Combine Logistic to Pray and Impacts of Harvesting on Prey-Predator Model

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Abstract

The present article argues about a prey-predator model where the two species are harvested autonomously with constant or variable rates that have been well thought-out. According to the theory of logistic growth, the prey population is increasing. The Lotka-Volterra predator-prey model is modified and adapted into this model. The model has reached its equilibrium points, and the dynamical behavior of the suggested system was examined to display the phase portrait. Model simulations are analyzing, bifurcation diagrams are examined, and the behavior of the solutions' locally and globally is examined. The impacts of logistic and harvesting on the steadiness of stability states are inspected. The main purpose of the article is to implement a mathematical investigate of the model, and it can be obtained that the effect of activities in harvesting is more effective on the dynamical behaviour in the current model. At the end, solution curves are plotted by utilizing the Maple program to explore the dynamical behaviour of the model regarding to the time.

Keywords: Logistic, Harvesting, Predator-Prey, Lotka-Volterra, Jacobian Matrix, Linearization.

1 Introduction

A.J. Lotka and V. Volterra was originally proposed the predator-prey's problem, which is traditional and well-known problem [1]. The Italian mathematician Vito Volterra, in order to talk about the population dynamics of interacting species and predator-prey relationships in the 1920s, he projected a system of differential equations. He wanted to look at the growth of predators, or the decline in prey.

The mathematical modelling works on the study of increasing the population rates, and the environmental diseases and the ecotoxicological problems [2]. One of the main investigations is the model of harvest-population. Nowadays, the problems are identified with harvesting of multispecies fisheries have bed drawing consideration of researchers. The problem of combining harvesting of both independent species ecologically, the rate of harvesting can be used as the parameter's control [3]. Moreover, the investigation of harvested of dynamic's population is more realistic. Rahmani and Saraj [4] studied the efficiency of harvested factor in predator-prey model without having logistic factor in prey or predator. A model with two predators fighting for a single prey species subjected to logistic growth was provided by Ang and Safuan [3] in order to examine the effects of harvesting on a single predator species up to the model's critical point. The effects of harvesting operations on prey-predator populations may be determined by logistical considerations. Ghosh, Chandra [2] improved a fishery model of a prey- predator to study the research of the relation between the population's concentration, as well as the equilibrium of fish populations in areas where harvesting activities expose predator species. Different harvesting efforts and activities are encountered for prey and predatory species [5–6].

The primary goals of this research are to investigate the dynamical behaviors of a predator-prey model in the harvesting environment. The prey population establishes the law of logistic growth, and both species have varying harvesting rates. The effects of harvesting on the two species are investigated in order to establish further assumptions about persistence and extinction behaviors. A mathematical model via Jacobian matrix to analysing equilibrium points is used. A numerically study of the model through Maple software to stability analysis, and bifurcation results was used.

The remaining sections of the paper as follows. Next section is about adding logistic factor to the model, and the non-dimensional model and its stability is presented. In section 3 consider the model with constant and effort harvest factor and discuss equilibria and their stability are investigated, respectively. Finally, in section 4 numerical simulation results and analysis discussed. And the paper end with and conclusion.

2 Methods and Materials

The dynamical interaction between predator and prey species is defined using the traditional Lotka-Volterra model. The linear differential equations for the Lotka-Volterra model are as follows.

$$\begin{cases}\frac{dx}{dt} = ax - bxy \\ \frac{dy}{dt} = cxy - dy\end{cases}$$

The functions $x(t)$,and $y(t)$ express the populations of prey, and predator at time t respectively [3, 7]. Each parameter (a, b, c, d) are positive constants. The natural growth rate of the prey in the absence of predator represented by the parameter a. The effect of the predator on the prey species is represented by parameter b. Furthermore, predators will consume prey if b is the only factor affecting the prey species. The effect of prey on the predator species is shown by parameter c. In addition, the predator population will increase according to the amount of food available if c is the only factor affecting population growth. The biological mortality rate of the predator in the absence of prey was denoted by the parameter d. In the case of $b = 0$ it is assumed that the prey species would growth indefinitely in the absence of predators. The classic Lotka-Volterra assumptions are unrealistic, where one of the alternative solutions is to take on logistic growth behaviour in the prey population which is discussed in this section [8].

