Machine Scheduling Problem for Solving Tri-Objective Function Using Local Search Algorithm

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Abstract.

This study introduces the multi-objective single-machine approach. Reducing the three criteria of maximum earliest time E_{max}, tardiness (ΣT_j), and total completion time (ΣC_j) will solve the machine scheduling problem (MSP). It is an NP-hard problem.

In this paper's theoretical section, we give the mathematical formulation of. Next, we'll look at how the dominance rule may help you make the most informed decisions. In the practical part, the Branch and Bound method is one of the most important exact approaches. A collection of optimal determination issues for $1//F(\Sigma C_j + \Sigma T_j + E_{max})$ are generated, and the provided MSP goal is solved. In a fair amount of time, the BAB approach finds the best resolution issues to the problem. In addition, we provide two heuristic methods to address the Issue and provide appropriate estimates. The two suggested determination issues work well, according to the results of the practical investigation.

Keywords. Branch and Bound, Multi-Objective, Total Completion Time, Heuristic Approaches.

المستخلص

تُقدّم هذه الدراسة نهج الآلة الواحدة متعدد الأهداف. سيؤدي تقليل المعايير الثلاثة لأقصى وقت مبكر E_{max} ، والتأخير (ΣT_j) ، وإجمالي وقت الإكمال (ΣT_j) إلى حل مشكلة جدولة الآلة .(MSP) وهي مشكلة من نوع.NP-hard

في القسم النظري من هذه الورقة، نُقدّم الصياغة الرياضية لـ. بعد ذلك، سننظر في كيفية مساعدة قاعدة الهيمنة في اتخاذ القرارات الأكثر استنارة. في الجزء العملي، تُعد طريقة الفرع والحد واحدة من أهم الطرق الدقيقة. يتم توليد مجموعة من مشكلات التحديد الأمثل لـ$1//F(\Sigma C_j + \Sigma T_j + E_{max})$ ، ويتم حل هدف MSP المُقدّم. في فترة زمنية معقولة، يجد نهج BABأفضل مشكلات الحل للمشكلة. بالإضافة إلى ذلك، نُقدّم طريقتين استدلاليتين لمعالجة المشكلة وتقديم تقديرات مناسبة. تعمل مشكلتا التحديد المقترحتان بشكل جيد، وفقًا لنتائج البحث العملي.

1-Introduction

The difficulty of allocating a group of tasks to a group of machines under a specific time constraint is known as the scheduling Issue [1-3]. A given number of jobs, n, each requiring a set number of operations to be planned on one or more machines over a particular amount of time about a specific target to be lowered, may be characterized as the machine scheduling Issue (MSP) [4] and [16].

A single objective Issue has been the focus of most research until the late 1980s. Actually, in order to provide decision makers about more practical options, scheduling choices should consider a number of elements. Modeling and solving scheduling Issues (SP) becomes more difficult when several objectives (criteria) are required. In many cases, using the same choice variables won't optimize several goals. There is hence a trade-off between several goals. A Multi Objective Scheduling Issue (MOSP) is the term for this type of Issue [5, 6].

The multicriteria goal's function $1//(\sum C_j + R_L + T_{max})$ is used to solve the Tricriteria Machine Scheduling Issue (SMSP), which is done about BAB and a few heuristic strategies [7]. Effective determination Issues to the Issues at hand are demonstrated in a number of particular examples. To find good or optimal determination Issues, they employed both heuristic and exact techniques to address the $1//(\sum C_j + R_L + T_{max})$ Issue [8].

Determine the sequence that minimizes this MOF by investigating the $1//(\sum(E_j + T_j + C_j + U_j + V_j)$ Issue [9] and [16]. They give a BAB determination Issue for this issue. Additionally, they employ fast LSMs, yielding almost perfect Outcomes. They report on computation experience and evaluate the performance of exact and LSMs on a variety of test Issues.

In order to decrease a multi-objective function, this article discusses arranging the number of tasks (n) on one machine. This may be expressed as follows: A separate machine, capable of performing one job at a time, will manage the whole load. A processing time and a due date are assigned to every job. All tasks are completed and prepared for execution at time zero. The objective is to resolve the $1//F(\Sigma C_j + \Sigma T_j + E_{max})$ issue.

