Solving Multi-Objective Linear Programming Problem by Using Revised Simplex Method and S.N. Advanced Approach

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Abstract:

This study introduces a Revised Simplex approach for solving linear programming problems (SLPP) and proposes the Shaida, Nejmaddin, Advanced(S. N. Advanced Technique), which converts multi-objective linear programming problems (MOLPP) into single-objective linear programming problems (SOLPP). An algorithm was developed specifically for the S.N. Advanced Technique to address MOLPPs, and an updated version of Chandra Sen's method was integrated with a new algorithm for solving such problems. This is the first time this road has been worked on, and the first time the algorithm for the Chandra Sen's method has been developed. The proposed methods were validated through extensive numerical testing, demonstrating their effectiveness and competitiveness compared to existing approaches, highlighting their practicality and efficiency in addressing complex optimization challenges.

Keyword: Revised simplex method, S.N.Advanced technique, Multi objective function, applications.

1. Introduction

Linear programming, first introduced in the late 1940s, has become a fundamental technique for optimization in fields such as economics, engineering, logistics, and communications. The development of the Simplex Method marked a significant milestone, revolutionizing optimization practices. However, the classical Simplex Method encountered limitations in computational efficiency and numerical stability, particularly for large-scale problems.

To address these challenges, Fiasco and McCormick proposed the Revised Simplex Method (RSM) in 1965 [1]. By 1983, multi-objective optimization (MOO) had been developed as an effective approach to addressing problems that have several objectives. Its principles and applications have been thoroughly investigated [2]. However, MOO is a computationally expensive approach, and the computational complexity increases with the number of objectives. For the above reasons, numerous solution techniques have been proposed [4] – [15]. RSM was developed to improve the classical simplex algorithm to include more sophisticated characteristics such as rotational updates and sparse matrix techniques for large systems without sacrificing the ability to find optimal solutions at polyhedron vertices.

In 2006, a new approach was proposed to transform (MOLPP) problems into (SOLPP) [4]. This was followed by further developments, including the use of mean and median values in 2010 [6]. The research has been carried out to investigate both theoretical and applied aspects of solution methods. Some other methods of solving multi-objective linear programming problems (MOLPP) include Sulaiman and Sadiq proposed Mean and Median Method [4], Sulaiman & Ameen applied Optimal Transformation Technique [7], Harmonic Mean Method introduced by Sulaiman and Mustafa [8], and Nahar and Alimin proposed New Statistical Average Method [10].

As problem sizes increase, shortcomings in solution generation and slower algorithmic convergence become more apparent. To address these problems, this paper proposes an improved transformation strategy for MOLPP. The proposed method is compared with other methods such as Chandra Sen’s method and other transformation methods to show that it is more efficient and simpler.

This paper proposes the use of the Revised Simplex Method to solve MOLPP by converting multi-objective problems into single-objective problems. This approach has been improved over the years, and the most recent contributions are those of Nejmaddin A. Sulaiman and Shaida O. Muhammad in 2024 [18]. The newly suggested S.N. Advanced Transformation Technique provides a very fast solution that can be used for a wide range of problems. Detailed comparisons, physical interpretations, and data analyses underscore the method’s practicality and effectiveness, confirming its potential as a valuable tool for optimization.

2. Multi-Objective Linear Programming Problem (MOLPP)

MOLPP is a branch of multiple-criterion decision-making that focuses on mathematical OP where more than one goal function needs to be enhanced at the same time.

Mathematically **(MOLPP)** defined as follows:

$$\text{Max or Min } f_i = C_i^T x$$

Subject to

$$Ax \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} B$$

$$x \geq 0$$

$i = 1, \dots, r$ for max. and $i = r + 1, \dots, s$ for min.

where x ={x_1 , x_2, x_3,… x_n } of decision variables, P is $a\ (n \times n)$ symmetrical matrix of constants. A is $(m \times n)$ matrix of constants, B = mx1 matrix and C=$1 \times n$ matrix are n-dimensional vectors' of coefficients[6].

3. Shaida, Nejmaddin, Advanced (S. N. Advanced Technique)

(S.N.Advanced) this is one of the new methods that no one has used yet and we are the first to work on it. Therefore,the name advanced is a change in his work and a way forward and the latest way to analyze Multi-Objective Optimization Problem.

The different idea of using this function is that the optimal point is different from the others.

4. Approaches for solving MOLPP

4.1. Chandra Sen's Method

A multi-objective programming is structured and enhanced subject to shared constraints.