The modification of the traditional system of differential equations, by adding the law of logistic growth on prey species as follow:

$$\begin{cases}\frac{dx}{dt} = ax(1 - \frac{x}{K}) - bxy \\ \frac{dy}{dt} = cxy - dy\end{cases} \qquad (1)$$

The parameter (K) means environmental carrying capacity of the prey species. In the absence of predator populations, the prey population grows logistically. All parameters are assumed to have positive values. Introducing the new variable to simplify the model for overall understanding, and reduce the number of parameters. We introduce new variables u and v and let

$$u = \frac{d}{c}x , \quad v = \frac{b}{aF}y \quad \text{and} \quad \tau = aFt \quad \text{where} \quad F = (1 - \frac{N}{K})$$

The dimensional system (1) becomes.

$$\frac{du}{d\tau} = u - uv$$

$$\frac{dv}{d\tau} = -\alpha v + \alpha uv \qquad (1) \qquad \text{Where} \quad \alpha = \frac{c}{aF}$$

In system (2) α: is a quotient between the growth rate of v to times of the growth rate of u and environmental limit carrying capacity of u.

Now, by using the Jacobian matrix which is one of the useful technique, we can discuss the behavers of equilibrium points, and the stability of system (2). By applying the Jacobean matrix to the system, we have

$$J = \begin{bmatrix} 1-v & -u \\ \alpha v & -\alpha + \alpha u \end{bmatrix}$$

The system (2) has two steady states in orbit of solutions of the model, as occurs: (0,0) and (1,1). Now, by using Jacobean matrix then simplify eigenvalues in equilibrium point (0,0) are identified as $\lambda_1 = 1, \lambda_2 = -\alpha$. As regards to equilibrium point (0,0) is a saddle point for the system. However, by linearize at the equilibrium (1,1) Jacobian matrix obtained as

$$J_{0,0} = \begin{bmatrix} 0 & -1 \\ \alpha & 0 \end{bmatrix}$$

So, at this point the phase portrait has a nonhyperbolic critical point as shown in finger (1)

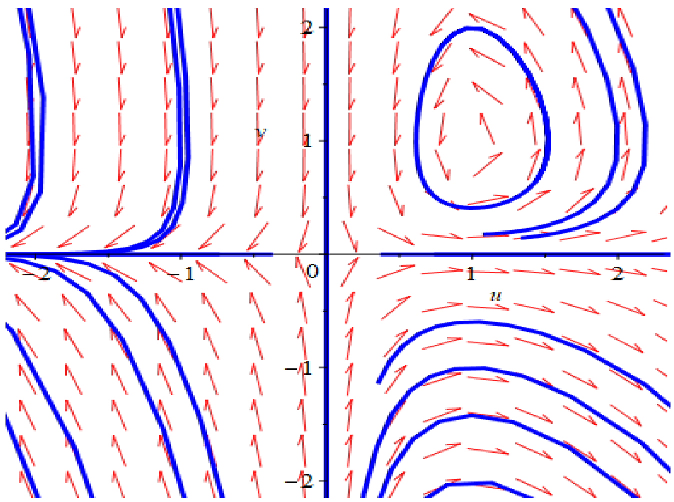

Figure 1: Phase portrait of system (1) where $\alpha = 3$.

The above figure shows the phase portrait trajectories of equilibrium point (0,0) is a saddle point, and the system has non-hyperbolic critical point at equilibrium point (1,1). See appendix (A) for maple codes to plot phases portrait of the system. It illustrated that both of the species are induced by a harvesting effort. The behaviour of the solutions in the global and local stability aspect have been examined in section 3.

