mathcal{T}^*)$. As $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, so $\mathfrak{F}^{-1}(\mathcal{W})$is $\omega_{\delta\text{-}\beta}$-closed in $(\mathcal{X}, \mathcal{T})$, thus closed. As $\mathfrak{F}$ is closed & surjective, so $\mathfrak{F}\left(\mathfrak{F}^{-1}(\mathcal{W})\right) = \mathcal{W}$is closed in $(\mathcal{Y}, \mathcal{T}^*)$, consequently $(\mathcal{Y}, \mathcal{T}^*)$ is locally indiscrete.

Definition3.22: $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is slightly $\omega_{\delta\text{-}\beta}$-continuous if $\mathfrak{F}^{-1}(\mathcal{W})$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$ $\forall$ clopen sets $\mathcal{W}$ of $(\mathcal{Y}, \mathcal{T}^*)$.

Remark 3.23: Every contra $\omega_{\delta\text{-}\beta}$-continuous is slightly $\omega_{\delta\text{-}\beta}$-Cont, but the converse not true as shown in example 3.17 .

Definition 3.24: A map $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is called weakly $\omega_{\delta\text{-}\beta}$-Cont, if $\forall\ p \in \mathcal{X}$ and all open set $\mathcal{W}$ in $(\mathcal{Y}, \mathcal{T}^*)$ including $\mathfrak{F}(p)$ $\exists$ $\omega_{\delta\text{-}\beta}$-open set $\mathcal{N}$ in $(\mathcal{X}, \mathcal{T})$ containing p (s. t) $(\mathcal{N}) \subseteq Cl(\mathcal{W})$.

Recall that for a mapping $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$, the subset $\{(p, \mathfrak{F}(p)): p \in \mathcal{X}\} \subseteq \mathcal{X} \times \mathcal{Y}$ is said to be the graph of $\mathfrak{F}$ and is denoted by $\mathbb{G}(\mathfrak{F})$ **.**

Proposition3.25: Suppose $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is a map & $\mathcal{G}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{X} \times \mathcal{Y}, \mathcal{T} \times \mathcal{T}^*)$. The graph of $\mathfrak{F}$, described by $\mathcal{G}(p) = (p, \mathfrak{F}(p))\ \forall\ p \in \mathcal{X}$. If $\mathcal{G}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, then $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont.

Proof: Assume $\mathcal{N}$ be an open in $(\mathcal{Y}, \mathcal{T}^*)$, so $\mathcal{X} \times \mathcal{N}$ is open in $(\mathcal{X} \times \mathcal{Y}, \mathcal{T} \times \mathcal{T}^*)$. Since $\mathcal{G}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, so $\mathcal{G}^{-1}(\mathcal{X} \times \mathcal{N}) = \mathfrak{F}^{-1}(\mathcal{N})$ is $\omega_{\delta\text{-}\beta}$-closed in $(\mathcal{X}, \mathcal{T})$. This shows $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont.

Definition 3.26: A subset $\mathcal{H}$ of $(\mathcal{X}, \mathcal{T})$ is said to be $\omega_{\delta\text{-}\beta}$-dense in $\mathcal{X}$ if $Cl_{\omega_{\delta-\beta}}(\mathcal{H}) = \mathcal{X}$.

Theorem 3.27: Let $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ be contra $\omega_{\delta\text{-}\beta}$-continuous and $\mathcal{G}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ be C-ω-Cont. *If* $(\mathcal{Y}, \mathcal{T}^*)$ is urysohn, then next statements satisfied:

$(\boldsymbol{i})$ $\mathcal{A} = \{p \in \mathcal{X}: \mathfrak{F}(p) = \mathcal{G}(p)$ is $\omega_{\delta\text{-}\beta}$-closed in $(\mathcal{X}, \mathcal{T})$.

$(\boldsymbol{ii})$ $\mathfrak{F} = \mathcal{G}$ on $(\mathcal{X}, \mathcal{T})$ when $\mathfrak{F} = \mathcal{G}$ on a $\omega_{\delta\text{-}\beta}$-Dense set $\mathcal{H} \subseteq \mathcal{X}$.

Proof: $(\boldsymbol{i})$ Suppose $p \in \mathcal{X} - \mathcal{A}$. Then $\mathfrak{F}(p) \neq \mathcal{G}(p)$. By supposition on $(\mathcal{Y}, \mathcal{T}^*)$, there exists open sets $\mathcal{W}$ & $\mathcal{D}$ in $(\mathcal{Y}, \mathcal{T}^*)$ (s. t) $\mathfrak{F}(p) \in \mathcal{W}, \mathcal{G}(p) \in \mathcal{D}$ & $Cl(\mathcal{W}) \cap Cl(\mathcal{D}) = \emptyset$. As $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, consequently $\mathfrak{F}^{-1}(Cl(\mathcal{W}))$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$ containing p. As $\mathcal{G}$ is C-ω-Cont, $\mathcal{G}^{-1}(Cl(\mathcal{D}))$ is ω-open in $(\mathcal{X}, \mathcal{T})$ containing p. Assume $\mathcal{N} = \mathfrak{F}^{-1}(Cl(\mathcal{W}))$ & $\mathbb{G} = \mathcal{G}^{-1}(Cl(\mathcal{D}))$ and put $\mathcal{H} = \mathcal{N} \cap \mathbb{G}$. In that case by Lemma 3.6, $\mathcal{H}$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$ including p.

At this time, $\mathfrak{F}(\mathcal{H}) \cap \mathcal{G}(\mathcal{H}) = \mathfrak{F}(\mathcal{N} \cap \mathbb{G}) \cap \mathcal{G}(\mathcal{N} \cap \mathbb{G}) \subseteq \mathfrak{F}(\mathcal{N}) \cap \mathcal{G}(\mathbb{G}) \subseteq Cl(\mathcal{W}) \cap Cl(\mathcal{D}) = \emptyset$. Implies $\mathcal{H} \cap \mathcal{A} = \emptyset$, (s.t) $\mathcal{H}$ is an $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$. Thus $p \notin Cl_{\omega_{\delta-\beta}}(\mathcal{A})$. Consequently $\mathcal{A}$ is $\omega_{\delta-\beta}C(\mathcal{X}, \mathcal{T})$.

