Orthogonal Reverse Derivations with Ideal of semiprime Sets.

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Abstract

I concluded in my research that the nature of relationship between the orthogonality of antiderivatives with the ideal on prime quasi-sets, and we defined the orthogonality of antiderivatives, and we studied the relationship with the ideal on prime quasi-sets, and we reached results and theories that link them.

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Reverse derivations, orthogonality, semiprime rings, ideals within semiprime rings.

1. Introduction

In the theory of ring structures, derivations play a fundamental role.The assignment operation f: R → R is called a derivative on the set R if it satisfies the product rule f(ab) = f(a)b + af(b) for all elements a, b ∈ R, as stated in references [6] and [7].

Building upon this foundational concept, researchers in [2] and [3] have investigated orthogonal derivations in sets. Two mappings F and H on a set R are considered orthogonal if they satisfy the condition F(a)RH(b) = 0 = H(b)RF(a) for all a, b ∈ R.

The study of antiderivatives on semi-primary sets was explored by Saman and Alemany [5], extending the classical theory of derivatives to this specialized context. Further advancing this area of research, the concept of perpendicularity for inverse derivations was introduced in [1]. Specifically, two reverse derivations F and H of a set R are termed orthogonal when F(x)RH(y) = 0 = H(y)RF(x) for all x, y ∈ R.

In the context of semi-primary sets, we study additive subsets that form ideals. Let R be a semi-primary set. A plural subset W of R is called a left (or right) ideal of R if RW ⊆ W (or WR ⊆ W), as indicated in [4].

It is important to note that for any subset W of R, the left-handed pesticide l(W) and the right-handed pesticide r(W) are defined as l(W) = {x ∈ R : xW = 0} and r(W) = {x ∈ R : Wx = 0} respectively. In this paper, we have established several theorems and results related to our topic, which is the perpendicularity of antiparallel derivatives to the ideal on semi-elementary sets.

2-The Results

Puzzle 21 [4]: Let R be a semi-prime set without twist 2, let W be a non-zero ideal of R, and let M and N be elements of R such that:

$$A_{nn1}\ (w)\ \text{if}\ myn + nym = 0\ \text{then}\ \ myn = nym = 0$$

proof: let p and p' be two orbitrary components of w, then by postulate

(MqN) q'(MqN) = – (NqM) p' *(MqN)*

= – *(N(qMp′)M)qN*

= *(M(pMp') N) qN*

= – *Mq (Mq'N) qN*

= –*Mq (Nq'M) qN*

= – *(MpN)* p*'(MpN)*

This implies 2(MqN) p' (MqN) = 0

Since R is 2-torison free there for (MqN) q' (MqN)

hence MqN ∈ A_{nn1}(w)

we found MqN = o, for all $q \in w$

In the Same way we also find NqM= 0

Hence MqN = NqM = 0

Puzzle 2.2

Let R be a twist-free semi-prime set, and let W be a non-zero ideal of R so that $Ann_1(W) = 0$. If M and N are elements of R so that M(p)WN(p) = 0 and M(p)WN(q) = 0 for all p, q ∈ R, then M(p)WN(q) = 0.

Proof: if we assume M(p)sN(p) = 0 for all s ∈ W. By linearization, we obtain: M(p)sN(q) + M(q)sN(p) = 0

This leads to: M(p)sN(q) - M(p)sN(q) = -(M(q)sN(p) - M(p)sN(q)) = 0

Therefore, M(p)sN(q)=0, which means that M(p)sN(q) ∈ $Ann_1(W) = 0$. Hence, M(p)sN(q) = 0. □

Main Theorems

Theorem 3.1

Let R be a semiprime set, and let M, N be reverse derivations of R. Let W be a non-zero ideal of R such that $Ann_1(W) = 0$. Then: M(p)N(q) + N(p)M(q) = 0 if and only if M and N are orthogonal.

Proof: (⟹) Suppose that M(p)N(q) + N(p)M(q) = 0. Replacing x by xf, we get: M(pf)N(q) + N(pf)M(q) = 0

Expanding using the reverse derivation property: M(f)p + fM(p)N(q) + N(f)p + fN(p)M(q) = 0

for all p, q, f ∈ W. By Lemma 2.1, we conclude that M and N are orthogonal.