3 Having Constant Harvest Factors to the Predator - Prey Model

3.1 Constant Harvest Factors

Consider Lotka -Volterra model that has constant harvest factor for prey and predator species.

$$\begin{cases} \dfrac{du}{d\tau} = u - uv - h \\ \dfrac{dv}{d\tau} = -\alpha v + \alpha uv - i \end{cases} \qquad (3)$$

Where α, is a quotient between the growth rate of v to times of the growth rate of u, and environmental limit carrying capacity of u and h, i are constant harvest factors. This model is known as a predator-prey model with constant harvesting. To the system above, we will analyse local stability. As the result we analyse points of equilibrium of the system, the u and v null lines are $u = -\frac{h}{v-1}$ and $v = \frac{i}{(u-1)\alpha}$ and it has two equilibrium points (u_1^*, v_1^*) and (u_2^*, v_2^*), where:

$$u_1^* = \frac{-\alpha h + i + \alpha + \sqrt{(h-1)^2\alpha^2 + 2i(h+1)\alpha + i^2}}{-\alpha h - i + \alpha + \sqrt{(h-1)^2\alpha^2 + 2i(h+1)\alpha + i^2}}, \quad u_2^* = \frac{\sqrt{(h-1)^2\alpha^2 + 2i(h+1)\alpha + i^2} + (h-1)\alpha - i}{\sqrt{(h-1)^2\alpha^2 + 2i(h+1)\alpha + i^2} + (h-1)\alpha + i},$$

$$v_1^* = \frac{-\alpha h - i + \alpha + \sqrt{(h-1)^2\alpha^2 + 2i(h+1)\alpha + i^2}}{2\alpha} \quad \text{And} \quad v_2^* = \frac{-\alpha h - i + \alpha - \sqrt{(h-1)^2\alpha^2 + 2i(h+1)\alpha + i^2}}{2\alpha}$$

After we substate equilibrium points (u_1^*, v_1^*) and (u_2^*, v_2^*) in Jacobian matrix given below

$$J = \begin{bmatrix} -v + 1 & -u \\ \alpha v & \alpha u - \alpha \end{bmatrix}$$

also, by lemma (3.1) we have negative real part eigenvalues to equilibrium point (u_1^*, v_1^*) so that this point is a stable critical point. And (u_2^*, v_2^*) is unstable equilibrium point. Thus, the phase portrait of system (3) is shown in finger (2)

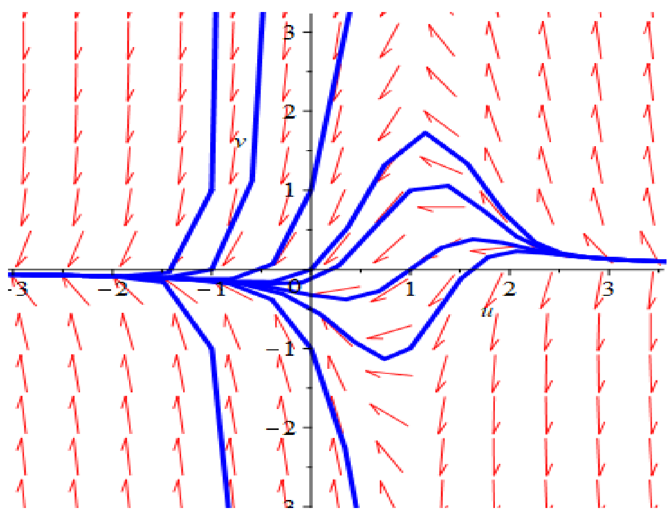

Figure 2: Phase portrait of system (3) where $\alpha = 20, h = 6, i = 5$

Lemma 3.1.1: To a real matrix N, If $tr(N)$, $\det(N)$ and $\det(N^2)$ are all negative, then all the eigenvalues of N have a negative real part. The proof can find in [9] and [10].

One of the most distinctive cases of the system (3) is

$$\begin{cases} \frac{du}{d\tau} = u - uv - h \\ \frac{dv}{d\tau} = -\alpha v + \alpha uv \end{cases} \quad (4)$$

Where we have only harvesting factor on pray population. And the second specially case of model (3) is

$$\begin{cases} \dfrac{du}{d\tau} = u - uv \\ \dfrac{dv}{d\tau} = -\alpha v + \alpha uv - i \end{cases} \quad (5)$$

It is defined as a predator-prey system with a constant harvest factor for predators. The analysis of the above models can be obtained by the same method.