$(\boldsymbol{ii})$ Presume $\mathcal{A} = \{p \in \mathcal{X}: \mathfrak{F}(p) = \mathcal{G}(p)\}$. Since $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, $\mathcal{G}$ is C-ω-Cont and $(\mathcal{Y}, \mathcal{T}^*)$ is urysohn, via part $(\boldsymbol{i})$, $\mathcal{A}$ is $\omega_{\delta\text{-}\beta}$-closed in $(\mathcal{X}, \mathcal{T})$. Through supposition, get $\mathfrak{F} = \mathcal{G}$ on $\mathcal{H}$, wherever $\mathcal{H}$ is $\omega_{\delta\text{-}\beta}$-Dense in$(\mathcal{X}, \mathcal{T})$. As $\mathcal{H} \subseteq \mathcal{A}$,so $\mathcal{H}$ is $\omega_{\delta\text{-}\beta}$-Dense with $\mathcal{A} \in \omega_{\delta\text{-}\beta}$-closed in $(\mathcal{X}, \mathcal{T})$, consequently $\mathcal{X} = Cl_{\omega_{\delta-\beta}}(\mathcal{H}) \subseteq Cl_{\omega_{\delta-\beta}}(\mathcal{A}) = \mathcal{A}$. Therefore $\mathfrak{F} = \mathcal{G}$ on $(\mathcal{X}, \mathcal{T})$.

Definition3.28: The graph $\mathbb{G}(\mathfrak{F})$ of a mapping $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is said to be contra $\omega_{\delta\text{-}\beta}$-closed if for each $(\mathcal{X}, \mathcal{Y}) \in (\mathcal{X} \times \mathcal{Y}) - \mathbb{G}(\mathfrak{F})$, there exist an $\omega_{\delta\text{-}\beta}$-open set $\mathcal{N}$ in $(\mathcal{X}, \mathcal{T})$ containing p and a closed set $\mathcal{W}$ in $(\mathcal{Y}, \mathcal{T}^*)$ containing q such that $(\mathcal{N} \times \mathcal{W}) \cap \mathbb{G}(\mathfrak{F}) = \emptyset$.

Definition 3.29: A space $(\mathcal{X}, \mathcal{T})$ is called $\omega_{\delta\text{-}\beta}$-$\mathcal{T}_1$ if for each pair of distinct points p & q $\in \mathcal{X}$, $\exists$ $\omega_{\delta\text{-}\beta}$-open sets $\mathcal{N}$ & $\mathcal{W}$ containing p & q, *resp*, (s. t) $q \notin \mathcal{N}$ & $p \notin \mathcal{W}$.

Theorem 3.30: Let a map $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ have a C-$\omega_{\delta\text{-}\beta}$-closed graph. Then $(\mathcal{X}, \mathcal{T})$ is $\omega_{\delta\text{-}\beta}$-$\mathcal{T}_1$ *if* $\mathfrak{F}$ injective.

Proof: Assume p & q be arbitrary two distinct points $(\mathcal{X}, \mathcal{T})$. So, $(p, \mathfrak{F}(q)) \in (\mathcal{X} \times \mathcal{Y}) - \mathbb{G}(\mathfrak{F})$. Consequently, there exists $\omega_{\delta\text{-}\beta}$-open $\mathcal{N}$ in $(\mathcal{X}, \mathcal{T})$ including p and a closed $\mathcal{M}$ in $(\mathcal{Y}, \mathcal{T}^*)$ containing

$\mathfrak{F}(q)$ (s. t) $(\mathcal{N}) \cap \mathcal{M} = \emptyset$, thus $\mathcal{N} \cap \mathfrak{F}^{-1}(\mathcal{M}) = \emptyset$. As a result, we have $q \notin \mathcal{N}$. Thus, $(\mathcal{X}, \mathcal{T})$ is $\omega_{\delta\text{-}\beta}$-$\mathcal{T}_1$.

Remark 3.31: The composition of two contra $\omega_{\delta\text{-}\beta}$-continuous mappings need not be contra $\omega_{\delta\text{-}\beta}$-continuous as shown the following example :

Example3.32: Assume $\mathcal{X} = \mathbb{R}$, $\mathcal{T} = \mathcal{T}_{coc}$ and $\mathcal{Y} = \{r, s\}$ with $\mathcal{T}^* = \{\emptyset, \mathcal{Y}, \{r\}\}$ and $\mathcal{T}^{**} = \{\emptyset, \mathcal{Y}, \{s\}\}$. Presume $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ described as:

$$\mathfrak{F}(p) = \begin{cases} r & p \in \mathbb{Q} \\ s & p \in \mathbb{R} - \mathbb{Q} \end{cases}$$

and $\mathcal{G}: (\mathcal{Y}, \mathcal{T}^*) \to (\mathcal{Y}, \mathcal{T}^{**})$ be the identity map. So therefore $\mathfrak{F}, \mathcal{G}$ are C-$\omega_{\delta\text{-}\beta}$-Cont, but $\mathcal{G} \circ \mathfrak{F}$ is not C-$\omega_{\delta\text{-}\beta}$-Cont.

Definition 3.33: A map $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is said to $\omega_{\delta\text{-}\beta}$-irresolute if the inverse image of every $\omega_{\delta-\beta}\Sigma(\mathcal{Y}, \mathcal{T}^*)$ set is $\omega_{\delta-\beta}\Sigma(\mathcal{X}, \mathcal{T})$ set .

Theorem 3.34: For $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ and $\mathcal{G}: (\mathcal{Y}, \mathcal{T}^*) \to (\mathcal{Z}, \mathcal{T}^{**})$, the next statements hold:

(i) $\mathcal{G} \circ \mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-continuous, *if* $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont & $\mathcal{G}$ is C-cont.

(ii) $\mathcal{G} \circ \mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, if $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-cont and $\mathcal{G}$ is C-cont.

(iii) $\mathcal{G} \circ \mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, if $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-irresolute and $\mathcal{G}$ is C-$\omega_{\delta\text{-}\beta}$-cont.

Proof: its follows immediately from their respective definitions thus excluded .

Definition 3.35: $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is called:

(i) $\omega_{\delta\text{-}\beta}$-open if the image of every open in $\mathcal{X}$ is a $\omega_{\delta\text{-}\beta}$-open in $\mathcal{Y}$.