Section two demonstrates various machine scheduling Issue (MSP) principles. Section 3 will cover the mathematical formulation of $1//F(\Sigma C_j + \Sigma T_j + E_{max})$. Section 4 presents special cases. Section 5 illustrates our proposed determination Issues. Section six discusses the practical and comparative outcomes. In section seven, we shall present the most important conclusions and recommendations.

1- Machine Schedule Issue Concept

2.1 Important Notations

This research employs many notations:

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- n: number of jobs.
- p_j: processing time of jobs j.
- d_j: due date of jobs j.
- C_j: completion time of job j, where $C_j = \Sigma_{k=1}^{j} p_k$
- ΣC_j: total completion time.
- L_j: lateness of job j, $L_j = C_j - d_j$.
- T_j: tardiness of job j, $T_j = \max\{L_j, 0\}$.

2.2 Important Definition Machine Scheduling Issue

A number of terms are necessary in this study:

Shortest Processing Time [10]**:** Using a non-decreasing order of processing time (p_j). The Issue $1 // \Sigma C_j$ is resolved by using this process.

Minimum Slack Time [11]*:* Jobs are arranged according to the slack time $s_j = d_i - p_j$, which means that $(s_1 \leq s_2 \leq s_3 \leq \cdots \leq s_n)$,in non − decreasing order.Applying this rule will minimize E_{max}.

Definition (1) [12] and [14]*:* The matrix A(G) represents the adjacency matrix of an n-vertices graph, whose i, j^{th} element is 1 in the instance that V_i and V_j have a minimum one edge, and nothing else.

Emmon's Theorem (1) [13]: For the $1//(\Sigma C_j, \Sigma T_j)$ issue, if $p_i \leq p_j$ and $d_i \leq d_j$, then task i occurs first.

2- Mathematical Formulation of Multi-Objectives Function

The following is the formulation of the suggested Issue: When considering a particular schedule, $\boldsymbol{\sigma} = (1,2,\ldots,n)$, where n is how many tasks SCSTEP completed.

$$V = \min\left(\sum_{j=1}^{n} C_{\sigma(j)} + \sum_{j=1}^{n} T_{\sigma(j)} + E_{max}\right)$$

S.t:

$$C_1 = p_{\sigma(1)},$$

$$C_j = C_{(j-1)} - p_{\sigma(j)}, j = 2,3,\ldots,n$$

$L_j = C_j - d_{\sigma(j)}, j = 1,2, \dots, n$ ...(SCSTEP)

$T_j \geq C_j - d_{\sigma(j)}, j = 1,2, \dots, n$

$E_j \geq d_{\sigma(j)} - C_j, j = 1,2, \dots, n$

$E_{max} \geq \max\{d_{\sigma(j)} - C_j, 0\}, j = 1,2, \dots, n$

$C_j, T_j \geq 0, E_j \geq 0, j = 1,2, \dots, n$

The SCSTEP challenge aims to minimize the overall number of completion times, total tardiness and maximum Earliest$\left(\sum_{j=1}^{n} C_{\sigma(j)} + \sum_{j=1}^{n} T_{\sigma(j)} + E_{max}\right)$ by scheduling the tasks on one machine. $\sigma \in S$ (S represents a set of all possible determination Issues), is the Issue's objective. As a result, the complexity of our issue is n! due to the ΣT_j function, it is considered an NP-hard issue.

3- Special Cases for SCSTEP -Issue

Case (1): For Issue (SCSTEP) if $p_j = p$ and $d_j = d, \forall j$ if so, we offer a special determination Issue:

1- $d \leq C_j, \forall j$ then $(\Sigma C_j + \Sigma T_j + E_{max}) = n(p(n+1) - d)$

2- $d > C_j, \forall j$ then $(\Sigma C_j + \Sigma T_j + E_{max}) = p\frac{n^2+n-2}{2} + d$

Proof: due to $p_j = p$, then $C_j = jp$, then

$$\Sigma C_j = p\frac{n(n+1)}{2} \quad \cdots (1)$$

For $d_j = d$ if:

1- $d \leq C_j, \forall j$ that's mean all jobs are tardy and $L_j = C_j - d = jp - d$, then