The mathematical representation is given as:

$$\text{Optimize } Z = [\text{Max } z_1, \text{Max } z_2, \dots, \text{Max } z_r, \text{Min } z_{r+1}, \dots, \text{Min } z_s]$$

Subject to

$$Ax \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b$$

$$x \geq 0 \quad \text{.................(*)}$$

In this approach, each objective function must be either maximized or minimized independently using the Simplex method or another technique. This process results in finding the optimal values for each goal function individually as:

$$Z_{\text{optima}} = \{\alpha_1, \alpha_2, \dots, \alpha_s\}$$

The ideal value of the goal function $\alpha_i (i = 1,2, \dots, s)$ may be positive or negative.

These values are combined to create a single goal function by addition (the supreme values) and deducting (the smallest values) for each outcome, then separating each z_i by α_i . The function is expressed as:

$$\text{Max } Z = \sum_{i=1}^{r} \frac{z_i}{|\alpha_i|} - \sum_{i=r+1}^{s} \frac{z_i}{|\alpha_i|}$$

Subject to

$$Ax \begin{bmatrix} \leq \\ = \\ \geq \end{bmatrix} b$$

$$x \geq 0$$

$\alpha_i \neq 0$ for i=1, 2,⋯, s .

where, α_i is the optimum value of ith goal function.

The resulting single-objective optimization problem is then answered using the Revised simplex technique, a technique identified as Chandra Sen's method.

4.2. S. N. Advanced Transformation Approach

A multi-objective optimization problem can be defined as:

$$\text{Max or Min } [z_1, z_2, \dots, z_s]$$

S. to

$$Ax\{\geq, =, \leq\}b, x \geq 0 \quad \text{................(**)}$$

Imagine we optimize each objective function separately and acquire the corresponding values:

$$\text{Max } z_1 = \alpha_1$$

$$\text{Max } z_2 = \alpha_2$$

.
.
.

$$\text{Min } z_{r+1} = \alpha_{r+1}$$

.
.
.

$$\text{Min } z_s = \alpha_s$$

Wherever α_i are the values of goal functions.

We need a combination of different decision criteria to represent the best consensus solution. The identified set of different choice variables can be determined using the following objective function.

Using our transformation algorithm above, we can gain a single goal function like this:

$$\text{Max Z} = \frac{\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i}{O_{AT}}$$

Let $O_{AT} = m$= min$\{m_1, m_2, \dots, m_r, m_{r+1}, \dots, m_s\}$; where $m_i = |\alpha_i|$, i=1,…,s

Under the same constraints as outlined in (**).

5. Algorithms

5.1 Algorithm for Chandra Sen's Approach

Step 1: Determine the value of each goal function that needs to be either max. or min.

Step 2: Solve the 1st goal function using Revised simplex technique.

Step 3: Label the optimal value of the 1st objective function $z_1$by α_1.

Step 4: Replication step-2 for i=2,3,…,s .

Step 5: Determine Optimal Z= [Max z_1, Max z_2,…,Max z_r,Min z_{r+1},…,Min z_s].

Step 6: Select Z $_{optima}$ = $\{\alpha_1, \alpha_2, \dots, \alpha_s\}$.

Step 7: Optimize the collective purposes function as:

$$\text{Max } Z = \sum_{i=1}^{r} \frac{z_i}{\alpha_i} - \sum_{i=r+1}^{s} \frac{z_i}{\alpha_i}$$

Below the same restraints by repetition Steps 2 & 3.

5.2 Algorithm for S. N. Advanced Technique

Step 1: Determine the value of individual goal function that needs to be either max. or min.

Step 2: Solve the 1st goal function using RSM.

Step 3: Label the optimal value of the first goal function z_1 by α_1.

Step 4: Replication step-2 for i=2,3,…,s .

Step 5: Choice m=min $\{m_1, m_2, \dots, m_r, m_{r+1}, \dots, m_s\}$, where $m_i = |\alpha_i|$, i=1,…, s

Step 6: Choice m=min $\{m_1, m_2, \dots, m_r, m_{r+1}, \dots, m_s\}$ and calculate $O_{AT} = \frac{1}{1/m}$.

Step 7: Optimize the combined goal function as

$$\text{Max } Z = \frac{\sum_{i=1}^{r} z_i - \sum_{i=r+1}^{s} z_i}{O_{AT}}$$

Below the same restraints by repetition, Steps 2 & 3.