(ii) $\omega_{\delta\text{-}\beta}$-closed if the image of each closed set in $\mathcal{X}$ is a $\omega_{\delta\text{-}\beta}$-closed in $\mathcal{Y}$.

Theorem 3.36: Let $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ be surjective $\omega_{\delta\text{-}\beta}$-open and $\omega_{\delta\text{-}\beta}$-irresolute map and $\mathcal{G}: (\mathcal{Y}, \mathcal{T}^*) \to (\mathcal{Z}, \mathcal{T}^{**})$ any map. So $\mathcal{G} \circ \mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Z}, \mathcal{T}^{**})$ is C-$\omega_{\delta\text{-}\beta}$-Cont *iff* $\mathcal{G}$ is C-$\omega_{\delta\text{-}\beta}$-Cont .

proof: Assume $\mathcal{G} \circ \mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Z}, \mathcal{T}^{**})$ is C-$\omega_{\delta\text{-}\beta}$-Cont, and $\mathcal{M}$ is closed set in $(\mathcal{Z}, \mathcal{T}^{**})$. In that case $\mathfrak{F}^{-1}(\mathcal{G}^{-1}(\mathcal{M})) = (\mathcal{G} \circ \mathfrak{F})^{-1}(\mathcal{M})$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$. As $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-open & surjective, so $\mathcal{G}^{-1}(\mathcal{M}) = \mathfrak{F}\left(\mathfrak{F}^{-1}(\mathcal{G}^{-1}(\mathcal{M}))\right)$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{Y}, \mathcal{T}^*)$ and get $\mathcal{G}$ is C-$\omega_{\delta\text{-}\beta}$-Cont.

Conversely, Assume $\mathcal{G}$ is C-$\omega_{\delta\text{-}\beta}$-Cont, and $\mathcal{W}$ is a closed in $(\mathcal{Z}, \mathcal{T}^{**})$. In that case $\mathcal{G}^{-1}(\mathcal{W})$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{Y}, \mathcal{T}^*)$. As $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-irresolute,$\mathfrak{F}^{-1}(\mathcal{G}^{-1}(\mathcal{W})) = (\mathcal{G} \circ \mathfrak{F})^{-1}(\mathcal{W})$ is $\omega_{\delta\text{-}\beta}$-open in $(\mathcal{X}, \mathcal{T})$, as a result $\mathcal{G} \circ \mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Cont map.

Definition3.37: A map $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$ is called perfectly $\omega_{\delta\text{-}\beta}$-Cont if the inverse image of each open in $(\mathcal{Y}, \mathcal{T}^*)$ is $\omega_{\delta\text{-}\beta}$-clopen of $(\mathcal{X}, \mathcal{T})$.

Remark 3.38: It is obvious that:

(i) Every perfectly $\omega_{\delta\text{-}\beta}$-continuous mapping is $\omega_{\delta\text{-}\beta}$-continuous, but the converse not true as shown in Example 2.17 the mapping $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-continuous, but not perfectly $\omega_{\delta\text{-}\beta}$-continuous .

(ii) Every perfectly $\omega_{\delta\text{-}\beta}$-Cont map is C-$\omega_{\delta\text{-}\beta}$-Cont, but the converse not true as shown in Example 2.18 the map $\mathfrak{F}$ is C-$\omega_{\delta\text{-}\beta}$-Conti, but not perfectly $\omega_{\delta\text{-}\beta}$-Cont.

(iii) In spite of the fact that $\omega_{\delta\text{-}\beta}$-Cont and C-$\omega_{\delta\text{-}\beta}$-Cont are independent concepts, each $\omega_{\delta\text{-}\beta}$-Cont and C-$\omega_{\delta\text{-}\beta}$-Cont map is perfectly $\omega_{\delta\text{-}\beta}$-Cont.

Theorem 3.39: Following statements are equivalent for $\mathfrak{F}: (\mathcal{X}, \mathcal{T}) \to (\mathcal{Y}, \mathcal{T}^*)$:

(i) $\mathfrak{F}$ is perfectly $\omega_{\delta\text{-}\beta}$-Cont. (ii) $\mathfrak{F}$ is $\omega_{\delta\text{-}\beta}$-Cont and C-$\omega_{\delta\text{-}\beta}$-Cont .

Proof: Its follows immediately from their respective definitions thus omitted.

Remark.3.40: From the above definitions and remarks, we have the following implications :

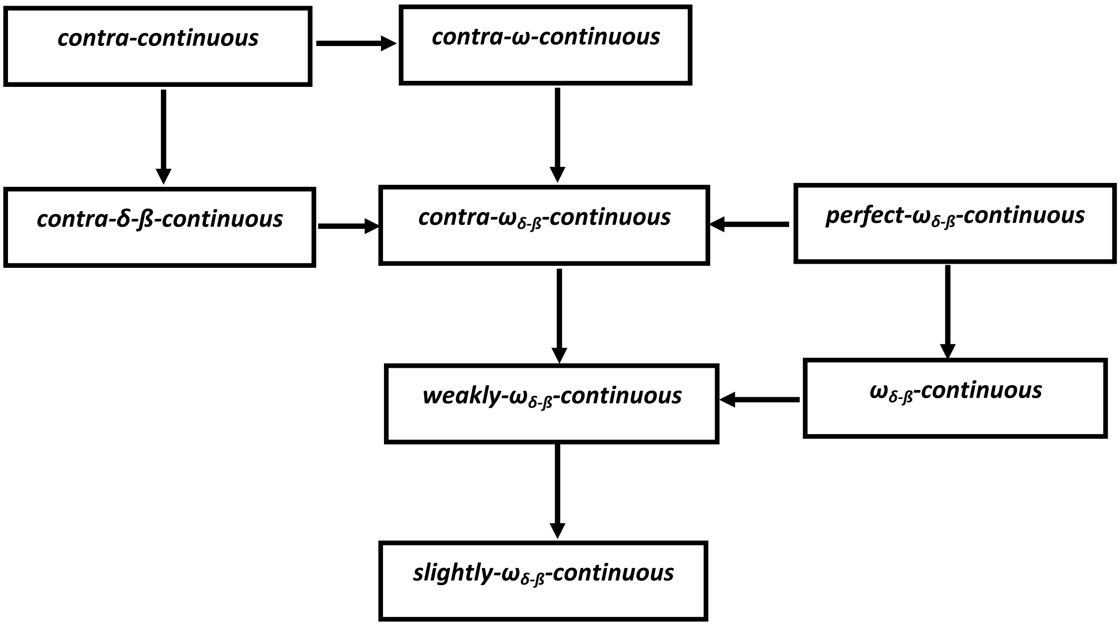

Diagram: contra-$\omega_{\delta\text{-}\beta}$-cont maps and other known structures of extended contra-ω-continuity

4- Some Applications of Contra $\omega_{\delta\text{-}\beta}$-Continuity

In this segment, apply the notion of contra $\omega_{\delta\text{-}\beta}$-continuity to verify some certain properties in connected and hyperconnected spaces .