3.2 Having Variable Harvest Factor to the Predator - Prey Model.

Considering the predator-prey model with having effort rate harvested factor for prey and predator. In the following model e_1, e_2 are positive constants that show effort rate to tong prey and predator, respectively.

$$\begin{cases} \dfrac{du}{d\tau} = u - uv - e_1 u \\ \dfrac{dv}{d\tau} = -\alpha v + \alpha uv - e_2 v \end{cases} \quad (6)$$

Two particular cases of the system (6) are known as a predator-prey model including prey harvesting (corresponding to prey population) and a predator-prey model with predator harvesting (corresponding to predator population), which can be written as follows, respectively.

$$\begin{cases} \dfrac{du}{d\tau} = u - uv - e_1 u \\ \dfrac{dv}{d\tau} = -\alpha v + \alpha uv \end{cases} \quad (7)$$

And

$$\begin{cases} \dfrac{du}{d\tau} = u - uv \\ \dfrac{dv}{d\tau} = -\alpha v + \alpha uv - e_2 v \end{cases} \quad (8)$$

To study the stability of the equilibrium points for the above system (6), by using the linearization technique. We find the Jacobean matrix for this purpose, which might be found as follows:

$$J = \begin{pmatrix} -e_1 - v + 1 & -u \\ \alpha v & \alpha u - \alpha - e_2 \end{pmatrix}$$

For equilibrium point (0,0) Jacobean matrix is given by:

$$J_{0,0} = \begin{pmatrix} -e_1 + 1 & 0 \\ 0 & -\alpha - e_2 \end{pmatrix}$$

So ($\lambda_1 = -e_1 + 1$) and ($\lambda_2 = -\alpha - e_2$) are eigenvalues of above matrix, as regarding to assumption $e_1 > 1$ thus equilibrium point (0,0) is a stable equilibrium point for the system (6).

At equilibrium point ($-e_1 + 1, 1 + \frac{e_2}{\alpha}$) Jacobean matrix is found out as follows:

$$J = \begin{pmatrix} -e_1 - \frac{e_2}{\alpha} & e_1 - 1 \\ \alpha + e_2 & -\alpha e_1 - e_2 \end{pmatrix}$$

therefor ($\lambda_1 = -e_1 - \frac{e_2}{\alpha}$) and ($\lambda_2 = -\alpha e_1 - e_2$) are eigenvalues of above matrix, thus equilibrium point ($-e_1 + 1, 1 + \frac{e_2}{\alpha}$) is a stable point for system (6).

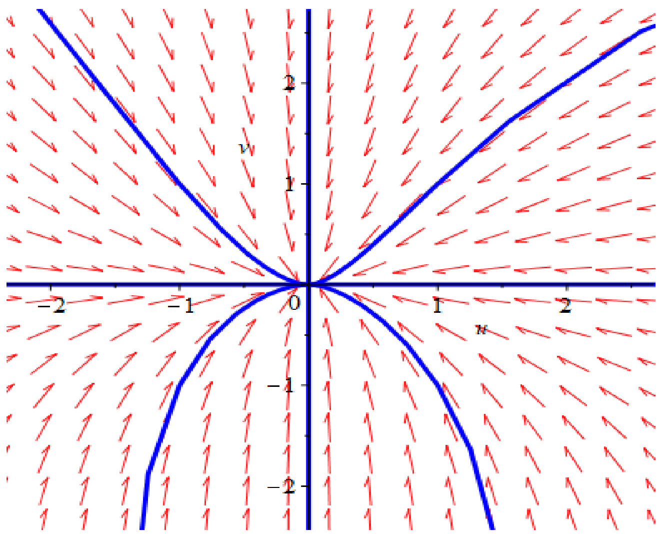

Figure 3: Phase portrait of system (6) where $\alpha = 0.75, h = 5, i = 6$

We identified the following proposition by substituting equilibrium points of the system (7) and (8) and simplifying them.

Proposition 3.2.1. To the system (7) these statements are true:

(i) The point (0,0) is a stable equilibrium point to the system.
(ii) Equilibrium point $(-e_1 + 1,1)$ is locally asymptotically stable in the system (7).