$T_j = \max\{jp - d, 0\} = jp - d$

$$\Sigma T_j = \Sigma jp - \Sigma d = p\frac{n(n+1)}{2} - nd \quad \cdots (2)$$

$E_j = \max\{d - C_j, 0\} = 0, \forall j$

$$\therefore E_{max} = 0 \quad \cdots (3)$$

From (1), (2) and (3) we obtain:

$$\Sigma C_j + \Sigma T_j + E_{max} = p\frac{n(n+1)}{2} + p\frac{n(n+1)}{2} - nd + 0 = pn(n+1) - nd = n(p(n+1) - d)$$

2 – $d > C_j, \forall j$ that's mean all jobs are early then $T_j = \max\{jp - d, 0\} = 0, \forall j$ then:

$$\Sigma T_j = 0 \quad \cdots (4)$$

$E_j = \max\{d - C_j, 0\} = d - C_j = d - jp$

$$E_{max} = d - p \quad \cdots (5)$$

From (1), (4) and (5) we obtain:

$$\Sigma C_j + \Sigma T_j + E_{max} = \left(p\frac{n(n+1)}{2} + 0 + d - p\right) = p\frac{n^2+n-2}{2} + d$$

Case (2): For the Issue (SCSTEP):

1- If $d_j > C_j$, $\forall j$, then the Issue changed to $(\Sigma C_j + E_{max})$,this Issue has just one determination Issue if SPT and MST are the same.
2- If $d_j < C_j$, $\forall j \in N$ then the Issue changed to $(\Sigma C_j , + \Sigma T_j)$,this Issue has just one determination Issue if SPT and EDD are the same.
3- If $d_j = C_j$, $\forall j \in N$ then the Issue changed to (ΣC_j) ,this Issue has a unique determination Issue according to the SPT rule.

Proof:

1- due to $d_j > C_j$, $\forall j$, then all jobs are early $T_j = 0$, $\forall j$, then $\Sigma T_j = 0$ then

$$\Sigma C_j + \Sigma T_j + E_{max} = \Sigma C_j + E_{max}$$

2- due to $d_j < C_j$ then all jobs are tardy for all j ,this Issue solved in $p-$ type if $\sigma \equiv SPT \equiv MST$,then σ will minimize both ΣC_j and ΣT_j about unique Significant

$$\Sigma C_j + \Sigma T_j + E_{max} = \Sigma C_j + \Sigma T_j + 0 = \Sigma C_j + \Sigma T_j$$

3- due to $d_j = C_j$ for all j in the SPT schedule (this means $T_j = 0$, $E_j = 0$, $\forall j$ then $\Sigma T_j = 0$ and $E_{max} = 0$ then

$$\Sigma C_j + \Sigma T_j + E_{max} = \Sigma C_j$$

Now if σ =SPT Significant is applied, then we obtain optimal determination Issue depend on ΣC_j only

Case (3): For the Issue (SCSTEP) if $p_j = p$ for $\forall j$, Consequently, if $\sigma = EDD$, we could have an efficient determination Issue.

proof:

$$E_j = \max\{d_j - C_j, 0\} = \max\{d_j - jp, 0\}$$

If $d_1 = p$ and $jp \geq d_j$,then all jobs are late and $E_j = 0$, $\forall$ and

$$E_{max} = 0 \qquad \cdots (6)$$

Then by using Relations (1) and (6), the Issue changes to $\left(\Sigma C_j + \Sigma T_j + 0\right) = \left(p\frac{n(n+1)}{2} + \Sigma T_j\right) = \left(p\frac{n(n+1)}{2} + \Sigma T_j\right)$

If we apply $\sigma = EDD$ for the Issue, then the $\Sigma T_j(\sigma) \leq \Sigma T_j(\pi)$ where π any

sequence and due to $p\frac{n(n+1)}{2}$ is constant then $p\frac{n(n+1)}{2} + \Sigma T_j(\sigma)$ has an optimal determination Issue for

the Issue.

Case (4): For the Issue (SCSTEP) if $d_j = d$, $\forall j$:

1- If $d \leq C_j, \forall j$ then $\left(\Sigma C_j + \Sigma T_j + E_{max}\right) = \Sigma C_j + n(C_n - d)$.
2- If $d > C_j, \forall j$ then $\left(\Sigma C_j + \Sigma T_j + E_{max}\right) = \Sigma C_j + d - p$

The issues described above can be solved using the SPT rule.