Definition 4.1: A topological space $(\mathcal{X},\mathcal{T})$ is called $\omega_{\delta\text{-}\beta}$-disconnected if there exist disjoint $\omega_{\delta-\beta}\Sigma(\mathcal{X},\mathcal{T})$ sets $\mathcal{H}$ and $\mathcal{K}$ such that $\mathcal{H}\cup\mathcal{K}=\mathcal{X}$ (i. e){$(\mathcal{X},\mathcal{T})$ is $\omega_{\delta\text{-}\beta}$-connected if $\mathcal{X}$ is not the union of two disjoint non-empty $\omega_{\delta\text{-}\beta}$-open sets}.

Proposition 4.2: Let $\mathfrak{F}$: $(\mathcal{X},\mathcal{T})\to(\mathcal{Y},\mathcal{T}^*)$ from $(\mathcal{X},\mathcal{T})$ into any T_0-space $(\mathcal{Y},\mathcal{T}^*)$ be a constant map, then$(\mathcal{X},\mathcal{T})$ *is* $\omega_{\delta\text{-}\beta}$-connected

proof: Assume $(\mathcal{X},\mathcal{T})$ not $\omega_{\delta\text{-}\beta}$-connected and each C-$\omega_{\delta\text{-}\beta}$-Cont map from $(\mathcal{X},\mathcal{T})$ into T_0-space $(\mathcal{Y},\mathcal{T}^*)$ is constant. As $(\mathcal{X},\mathcal{T})$ is not $\omega_{\delta\text{-}\beta}$-connected, $\exists$ proper nonempty $\omega_{\delta\text{-}\beta}$-clopen subset $\mathcal{H}$ of $(\mathcal{X},\mathcal{T})$. Presume $\mathcal{Y}=\{1,2\}$ & $\mathcal{T}^*=\{\emptyset,\mathcal{Y},\{1\},\{2\}\}$ topology on $\mathcal{Y}$, with $\mathfrak{F}$: $(\mathcal{X},\mathcal{T})\to(\mathcal{Y},\mathcal{T}^*)$ is map (s. t) $\mathfrak{F}(\mathcal{H})=\{1\}$ and $\mathfrak{F}(\mathcal{X}-\mathcal{H})=\{2\}$. In that case $\mathfrak{F}$ is not constant and C-$\omega_{\delta\text{-}\beta}$-Cont (s. t) $(\mathcal{Y},\mathcal{T}^*)$ is T_0-space. This is contradiction. Thus $(\mathcal{X},\mathcal{T})$ should be $\omega_{\delta\text{-}\beta}$-connected.

Definition 4.3: [25] A space $(\mathcal{X},\mathcal{T})$ is said to be hyper connected if the closure of every open set is the entire set $\mathcal{X}$.

Remark 4.4: A C-$\omega_{\delta\text{-}\beta}$-Cont surjection does not necessarily preserve hyper connectedness, as shown in the next example:

Example 4.5: Assume $\mathcal{X}=\{r,s,t\}$ & $\mathcal{T}=\{\emptyset,\mathcal{X},\{r\}\}$ and $\mathcal{T}^*=\{\emptyset,\mathcal{X},\{s\},\{t\},\{s,t\}\}$. The identity map $\mathfrak{F}$: $(\mathcal{X},\mathcal{T})\to(\mathcal{X},\mathcal{T}^*)$ is C-$\omega_{\delta\text{-}\beta}$-Cont and $(\mathcal{X},\mathcal{T})$ is hyper connected, but $(\mathcal{X},\mathcal{T}^*)$ is not hyper connected.

Definition 4.6: A Top-sp $(\mathcal{X},\mathcal{T})$ is called:

(i)Weakly Hausdorff [26] if all element of $\mathcal{X}$ is an intersection of regular closed disjoint clopen sets.

(ii)Ultra Hausdorff [27] if each two distinct points of $\mathcal{X}$ can be separated by disjoint clopen sets .

(iii)Ultra normal [27] if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.

(iv)Ultra $\omega_{\delta\text{-}ß}$-normal if all pair of non-empty disjoint closed sets can be separated by disjoint $\omega_{\delta\text{-}ß}$-open sets.

Theorem.4.7: Following properties holds for a C-$\omega_{\delta\text{-}ß}$-Cont injection map $\mathfrak{F}: (\mathcal{X},\mathcal{T}) \to (\mathcal{Y},\mathcal{T}^*)$:

(i) $(\mathcal{X},\mathcal{T})$ is $\omega_{\delta\text{-}ß}$-$\mathcal{T}_1$-space if $(\mathcal{Y},\mathcal{T}^*)$ is weakly Hausdorff.

(ii) $(\mathcal{X},\mathcal{T})$ is $\omega_{\delta\text{-}ß}$-normal if $(\mathcal{Y},\mathcal{T}^*)$is ultra normal & $\mathfrak{F}$ is closed.

Proof:(i) Assume $(\mathcal{Y},\mathcal{T}^*)$ is weakly Hausdorff. For every distinct points p & q in $(\mathcal{X},\mathcal{T})$, $\exists$ regular closed $\mathcal{H},\mathcal{K}$ of $(\mathcal{Y},\mathcal{T}^*)$ (s. t) $\mathfrak{F}(p) \in \mathcal{H}, \mathfrak{F}(q) \notin \mathcal{H}, \mathfrak{F}(p) \notin \mathcal{K}$ & $\mathfrak{F}(q) \in \mathcal{K}$. Since $\mathfrak{F}$ is C-$\omega_{\delta\text{-}ß}$-Cont, so $\mathfrak{F}^{-1}(\mathcal{H})$ & $\mathfrak{F}^{-1}(\mathcal{K})$ are $\omega_{\delta\text{-}ß}$-open sets in $(\mathcal{X},\mathcal{T})$ (s. t) $p \in \mathfrak{F}^{-1}(\mathcal{H}), q \notin \mathfrak{F}^{-1}(\mathcal{H}), p \notin \mathfrak{F}^{-1}(\mathcal{K})$ & $q \in \mathfrak{F}^{-1}(\mathcal{K})$. This explains $(\mathcal{X},\mathcal{T})$ is $\omega_{\delta\text{-}ß}$-$\mathcal{T}_1$.