5. Example:

Assume the following Multi-Objective Linear Programming problem:

Max $Z_1 = 5x_1 + 3x_2$

Max $Z_2 = 9x_1 + 5x_2$

Max $Z_3 = 3x_1 - 4x_2$

Max $Z_4 = 3x_1 + 2x_2$

Subject to

$x_1 + x_2 \leq 3$

$2x_1 + x_2 \leq 5$

$x_1 \leq 2$

$x_1 , x_2 \geq 0$

Solution:

1- The given problematic in the revised simplex method for first objective function form may be expressed by presenting the slack variables s_1, s_2 and s_3 as:

$$\text{Max } Z_1 = 5x_1 + 3x_2$$

Subject to

$x_1 + x_2 \leq 3$

$2x_1 + x_2 \leq 5$

$x_1 \leq 2$

$x_1 , x_2 \geq 0$

$$\text{Max } Z = 5x_1 + 3x_2 + 0s_1 + 0s_2+0s_3$$

s.t. $x_1+ x_2 + s_1 = 3$

$2x_1 + x_2 + s_2 = 5$

$x_1+s_3=2$

$x_1 , x_2 , s_1 , s_2 , s_3 \geq 0$

And $Z - 5x_1-3x_2+0s_1+0s_2+0s_3= 0$

$x_1+x_2 + s_1 = 3$

$2x_1+x_2 + s_2 = 5$

$x_1 \quad +s_3=2$

$x_1 , x_2 , s_1 , s_2 , s_3 \geq 0$

The system of constraint equations may be represented in the following matrix form:

$$\begin{bmatrix} 1 & -5 & -3 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 5 \\ 2 \end{bmatrix}$$

Determine to first table in Revised simplex method (RSM):

Basic variable	B_0	B_1	B_2	B_3	x_B	x_k	Ratio x_B/x_k		a_1	a_2
Z	1	0	0	0	0	-5	-		-5	-3
s_1	0	1	0	0	3	1	3		1	1
s_2	0	0	1	0	5	2	2.5		2	1
← s_3	0	0	0	1	2	1	2←		1	0

$$Z=(1,0,0,0) * \begin{bmatrix} -5 & -3 \\ 1 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \min\{-5,-3\}=-5$$

Final table in Revised simplex method (RSM):

Basic variable	B_0	B_1	B_2	B_3	x_B	x_k	Ratio x_B/x_k		a_1	a_2
Z	1	3	0	2	13				0	0
x_2	0	1	0	-1	1				0	1
s_2	0	-1	1	-1	0				0	0
← x_1	0	0	0	1	2				1	0

$$Z=(1,3,0,2) * \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} = \min\{2, 3\} \geq 0$$. It is optimal solution.

$x_1 = 2$, $x_2 = 1$ and $Z_1=Q_1=13$

Thus Max $Z_1 = 13$ at (2, 1).

- After discovery the value of apiece of individual goal function by using Revised simplex method (RSM), the numerical outcomes are given in **Table 1**.

Table1. Numerical results for given example.

I	Z_i	$x_i(x_1,x_2)$	Q_i	OA_i
1	13	(2,1)	13	13
2	23	(2,1)	23	23
3	6	(2,,0)	6	6
4	8	(2,1)	8	8

- **Applied Chandra Sen's Approach of this example :**

By Sen's,Ch. Approach, and by Table 1.

$$\text{Max } Z = \sum_{k=1}^{2} \frac{Z_k}{|Q_k|} - \sum_{k=3}^{4} \frac{Z_k}{|Q_k|}$$

$$\text{Max } Z = \frac{Z_1}{Q_1} + \frac{Z_2}{Q_2} - \frac{Z_3}{Q_3} - \frac{Z_4}{Q_4}$$

$$= \frac{5x1 + 3x2}{13} + \frac{9x1 + 5x2}{23} - \frac{3x1 - 4x2}{6} - \frac{3x_1 + 2x_2}{8} = 1.64\, x_1 + 0.04\, x_2$$

Using Sen's, Ch. Approach, the structure converts,

Max $Z = 1.64\, x_1 + 0.04\, x_2$

Subject to

$x_1 + x_2 \leq 3$

$2x_1 + x_2 \leq 5$

$x_1 \leq 2$

$x_1 , x_2 \geq 0$

Solving by Revised simplex method (RSM):