Proposition 3.2.2. Following statements about system (8) are holds:

(i) The point (0,0) is a saddle equilibrium point for system the (8).
(ii) Let ($\frac{e_2}{\alpha} > 0$) the equilibrium point ($1, e_2 + 1$) is locally asymptotically stable in the system (8).

Now, we will mix two harvest factors which is called complexed harvest model.

3.3 Complex Harvested Model:

Consider following system with diseases occurs in an environment or there is interspecific rivalry for the predator-prey model. In the present system, harvested factors are u^2 and v^2 for prey and predator population, separately. However, we may also assume that the following system is a model with having interaction interspecies between prey, and predator species. which may be written as follow:

$$\begin{cases} \dfrac{du}{d\tau} = u - uv - e_1u^2 \\ \dfrac{dv}{d\tau} = -\alpha v + \alpha uv - e_2v^2 \end{cases} \quad (9)$$

To the above system, we can identify the following theorem.

Theorem 3.3.1. The system (9) has two steady states.

(i) The point (0,0) is saddle equilibrium point, i.e., stable provided ($\alpha > 0$)

(ii) This system is locally asymptotically unstable at its other equilibrium point.

Proof. One of the steady states is (0,0) and the next one is presented by (u^*, v^*).

By using linearization method to show its eigenvalues at equilibrium point (u*, v*) are $\lambda_1 = \frac{3e_1(\alpha + e_2)}{e_1e_2 + \alpha}$

and $\lambda_2 = \frac{\alpha(-2\alpha + e2(e1-3))}{e1*e2+\alpha}$, and so, the point is locally asymptotically unstable □.

The behaver of the system is shown in figure (3) provided ($\alpha = 0.8, e_1 = 2, e_2 = 3$)

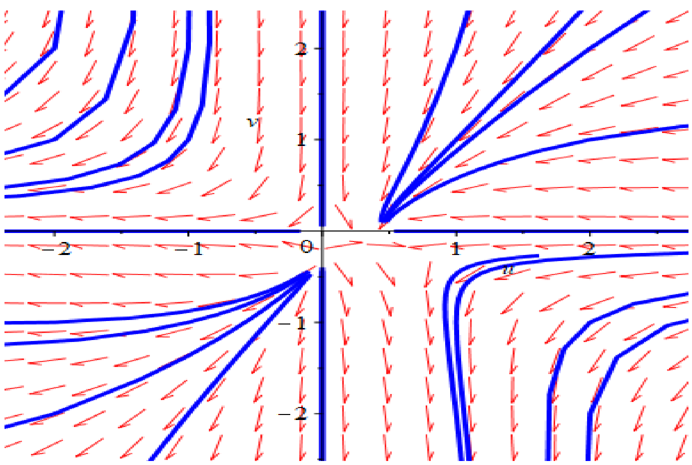

Figure 4: Phase portrait of system (9) where $\alpha = 0.8, e_1 = 2,\ and\ e_2 = 3$

4 Results and Discussions

In the proves systems, we have considered that each of the two species is harvested by constant and effort harvest. In section (3), we calculate the equilibrium points and discussed the state stability of them, also phase portrait of the system (1), (3), (6), and (9) are shown. Here, we sketch the solution curves of different harvest strategies, constant and effort harvest affect. However, for all types of the systems quantitatively relevant data are rarely available. Therefore, we are taking out some hypothetical data, and to explain the results we consider some numerical illustrations and the related solution curves are studied of the models.

Case 4.1. In the suitable units. Instead, the parameter values as $\alpha = 0.78$, the solution curves for Prey-Predator models in the system (2) as follows.