(ii) Suppose $\mathcal{M}_1$ & $\mathcal{M}_2$ are disjoint closed subsets of $(\mathcal{X},\mathcal{T})$. Since $\mathfrak{F}$ is closed and injective, so $\mathfrak{F}(\mathcal{M}_1)$ & $\mathfrak{F}(\mathcal{M}_2)$ are disjoint closed of $(\mathcal{Y},\mathcal{T}^*)$. As $(\mathcal{Y},\mathcal{T}^*)$ ultra normal,$\mathfrak{F}(\mathcal{M}_1)$ & $\mathfrak{F}(\mathcal{M}_2)$ are separated via disjoint clopen $\mathcal{W}_1$& $\mathcal{W}_2$, resp. As $\mathfrak{F}$ is C-$\omega_{\delta\text{-}ß}$-Cont, so $\mathcal{M}_i \subseteq \mathfrak{F}^{-1}(\mathcal{W}_i)$ & $\mathfrak{F}^{-1}(\mathcal{W}_i)$ is $\omega_{\delta\text{-}ß}$-open of $\mathcal{X}$ (s. t) $i = 1,2$ and $\mathfrak{F}^{-1}(\mathcal{W}_1) \cap \mathfrak{F}^{-1}(\mathcal{W}_2) = \emptyset$. Consequently, $(\mathcal{X},\mathcal{T})$ is $\omega_{\delta\text{-}ß}$-normal.

Conclusion

Several generalized forms of contra continuity stand among the most significant notions and most researched points in the whole of mathematical sciences and various interesting problems arise when one considers generalized forms openness and closeness . Therefore, in this article a new class of generalized contra ω-continuity namely, contra $\omega_{\delta\text{-}ß}$-Cont maps have been studied and investigated via new idea of generalized ω-open and δ-$ß$-open sets. Additionally, various interesting characterizations and vital characterizations related to this kind of generalized contra continuous mapping have been discussed.

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Modified Extended BDF for Numerical Solution of second order ODE

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Abstract

We introduce a general framework in this paper that allows two-order ordinary differential equations solved using modified extended backward differentiation formulas. This approach improves the classical stability areas backward differentiation formulae methods and is distinguished by the use of solution knowledge at a later position. We also examine the numerical scheme's consistency, which is supplied by a few theorems. Benefitable, the developed theory will be modified for right-sided discontinuity ordinary differential equation numerical solutions.

Keywords: Stability, modified techniques, ordinary differential equations, and backward differentiation formulas.

Introduction

The computational solutions for the subsequent ordinary differential equations (ODEs) of the second order are the focus of this paper:

$$\begin{cases} y'' = f\left(y(t)\right) \\ y\left(t_0\right) = y_0 \\ y'\left(t_0\right) = y'_0 \end{cases} \tag{1}$$

Although the issue can be transformed into a comparable system of first-order ODEs, doing so would result in an increase in dimensionality, which makes direct integration of the second-order system impractical. Ordinary differential equations (ODEs) are often obtained by reducing partial differential equations (PDEs), possess numerous uses in the fields of basic research, engineering, and medicine. ODE solutions are typically estimated using conventional numerical techniques for initial value problems (IVPs) and boundary value problems (BVPs). A broad framework for first order differential equations and the numerical solution of particular second order ODE-based issues has been presented in the papers [1, 2, 3]. Jia-Ling Lin et al. [4] used to find comprehensive solutions to second order differential equations Julia sets of solutions: Lower order and restricting directions. The growth characteristic and pole arrangement of meromorphic solutions f of specific intricate differential equations with rational function coefficients have been studied in [5, 6]. The relationship between the low order of the coefficients and the expansion of the differential equation solution is explained [7].

Transcendental complete solutions of complicated differential equations with coefficients of exponential growth were studied by Korhonen et al. [8]. In [9], first- and equations for ordinary differentials of second order are solved using wavelet neural networks (WNNs). A technique known as the S-function method was created by Duarte et al. [10] and has shown effective in solving a number of classes of rational second order ordinary differential equations to both Derounian and canonical Lie techniques. In [11], study is given to the second order linear delay differential equation., which shows up in the fluctuation phenomena of several models with real-world applications. Yan as well as others [12] expands the use of block universal Störmer-Cowell techniques (BGSCMs) to numerically solve second-order nonlinear delay differential algebraic equations (SNDDAEs) with index-1. Cash [13–15] focuses his convergence analysis on first order ODEs When looking at a novel set of techniques to solve differential equations numerically. An expansion according to the modified extended backward differentiation formulas (MEBDF).) is offered by these techniques.

Classical methods for second order ODEs regarded as GLMs

Multistep linear techniques

Second order ODEs using linear multistep techniques [16], as defined by

$$y_n = \sum_{j=1}^{k} \alpha_j y_{n-j} + h^2 \sum_{j=0}^{k} \beta_j f(y_{n-j}), \qquad (2)$$

are thought of as GLMs with $r = 2k, s = 1, Y^{[a]} = [y_n]$,

$$y^{[n-1]} = [y_{n-1} \quad y_{n-2} \quad \dots \quad y_{n-k} \quad h^2 f(y_{n-1}) \quad h^2 f(y_{n-2}) \quad \dots \quad h^2 f(y_{n-k})]^T,$$

and we will have the reduced tableau,

$$\left[\begin{array}{c|c} A & U \\ \hline B & V \end{array}\right] = \left[\begin{array}{c|cccccccc} \beta_0 & \alpha_1 & \dots & \alpha_{k-1} & \alpha_k & \beta_1 & \dots & \beta_{k-1} & \beta_k \\ \hline \beta_0 & \alpha_1 & \dots & \alpha_{k-1} & \alpha_k & \beta_1 & \dots & \beta_{k-1} & \beta_k \\ 0 & 1 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 0 & \dots & 0 & 0 \\ 1 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 1 & 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & 1 & \dots & 1 & 0 \end{array}\right],$$