Proof:

1- If $d \leq C_j, \forall j$ then all jobs are late

$\therefore L_j = C_j - d \ , \forall j$ then

$T_j = \max\{C_j - d, 0\} = C_n - d$ then , $\Sigma T_j = \Sigma C_n - \Sigma d$ then

$$\Sigma T_j = n(C_n - d) \qquad \cdots (7)$$

$E_j = \max\{d - C_j + 0\} = 0, \forall j$, then

$$Emax = 0 \qquad \cdots (8)$$

From (7) and (8) we obtain:

$\Sigma C_j + \Sigma T_j + E_{max} = \Sigma C_j + n(C_n - d)$

SPT rule can solve this Issue due to $n(C_n - d)$is constant.

2- If $d > C_j, \forall j$,then $T_j = 0, \forall j$, then all jobs are early ,

$$\Sigma T_j = 0 \qquad \cdots (9)$$

$E_j = \max\{d - C_j, 0\} = d - C_1$ then

$$E_{max} = d - p_{\sigma(1)} \qquad \cdots (10)$$

From (9) and (10) we obtain:

$\Sigma C_j + \Sigma T_j + E_{max} = \Sigma C_j + 0 + d - p_1 = \Sigma C_j + d - p_{\sigma(1)}$

This issue can be resolved using the σ =SPT rule due to $d - p_{\sigma(1)}$ is constant.

In Table 1, Instances are provided to clarify the special cases for the (SCSTEP) Issue, by calculating the objective functions(F).

Table 1. Special Cases of Issue (SCSTEP)In the following examples.

Case	p_j and d_j	Conditions	F
Case 1	$p = d = 5$ $p = 5, d = 7, p < d$	$p_j = p$ and $d_j = d, \forall j$	(130) (130)
Case 2	$p_j = 4,3,2,2,1, d_j = 14,9,5,6,2,3$ $p_j = 3,4,5,6,8,9, d_j = 3,5,6,7,9,10$ $p_j = 8,5,2,6,4,3, d_j = 30,14,2,21,10,6$	$d_j \geq C_j$ $d_j \leq C_j$	(39) (126) (83)
Case 3	$p_j = 3, d_j = 3,4,6,7,8,9$	$p_j = p$ for $\forall j$	(72)
Case 4	$p_j = 5,4,3,2,1,1, d_j = 6$ $p_j = 7,65,4,3,2,1, d_j = 7$	$d_j = d$	(62) (123)

The multi-objective function of the Issue (SCSTEP) is denoted by (F).

4- Propose Approaches for Solving SCSTEP-Issue

4.1 Exact Approaches for SCSTEP-Issue

Below this part, we use the classic Branch and Bound (BAB) approach, often known as BAB, to try to identify a set of Pareto optimal determination Issues for the SCSTEP issue. The following are the steps involved in BAB:

Algorithm: BAB Method

Step (1): **INPUT** n, p_j and d_j for $j = 1,2, \dots, n$ . .

Step (2): **SET** $S = \phi$, define $F(\sigma) = \Sigma C_{\sigma(j)} + \Sigma T_j(\sigma) + E_{max}(\sigma)$, for any σ.

Step (3): The upper bound (UB) is obtained by applying the $\sigma = SPT$ rule. Determine $F(\sigma)$, which is equal to $1, 2, \dots, n$. Lastly, place the upper bound UB= $F(\sigma)$ at the parent node of the search tree.

Step (4): Set a lower bound LB **(δ)** every node in the BAB tree of searches and component Job sequence. The LB **(δ)** calculation combines the costs of the sequence jobs **(δ)**

About those produced by sequencing the tasks in the SPT rule.

Step (5): When LB ≤ UB, begin at every node and branch out.

Step (6): **STOP**.

5- Heuristic Approaches for SCSTEP-Issue

Start by applying the heuristic strategy, where the goal function is calculated and the Issue is resolved by the SPT rule. Next, use the SPT rule to organize the remaining jobs and find out what the goal is. Up until the completion of n series, this process is repeated. The following are the main steps of $SCSTE_{max}P$:

Algorithm (2): $SCSTE_{max}P$ Heuristic Method

Step (1): INPUT n, p_j and d_j $, j = 1,2,\dots,n,\ \ \delta = \phi$.