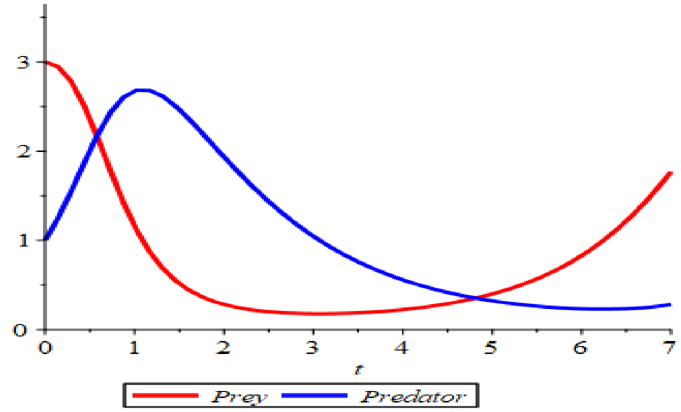

Figure 5: Solution curves of system (2) to the prey–predator model for a period t = 0–7 weeks.

See appendix (B) for maple codes to sketch the solution curves of the system.

Case 4.2. In this illustration, as in the above case, for the parameter α we set $\alpha = 0.78$ *and* $i = 1$, the solution curves for Prey-Predator models in the system (5) as follows.

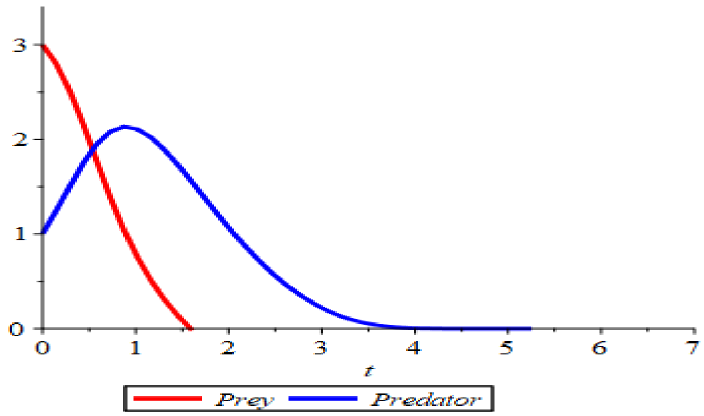

Figure 6: Solution curves of system (5) to the prey–predator model for a period t = 0–7 weeks.

But if we take $e_1 = 1$ in the system (8) then the solution curves shown below.

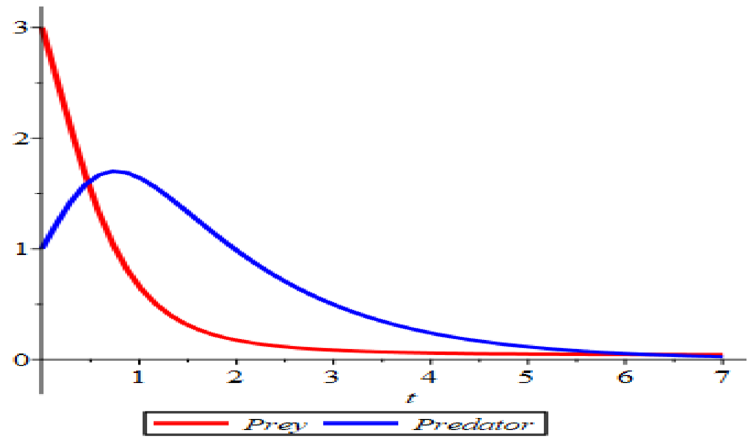

Figure 7: Solution curves of system (8) to the prey–predator model for a period t = 0–7 weeks.

Now, we compare the two harvest strategies constant and effort harvesting of the figures (6) and (7), By looking at the stationary states, from its figures, the population size of both the predator-prey population decline as the harvesting impacts on prey increases. This occurs because of the harvesting impacts which reduces the amount of prey in the environment and therefore reduce predator-on-prey food supplies, leading both populations to decline.

5 Conclusion

In the presented research, we studied the impact of various harvest rates on Lotka-Volterra predator-prey population which the pray population is governed by the logistic law of growth to hesitate effect of harvest rate, as a realistic parameter. Therefore, in this project, we have shown that the harvest rate has a positive impact on the Lotka-Volterra model predator-prey population increasing global stability. In addition, we have seen that the impact of harvesting activity is more significant in the system. The purpose of this impact can be explained by combining two types of prey and predator. The dynamical behaviour of the unexploited system is studied. It is found to possess two equilibria (0,0) and (u*, v*). This assumption may be used to describe the positive efficacy of controlled use of nature.