The Nemerov method is a well-known illustration of the linear multistep method (see, for example, [17])

$$y_n = 2y_{n-1} - y_{n-2} + h^2\left(\frac{1}{12}f(t_n, y_n) + \frac{5}{6}f(t_{n-1}, y_{n-1}) + \frac{1}{12}f(t_{n-2}, y_{n-2})\right). \qquad (3)$$

It corresponds to the GLM with an order four technique $r = 4, s = 1, Y^{[s]} = [y_n]$.

$$y^{[n-1]} = [y_{n-1} \quad y_{n-2} \quad h^2 f(y_{n-1}) \quad h^2 f(y_{n-2})]^T,$$

and reduced tableau,

$$\begin{bmatrix} A & U \\ B & V \end{bmatrix} = \left[\begin{array}{c|cccc} \frac{1}{12} & 2 & -1 & \frac{5}{6} & \frac{1}{12} \\ \hline \frac{1}{12} & 2 & -1 & \frac{5}{6} & \frac{1}{12} \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right].$$

The Runge-Kutta-Nyström technique

The class of Runge–Kutta–Nyström techniques (see to [16]) which is in the form

$$\begin{aligned} Y_i &= y_{n-1} + c_i\, hy'_{n-1} + h^2 \sum_{j=1}^{s} a_{ij} f(Y_j),\ i = 1, \dots, s, \\ hy'_n &= hy'_{n-1} + h^2 \sum_{j=1}^{s} b'_j f(Y_j), \qquad (4) \\ y_n &= y_{n-1} + hy'_{n-1} + h^2 \sum_{j=1}^{s} b_j f(Y_j), \end{aligned}$$

give an addition to Runge-Kutta methods' second order ODEs (1) (see, for example, [1,17]), involving the reliance on the current grid point's approximation to the first derivative. These techniques may be recast to fit the tableau as GLMs with r =1..

$$\begin{bmatrix} A & P & U \\ C & R & W \\ B & Q & V \end{bmatrix} = \begin{bmatrix} A & c & e \\ br^T & 1 & 0 \\ b^T & 1 & 1 \end{bmatrix},$$

where the unit vector in the case of $\mathbb{R}^s$, as well as the vectors used for input

$$y^{[n-1]} = [y_{n-1}],\ y'^{[n-1]} = [y'_{n-1}].$$

Coleman's mixed techniques

We currently examine the subsequent category of techniques.

$$Y_i = (1 + c_i)y_{e-1} - c_i y_{n-2} + h^2 \sum_{j=1}^{s} a_j f(Y_j),\ i = 1, \dots, s,$$
$$y_e = 2y_{n-1} - y_{n-2} + h^2 \sum_{j=1}^{s} b_j f(Y_j), \tag{5}$$

initially brought up through Coleman in [5], and are known as two-phase hybrid techniques. These techniques able to thought of as GLMs that match the smaller portrait.

$$\begin{bmatrix} \mathrm{A} & \mathrm{U} \\ \mathrm{B} & \mathrm{V} \end{bmatrix} = \begin{bmatrix} A & \mathrm{e} + \mathbf{c} & -\mathrm{c} \\ \mathbf{b}^T & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix},$$

and defined by the vector that was entered. $y^{[n-1]} = [y_{n-1} \quad y_{n-2}]^T$.

Runge-Kutta-Nyström techniques in two steps

The two-step technique family Runge, Kutta, and Nyström offers an additional fascinating group of second-order ODE numerical techniques [6].

$$Y_i^{[n-1]} = y_{n-2} + hc_i y'_{n-2} + h^2 \sum_{j=1}^{s} a_{ij} f\left(Y_j^{[n-1]}\right),\ i = 1, \dots, s,$$
$$Y_i^{[n]} = y_{n-1} + hc_i y'_{n-1} + h^2 \sum_{j=1}^{s} a_{ij} f\left(Y_j^{[n]}\right),\ i = 1, \dots. s, \tag{6}$$
$$hy'_n = (1-\theta)hy'_{n-1} + \theta hy'_{n-2} + h^2 v'_j f\left(Y_j^{[n-1]}\right) + h^2 w'_j f\left(Y_j^{[n]}\right),$$

$$y_n = (1-\theta)y_{n-1} + \theta y_{n-2} + h\sum_{j=1}^{s} v'_j y'_{n-2} + h\sum_{j=1}^{s} w'_j y'_{n-1}$$
$$+h^2 \textstyle\sum_{j=1}^{s} v_j f\left(Y_j^{[n-1]}\right) + h^2 \sum_{j=1}^{s} w_j f\left(Y_j^{[n]}\right).$$

Both the grid points containing the answer and its first derivative, in addition to two consecutive stage value approximations, are crucial to these strategies. One can depict using the tableau to represent two-step Runge-Kutta-Nyström techniques as GLMs with r = s+2 and r' = 2..

$$\begin{bmatrix} A & P & U \\ C & R & W \\ B & Q & V \end{bmatrix} = \begin{bmatrix} A & c & 0 & e & 0 & 0 \\ w^T & (1-\theta) & \theta & 0 & 0 & {}^r r^T \\ 0 & 1 & 0 & 0 & 0 & 0 \\ w^T & wr^T e & v^T e & (1-\theta) & \theta & v^T \\ 0 & 0 & 0 & 1 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

while the input vectors coincide with r=s+2 and r'=2

$$y^{[n-1]} = \begin{bmatrix} y_{n-1} & y_{n-2} & h^2 f\left(Y^{[n-1]}\right) \end{bmatrix}^T, y^{[[n-1]} = [y'_{n-1}\ y'_{n-2}]^T.$$

Prior levels of values have additionally been employed with reference to Dual-Step Runge-Kutta-Nyström Pseudo-Iterated Parallels processes.

$$\begin{aligned} V_n &= y_{n-1}\mathrm{e}_v + hy_{n-1}c_v + h^2 \mathrm{A}_{vv} f(V_{n-1}) + h^2 \mathrm{A}_{ww} f(W_{n-1}), \\ W_n &= y_{n-1}\mathrm{e}_w + hy_{n-1}c_w + h^2 \mathrm{A}_{wv} f(V_n) + h^2 \mathrm{A}_{ww} f(W_n), \\ hy_n &= hy_{n-1} + h^2 \mathrm{d}_v^T f(V_n) + h^2 \mathrm{d}_w^T f(W_n), \\ y_n &= y_{n-1} + hy_{n-1} + h^2 \mathrm{b}_v^T f(V_n) + h^2 \mathrm{b}_w^T f(W_n), \end{aligned}$$