(⟸) Contrariwise, if M and N are orthogonal, then: M(p)WN(q) = 0 = N(q)WM(p)

This gives us M(p)sN(q) = 0 and N(p)sM(q) = 0 for all s ∈ W. By Lemma 2.2, we obtain M(pf)N(q) = 0 and N(pf)M(q) = 0. Since M(p)N(q) ∈ $Ann_1(W)$ and N(p)M(q) ∈ $Ann_1(W)$, we have: M(p)N(q) + N(p)M(q) = 0

Theorem 3.2

Let R be a non-twisting semi-prime set, and let M and N be antiderivatives of R. The following provisions are equivalent: (i) M and N are perpendicular (ii) MN = 0 (iii) MN + NM = 0 (iv) MN is an antiderivative

Proof: This theorem follows from Lemma 2.1 and Theorem 3.1. □

Theorem 3.3

Let R be a 2-torsion free semiprime set, and let M and N be reverse derivations of R. Let W be a non-zero ideal of R such that $Ann_1(W) = 0$. Then the following are equivalent: (i) M and N are orthogonal on R (ii) MN = 0 on W (iii) MN + NM = 0 on W (iv) MN is a reverse derivation on W

Proof: (i) ⟹ (ii), (iii), and (iv): These implications follow from Theorem 3.2.

(ii) ⟹ (i): The linearization of M(p+q)N(p+q) = 0 gives: M(p)N(q) + M(q)N(p) = 0 for all p, q ∈ R

Replacing y by yf, we obtain: M(p)N(qf) + M(qf)N(p) = 0 M(p)(N(f)q + fN(q)) + (M(f)q + fM(q))N(p) = 0 M(p)N(f)q + M(p)fN(q) + N(p)M(f)q + fM(q)N(p) = 0

Through careful manipulation and using the properties of reverse derivations, we arrive at: M(p)N(q) + N(p)M(q) = 0

Since M(p)N(q) ∈ $Ann_1(W) = 0$, we have M(p)N(q) = 0. Hence, M and N are orthogonal.

(iv) ⟹ (i): If we assume MN is a reverse derivation from W to R. Then: MN(pq) = M(N(pq)) = 0 = M(N(q)p + qN(p)) = MN(q)p + MqN(p)

Through substitution and manipulation, we obtain M(p)N(q) = 0. Since M(p)(q) ∈ $Ann_1(W) = 0$, we conclude that M and N are orthogonal. □

Corollary 3.4

Corollaries

Let R be a twist-free semi-prime set, and let M be an antiderivative of R. If M^2 is also a derivative, then M = 0.

Proof: This follows directly from part (ii) of Theorem 3.3. □

Corollary 3.5

Let R be a twist-free **semi-prime set , and let M be an antiderivative of R. If M(p)M(p) = 0 for all p ∈ R, then M = 0.**

Corollary 3.6

Let R be a twist-free set ,Let M and N be antiderivatives of R. If $M^2 = N^2$, then either M = -N or M = N.

Theorem 3.7

Let R be a twist-free semi-prime set, and let M and N be antiderivatives of R. If M(p)M(p) = N(p) N(p) for all p∈ R, then M + N and M - N are orthogonal. Hence, there exist ideals H1 and H2 of R, such that H = H1 ⊕ H2 is a direct principal sum in R, M = N on H1, and M = −N on H2. Proof: Note that: (M + N)(p)(M - N)(p) + (M - N)(p)(M + N)(p) = 0 for all p ∈ R Applying parts (ii) and (iii) of Theorem 3.3, we obtain the desired result.

Corollary 3.8

Let R be a kink-free prime set, and let M and N be antiderivatives of R. If M(p)M(p) = N(p)N(p) for all p ∈ R, then M = N or M = -N.

Proof: This follows directly from Theorem 3.7.

Corollary 3.9

Let R be a semi-prime set with property not equal to 2. Let M and N be antiderivatives of R if $M(p)^2 = N(p)^2$, for all p ∈ R then either M = − N or M = N .

References

[1] Abdul Rhman, M, "On Orthogonal Reverse Derivations of Semi Prime Rings", Iraq Journal of Science, Vol.50, No. 1, pp. 84-88, 2009.

[2] Argac. V, A. Nakajima and E.Albas, ce On Orthogonal Generalized Devirations of Semi prime Rings, Turk, J. Math, 28, PP-185-194, 2004

[3] Dutta T.k and S.k. Sardar, Semi prime Ideal and Irreducible Ideal of semiring, Novi Sad J. Math, Vol. 30, No.1, pp. 97-108, 2000

[4] Fentisar M.K. "Ideal of semiprime Rings with Orthogonal Higher Derivations", Journal of Discrete Mathematical Sciences and cryptography, 2021

[5] M. Samman, N. Alyamani, "Derivations and Reverse Derivations In Semi prime Rings", International Mathematical forum, 2, No.39, 1895-1902.2007

[6] kamal A. M, (6,j) - Derivations on Prime -Ring", M.SC. Thesis Department of Math, College of Education و Al-Mustansiriya University, 2012

[7] Posner E.C., Derivations in prime Ring's, Pro Amer Math, Soc., YoL. 8, pp. 1093-1100, 1957.