Step (2): Assign tasks according to the SPT rule (σ_1) and calculate $F_{11}\ (\sigma_1)$, where $\delta = \delta \cup \{ F_{11}\ (\sigma_1)\};$

Step (3): To obtain σ_j, place job i at the forefront of σ_{j-1} for $j = 2, \dots, n$. Then, compute $F_{2j}(\sigma_j)\ \delta = \delta \cup \{ F_{1j}\ (\sigma_j)\}$.

END;

$F_1 = \min\{F_{1j}(\sigma_j)\}$

Step (4): Determine $F_{21}\ (\pi_1),\ \delta = \delta \cup \{ F_{21}\ (\pi_1)\};$ then arrange jobs according to MST rule (π_1).

Step (5): To obtain π_j, place job j in the first position of π_{j-1} for $j = 2, \dots, n$. Then, compute $F_{2j}(\pi_j), \delta = \delta \cup \{F_{2j}(\pi_j)\}$.

$F_2 = \min\{F_{2j}(\pi_j)\}$

END;

$F = \min\{F_1, F_2\}$

Step (6): A set of efficient $SCSTE_{max}P$ -issue determination Issues is provided by filter set δ.

Step (7): OUTPUT best F

Step (8): STOP.

The concept behind the following heuristic technique is summed up by calculating the target function and identifying an ordered list about the fewest p_j and d_j which is not in disagreement about dominance rule. The following are the primary steps of $SCSTE_{max}P$:

Algorithm (3): DR- $SCSTE_{max}P$ Heuristic Method

Step (1): INPUT: n, p_j and d_j , $j = 1,2,\ldots,n$, $\delta = \emptyset$

Step (2): To get the DR adjacency matrix A, use theorem (1), where $N = \{1,2,\ldots,n\}$,

$\delta = \emptyset$

Step (3): If more than one job break tie randomly, then $\delta = \delta \cup \{\sigma_1\}$. Construct a pattern σ_1 about a minimal p_j which is not in disagreement about DR (matrix A).

Step (4): If more than one job break tie randomly, then $\delta = \delta \cup \{\sigma_2\}$. Determine a sequence σ_2 about a minimal d_j which is not in disagreement about DR (matrix A).

Step (5): Determine which order is set $\delta^{`}$ from δ.

Step (6): Calculate $F(\delta), F = \min\{F(\delta)\}$.

Step (7): OUTPUT best F.

Step (8): END.

6- Practical Outcomes of the Issue (SCSTEP)

We provide a few key abbreviations before displaying all of the outcome tables:

AT/S Average of CPU-Time per second.
AMAE Average of Minimum Absolute Error.
BS BS Significant of Issue (SCSTEP).
OP Optimal Significant of Issue (SCSTEP).
R $0 < \text{Real} < 1$.
F Objective Function of Issue (SCSTEP).

After demonstrating how the recommended approaches (exact and estimation) are put into practice, we must first demonstrate that we compare the outcomes of the suggested strategies about the full enumeration method (CEM), which tries every possible variant of events to find the most efficient one.

Example (1): Using the following table, let's utilize MSP about five jobs, processing time, and due date:

Table 2. The p_j , d_j, and s_j data for the Issue (SCSTEP)

	1	2	3	4	5
p_j	1	8	10	4	9
d_j	14	28	27	23	12
s_j	13	20	17	19	3

Shown in Figure (1) may be obtained using theorem (1).

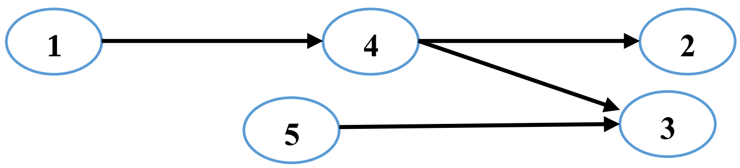

Figure (1): The DRs of the example (1)