In accordance with the tableau, these techniques can also be reconstructed as GLMs with r =2s+1 and r'=1.

$$\begin{bmatrix} A & P & U \\ C & R & W \\ B & Q & V \end{bmatrix} = \begin{bmatrix} 0 & 0 & c_v & e_v & A_{wv} & A_{vw} \\ A_{wv} & A_{ww} & c_w & e_w & 0 & 0 \\ d_v^T & d_w^T & 1 & 0 & 0 & 0 \\ b_v^T & b_w^T & 1 & 1 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 \end{bmatrix}$$

as well as the outlines $Y^{[n]} = [V_n \quad W_n]^T$, $y^{[n-1]} = [y_{n-1} \quad h^2 f(V_{n-1})\ h^2 f(W_{n-1})]^T$

and $y^{[n-1]} = [y'_{n-1}]$.

Extended BDF with modifications for second order ODEs

The differential equations for backward differentiation that has been changed (MEBDF) created by Cash signify a group of techniques for the numerical solution of first-order ODEs that we explore. The first modification deals with first order ODEs so that the techniques of order $p = k+1$ are represented through techniques known as extended BDF (EBDF).

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h\beta_k f_{n+k} + h\beta_{k+1} f_{n+k+1}, \qquad (7)$$

where $f_{n+k} = f(t_{n+k}, y_{n+k}), f_{n+k+1} = f(t_{n+k+1}, y_{n+k+1})$. The following succinctly describes the predictor corrector scheme in which this numerical approach is utilized as a corrector:

(i) Compute $\bar{y}_{n+k}$ using the traditional BDF approach as the solution.

$$\bar{y}_{n+k} + \sum_{j=0}^{k-1} \hat{\alpha}_j y_{n+j} = h\hat{\beta}_k \bar{f}_{n+k}, \quad (8)$$

with

$\bar{f}_{n+k} = f(t_{n+k}, \bar{y}_{n+k})$.

(ii) As the identical BDF solution progressed one step, compute $\bar{y}_{n+k+1}$

$$\bar{y}_{n+k+1} + \hat{\alpha}_{k-1}\bar{y}_{n+k} + \sum_{j=0}^{k-2} \hat{\alpha}_j y_{n+j+1} = h\hat{\beta}_k \bar{f}_{n+k+1}, \quad (9)$$

with

$\bar{f}_{n+k+1} = f(t_{n+k+1}, \bar{y}_{n+k+1})$

(iii) Throw away $\bar{y}_{n+k}$,add $\bar{f}_{n+k+1}$ to the EBDF technique, and find y_{n+k} as

$$y_{n+k} + \sum_{j=0}^{k-1} \alpha_j y_{n+j} = h\beta_k f_{n+k} + h\beta_{k+1}\bar{f}_{n+k+1}. \quad (10)$$

As [3] shows, if the EBDF technique (7) is k+1 in order, and the BDF approaches (8) and (9) belong to the order k., then the complete algorithmic (i)-(iii) based on (8)-(10) is arranged. *k+1*.In order to address this, he suggested in [4] an algorithm in which the final step (iii) was swapped out for an altered EBDF (MEBDF) approach that took the arrangement

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h\hat{\beta}_k f_{n+k} + h(\beta_k - \hat{\beta}_k)\bar{f}_{n+k} + h\beta_{k+1}\bar{f}_{n+k+1}. \quad (11)$$

The order of these procedures is k+1.
When we change (8) into (9), we get

$$\bar{y}_{n+k+1} = \hat{\alpha}_{k-1}\hat{\alpha}_0 y_n + \sum_{j=1}^{k-1} (\hat{\alpha}_{k-1}\hat{\alpha}_j - \hat{\alpha}_{j-1}) y_{n+j} - h\hat{\alpha}_{k-1}\hat{\beta}_k \bar{f}_{n+k} + h\hat{\beta}_k \bar{f}_{n+k+1}. \quad (12)$$

Our goal, which this thesis offers as a novelty, is now to carry out a comparable numerical approach Consider ODEs of second order (1). This can be condensed into the following:

(i) Use the following predictor approach to calculate $\bar{y}_{n+k}$.

$$\bar{y}_{n+k} + \sum_{j=0}^{k-1} \hat{\alpha}_j y_{n+j} = h^2 \hat{\beta}_k \bar{f}_{n+k},\ \bar{f}_{n+k} = f(t_{n+k}, \bar{y}_{n+k}). \quad (13)$$

(ii) Proceed $\bar{y}_{n+k+1}$ one step farther and compute the same predictor's solution.

$$\bar{y}_{n+k+1} + \hat{\alpha}_{k-1}\bar{y}_{n+k} + \sum_{j=0}^{k-2} \hat{\alpha}_j y_{n+j+1} = h^2 \hat{\beta}_k \bar{f}_{n+k+1}. \quad (14)$$

(iii) Employ the following corrector:

$$\sum_{j=0}^{k} \alpha_j y_{n+j} = h^2 \hat{\beta}_k f_{n+k} + h^2 (\beta_k - \hat{\beta}_k) \bar{f}_{n+k} + h^2 \beta_{k+1} \bar{f}_{n+k+1}. \quad (15)$$

Based on formulas (13), (14), and (15), we consider the numerical scheme as a hybrid form of GLM including the approximations, the vector $Y^{[n]}, f(Y^{[n]})$ of external approximations y ([n]) defined by, and s=3, r =k.