Appendix A : Maple codes to skitch phase portrait of the system (2)

```
> restart; with(DETools); with(plots); with(LinearAlgebra);
> sys := {diff(u(t), t) = u(t) - u(t)*v(t), diff(v(t), t) = -alpha*v(t) + alpha*u(t)*v(t)};
> ics := seq(seq([0, i, j], i = -2 .. 2), j = -2 .. 2);
> DEplot(subs({alpha = 3}, sys), {u(t), v(t)}, t = -1 .. 4, u = -3 .. 3, v = -3 .. 3, [ics], linecolor = blue);
```

Appendix B: Maple codes to skitch the solution curves the system (2)

```
> restart; with(DETools); with(LinearAlgebra); with(plots);
> sys1 := {diff(u(t), t) = u(t) - u(t)*v(t), diff(v(t), t) = -alpha*v(t) + alpha*u(t)*v(t)};
> ics := [[0, 3, 1]];
> figu := DEplot(subs({alpha = 0.78}, sys1), {u(t), v(t)}, t = 0 .. 7, u = 0 .. 4, v = 0 .. 4, ics, scene = [t, u(t)]);
> figv := DEplot (subs({alpha = 0.78}, sys1), {u(t), v(t)}, t = 0 .. 7, u = 0 .. 4, v = 0 .. 4, ics,
```

```
scene = [t, v(t)]);
> display([figu, figv], color = ["red", "blue"], legend = [Prey, Predator]);
```

Funding

This research received no external funding.

Acknowledgements

Authors contribution

Azhi Sabir contributed in the original draft preparation, methodology, formal analysis. Hiwa Abdullah contributed in the writing, validation, review and editing. Awder Rasul contributed in the computational simulations, formal analysis. Surme Rasul contributed in validation, methodology, formal analysis.

Conflict of interests

The authors declare no conflicts of interest.

References

1. Rahmani Doust, M. and S. GHolizade, *The Lotka-Volterra Predator-Prey Equations.* Caspian Journal of Mathematical Sciences (CJMS), 2014. **3**(2): p. 221-225.
2. Ghosh, M., P. Chandra, and P. Sinha, *A mathematical model to study the effect of toxic chemicals on a prey-predator type fishery.* Journal of Biological Systems, 2002. **10**(02): p. 97-105.
3. Ang, T.K., H.M. Safuan, and J. Kavikumar, *The impacts of harvesting activities on prey-predator fishery model in the presence of toxin.* Journal of Science and Technology, 2018. **10**(2).
4. Rahmani, D.M. and M. Saraj, *The logistic modeling population: Having harvesting factor.* Yugoslav Journal of Operations Research, 2015. **25**(1): p. 107-115.
5. Rahmani, D. M. & Saraj, M. *The logistic modeling population: Having harvesting factor.* Yugoslav Journal of Operations Research, 25, 107-115. 2015
6. Das, T., Mukherjee, R. & Chaudhuri, K.*Harvesting of a prey–predator fishery in the presence of toxicity.* Applied Mathematical Modelling, *33, 2282-2292.* 2009
7. Yalong Xue, Fengde Chen, Xiangdong Xie, Shengjiang Chen. An analysis of a predator-prey model in which fear reduces prey birth and death rates[J]. AIMS Mathematics, 2024, 9(5): 12906-12927. doi: 10.3934/math.2024630
8. Ilyasse Lamrani, Imad El Harraki, M. A. Aziz-Alaoui, Fatima-Zahrae El Alaoui. Feedback stabilization for prey predator general model with diffusion via multiplicative controls[J]. AIMS Mathematics, 2023, 8(1): 2360-2385. doi: 10.3934/math.2023122
9. McCluskey, C.C., P.J.J.o.D. van den Driessche, and D. Equations, *Global analysis of two tuberculosis models.* 2004. **16**(1): p. 139-166.
10. Saraj, M., M. Doust, and F. Haghighifar, *The stability of Gauss model; having harvested factor.* 2012.