The following (6) DRs may be seen from theorem (1): 1→2, 1→ 3,1→4, 1→6, 2→ 6, 4→2, 4→ 3, 4→ 6,5→3. There are seven possible sequences in Table 2, some or all of which are susceptible in addition to what already mentioned DRs. The following are the adjacency matrix A:

$$A(G) = \begin{bmatrix} 0 & 1 & 1 & 1 & a_{15} \\ 0 & 0 & a_{23} & 0 & a_{25} \\ 0 & a_{32} & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & a_{45} \\ a_{51} & a_{52} & 1 & a_{54} & 0 \end{bmatrix}. \text{where } a_{ji} = \begin{cases} 1, \text{if } a_{ji} = 0 \\ 0, \text{if } a_{ji} = 1 \end{cases}$$

Table 3. Under DR, optimal sequences for example (1)

EF-SQ						SCSTEP
Seq	**POS1**	**POS2**	**POS3**	**POS4**	**POS5**	$\Sigma C_j + \Sigma T_j + E_{max}$
1	1	4	2	5	3	129
2	1	4	5	2	3	116
3	1	4	5	3	2	114
4	1	5	4	2	3	114
5	1	5	4	3	2	113
6	5	1	4	2	3	114

where POS stands for position and EF-SQ for efficient sequence.

As can be seen in Table 3, the sequences (1–7) present the issue an optimal Significant for the Issue (SCSTEP).

Table (4) shows the Outcomes of solving the SCSTEP-Issue, n=3:10, using CEM and BAB.

Table (4): SCSTEP-Issue BAB and CEM Overviews, n = 3:10.

n	BAB		CEM	
	Optimal	Time	F	Time
	Av(F)	AT/s	Av(F)	AT/s
3	47.2	R	47.2	R
4	57	R	57	R
5	85	R	85	R
6	103.8	R	103.8	R
7	156.6	R	156.6	R
8	259.6	5.1824	259.6	R
9	299.4	45.8577	299.6	R
10	361.4	477.8011	361.8	R

Table (5): Discussion featuring SCSTE$_{max}$P and SCSTE$_{max}$P (DR) about CEM for SCSTEP -Issue, n=3:10.

n	CEM		SCSTE$_{max}$P		SCSTE$_{max}$P (DR)	
	OP	TIME	BS	AT/s	BS	AT/s
	Av(F)	AT/s	AMAE		AMAE	
3	47.2	R	8.8	R	49.8	R
4	57	R	9	R	58.4	R
5	85	R	14.6	R	89.4	R
6	103.8	R	23.6	R	112	R
7	156.6	R	32	R	166.8	R
8	259.6	5.1824	52.8	R	271.6	R
9	299.4	45.8577	62.2	R	305.4	R
10	361.4	477.8011	68.6	R	367	R

Table (6): Summary between SCSTE$_{max}$P and SCSTE$_{max}$P (DR) for SCSTEP -Issue, n=0:4000.

7-

n	SCSTE$_{max}$P		SCSTE$_{max}$P (DR)	
	BS AMAE	AT/s	BS AMAE	AT/s
30	574.2	R	3447	R
35	1120.6	R	4429.6	R
40	1367.8	R	6222.4	R
45	1408.6	R	7026.6	R
50	1900.8	R	9233.6	R
55	2143.4	R	1202.8	R
60	2685.6	R	1447.3	R
100	7527	R	37802	1.1889
150	19342	R	8475.2	2.0571
200	3179.1	R	1560.2	2.4966
400	1182.2	1.0088	6060.0	5.8263
500	1905.5	1.3040	9703.6	10.0629
600	2805.5	1.7679	1349.5	13.3215
700	3701.1	2.3175	1893.1	16.0939
800	4820.1	2.9208	2445.7	19.5225
900	5962.2	3.6512	3069.4	27.1716
1000	7895.9	4.4668	3846.0	44.6153
2000	3048.9	19.5043	1543.0	222.3472
4000	1240.5	109.6704	6153.6	963.4205

Conclusions and Future Works

1. According to this study, we use the BAB algorithm about DR to determine the best determination Issue up to n = 10 jobs. Outcomes from the BAB algorithm is compared to those from CEM.

2. We offer two effective and simple heuristic techniques for solving the SCSTEP Issue: SCSTE$_{max}$P and SCSTE$_{max}$P (DR), both of which perform well.

3. For further study, we suggest using some techniques such as (PSO algorithm, Genetic algorithm, etc.) to identify efficiency and approximate determination Issues for the SCSTEP Issues.

4. For more research, we recommend utilizing local search algorithms about multi-criteria and about multi-objective or together.

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