$$Y^{[n]} = \begin{bmatrix} \bar{y}_{n+k} \\ \bar{y}_{n+k+1} \\ y_{n+k} \end{bmatrix},\ f(Y^{[n]}) = \begin{bmatrix} \bar{f}_{n+k} \\ \bar{f}_{n+k+1} \\ f_{n+k} \end{bmatrix},\ y^{[n]} = \begin{bmatrix} y_{n+k} \\ y_{n+k-1} \\ \vdots \\ y_{n+1} \end{bmatrix}, \quad (16)$$

and using the supplied coefficient matrices A, U, B, and V

$$\mathbf{A} = \begin{bmatrix} \hat{\beta}_k & 0 & 0 \\ -\hat{\alpha}_{k-1}\hat{\beta}_k & \hat{\beta}_k & 0 \\ \beta_k - \hat{\beta}_k & \beta_{k+1} & \hat{\beta}_k \end{bmatrix},$$

$$\mathbf{U} = \begin{bmatrix} -\hat{\alpha}_{k-1} & -\hat{\alpha}_{k-2} & \cdots & -\hat{\alpha}_1 & -\hat{\alpha}_0 \\ \hat{\alpha}_{k-1}\hat{\alpha}_{k-1} - \hat{\alpha}_{k-2} & \hat{\alpha}_{k-1}\hat{\alpha}_{k-2} - \hat{\alpha}_{k-3} & \cdots & \hat{\alpha}_{k-1}\hat{\alpha}_1 - \hat{\alpha}_0 & \hat{\alpha}_{k-1}\hat{\alpha}_0 \\ -\alpha_{k-1} & -\alpha_{k-2} & \cdots & -\alpha_1 & -\alpha_0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} \beta_k - \hat{\beta}_k & \beta_{k+1} & \hat{\beta}_k \\ 0 & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{V} = \begin{bmatrix} -\alpha_{k-1} & -\alpha_{k-2} & \cdots & -\alpha_1 & -\alpha_0 \\ 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 1 & 0 \end{bmatrix}.$$

First, we examine the numerical scheme's consistency, which is given by the following theorems.

Theorem 1. Second order ODEs: a k-step MEBDF technique (3.1) is coherent solely in the event that the subsequent algebraic requirements are satisfied, according to the matching predictor-corrector system to formulas (5.7), (5.8), and (5.9).

$$\sum_{j=0}^{k} \hat{\alpha}_j = -1, \qquad (17)$$

$$\sum_{j=0}^{k} \hat{\alpha}_j - \hat{\alpha}_{j+1}\hat{\alpha}_{k-1} = 1 - \hat{\alpha}_{k-1}\alpha_0, \qquad (18)$$

$$\sum_{j=0}^{k-1} \alpha_j = -1, \qquad (19)$$

$$\sum_{j=0}^{k-2} (j-k+1)\,\hat{\alpha}_j = -c_1, \qquad (20)$$

$$\sum_{j=0}^{k-2} (j-k+1)(\hat{\alpha}_j - \hat{\alpha}_{j+1}\hat{\alpha}_{k-1}) = -c_2, \qquad (21)$$

$$\sum_{j=0}^{k-1} (j-k+1)\,\alpha_j = -c_3, \qquad (22)$$

$$2\beta_k + 2\beta_{k+1} + \sum_{j=0}^{k-1} (j-k+1)\,\alpha_j = 1. \qquad (23)$$

Theorem 2. For second order ODEs (1), a consistent k-step MEBDF technique is zero-stable if the matching predictor-corrector system to formulas (13), (14), and (15) is used.

$$\sum_{i=2}^{k-1} i(i-1)\,\alpha_i + k(k-1) \neq 0,$$

and how the polynomial's roots

$$p_k(t) = \sum_{i=0}^{k-1} \alpha_i\, t^i + t^k,$$

Never lie beyond the unit circle.

Together, The findings presented in This part offers the examination of convergence off the MEBDF. techniques for ODEs of second order (1), which is made possible by Theorem 1's general convergence result. The following new approach provides a MEBDF approach that is convergent, for example.

$$\begin{bmatrix} A & U \\ B & V \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 2 & 1 & 0 & 3 & -2 \\ -\beta_3 & \beta_3 & 1 & 2 & -1 \\ -\beta_3 & \beta_3 & 1 & 2 & -1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}, \tag{24}$$

which is dependable The vectors for consistency and pre-consistency are provided by $q_0 = [1,1]^T, q_1 = [0,-1]^T, q_2 = [0,1/2]^T$) and for any β_3, zero-stable and hence convergent. The numerical test on the classical test equation, as presented in Table 1, further supports this.

$$y''(x) = -\omega^2 y, \qquad x \in [0,\pi] \tag{25}$$

using starting values $y'(0) = 1, y(0) = 0$. Thus, the precise answer is,

$y(x) = \sin(\omega x)$.

The implementation of the MEBDF method (24) produced numerical results. $\beta_3 = 1/2$ to issue (25), where ω=1. The used fixed step size is h.

$\|err\|_\infty$ is the global error's infinity norm, and p is the approximate convergence order.

Table 1. The problem's numerical results (25).

h	$\|err\|_\infty$	P
$\frac{\pi}{2^5}$	$9.41e-2$	---
$\frac{\pi}{2^6}$	$4.27e-2$	1.14
$\frac{\pi}{2^7}$	$2.03e-2$	1.09
$\frac{\pi}{2^8}$	$9.89e-3$	1.07

Conclusions

Our study has concentrated on the theory of convergence of general linear methods (GLMs) for particular classes of secondary ordinary differential equations $y''(x) = f(y(t))$ (ODEs). We talk about how universal this method is. In fact, we have utilized this theory for the first time to retrieve the formulation and convergence characteristics of many approaches for (1). The results given in [9] have then been tailored specifically regarding the set of altered extended BDF techniques. It will be profitable to introduce and modify the existing theory for the numerical solution of ordinary differential equations having a discontinuous right side.

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تطوير BDF الممتد للحل العددي لمعادلات من الدرجة الثانيةODE

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الخلاصة

نقدم في هذا البحث إطارًا عامًا لحل المعادلات التفاضلية العادية من الدرجة الثانية مع صيغ التمايز العكسي الموسعة المعدلة التي توفر تحسينًا لمناطق الاستقرار في طرق صيغ التمايز العكسي الكلاسيكية وتتميز بمشاركة معرفة الحل في نقطة مستقبلية. كما قمنا أيضًا بتحليل اتساق المخطط العددي الذي توفره بعض النظريات. سيتم تكييف هذه النظرية المطورة بشكل مفيد للمعالجة العددية للمعادلات التفاضلية العادية ذات الجانب الأيمن المتقطع.

الكلمات المفتاحية: المعادلات التفاضلية العادية، صيغ التمايز العكسي، الاستقرارية، الطرق المطورة.