Max $Z = 1.64x_1 + 0.04x_2 + 0s_1 + 0s_2+0s_3$

s.t. $x_1+ x_2 + s_1 = 3$

$$2x_1 + x_2 + s_2 = 5$$

$$x_1+s_3=2$$

$$x_1 , x_2 , s_1 , s_2 , s_3 \geq 0$$

And $Z - 1.64x_1-0.04x_2+0s_1+0s_2+0s_3= 0$

$$x_1+x_2 + s_1 = 3$$

$$2x_1+x_2 + s_2 = 5$$

$$x_1 \quad +s_3=2$$

$$x_1 , x_2 , s_1 , s_2 , s_3 \geq 0$$

The system of constraint equations may be represented in the following matrix form:

$$\begin{bmatrix} 1 & -1.64 & -0.04 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 5 \\ 2 \end{bmatrix}$$

Determine to first table in Revised simplex method (RSM):

Basic variable	B_0	B_1	B_2	B_3	x_B	x_k	Ratio x_B/x_k	a_1	a_2
Z	1	0	0	0	0	-1.64	-	-1.64	-0.04
s_1	0	1	0	0	3	1	3	1	1
s_2	0	0	1	0	5	2	2.5	2	1
← s_3	0	0	0	1	2	1	2←	1	0

$$Z=(1,0,0,0) * \begin{bmatrix} -1.64 & -0.04 \\ 1 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \min\{-1.64,-0.04\} = -1.64$$

Final table in Revised Simplex Method (RSM):

Basic variable	B_0	B_1	B_2	B_3	x_B	x_k	Ratio x_B/x_k		a_1	a_2
Z	1	0.04	0	1.6	3.32				0	0
x_2	0	1	0	-1	1				0	1
s_2	0	0	1	-1	0				0	0
← x_1	0	0	0	1	2				1	0

Z=(1,0.04,0,1.6) * $\begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$ = min{1.6, 0.04} $\geq$ 0 . It is an optimal solution .

Max Z $_{Optimal}$ = 3.32 at (2,1)

- **Applied S.N.Advanced Technique of this example:**

Using the proposed Technique, and by Table 1.

m_i =min$|\alpha_i|$ where i=1,…,4 and let $Q_i = m_i$

m=min $\{m_1, m_2, m_3, m_4\}$

m= min $\{13,23,6,8\}$ = 6

Max Z = $\frac{\sum_{i=1}^{4} z_i}{O_{AT}} = \frac{5x1 + 3x2 + 9x1 + 5x2 + 3x1 - 4x2 + 3x1 + 2x2}{6}$ = 3.33 $x_1 + x_2$

Max Z = 3.33 $x_1 + x_2$

Subject to

$x_1 + x_2 \leq 3$

$2x_1 + x_2 \leq 5$

$x_1 \leq 2$

$x_1 , x_2 \geq 0$

Solving by Revised Simplex Method (RSM):

Max $Z = 3.33x_1 + x_2 + 0s_1 + 0s_2 + 0s_3$

s.t. $x_1 + x_2 + s_1 = 3$

$2x_1 + x_2 + s_2 = 5$

$x_1 \quad + s_3 = 2$

$x_1 , x_2 , s_1 , s_2 , s_3 \geq 0$

And $Z - 3.33x_1 - x_2 + 0s_1 + 0s_2 + 0s_3 = 0$

$x_1 + x_2 + s_1 = 3$

$2x_1 + x_2 + s_2 = 5$

$x_1 \quad + s_3 = 2$

$x_1 , x_2 , s_1 , s_2 , s_3 \geq 0$

The system of constraint equations may be represented in the following matrix form:

$$\begin{bmatrix} 1 & -3.33 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} Z \\ x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 5 \\ 2 \end{bmatrix}$$

Determine to first table in Revised Simplex Method (RSM):

Basic variable	B_0	B_1	B_2	B_3	x_B	x_k	Ratio x_B/x_k		a_1	a_2
Z	1	0	0	0	0	-3.33	-		-3.33	-1
s_1	0	1	0	0	3	1	3		1	1
s_2	0	0	1	0	5	2	2.5		2	1
← s_3	0	0	0	1	2	1	2←		1	0

$$Z=(1,0,0,0) * \begin{bmatrix} -3.33 & -1 \\ 1 & 1 \\ 2 & 1 \\ 1 & 0 \end{bmatrix} = \min\{-3.33,-1\} = -3.33$$

Final table in Revised Simplex Method (RSM):

Basic variable	B_0	B_1	B_2	B_3	x_B	x_k	Ratio x_B/x_k		a_1	a_2
Z	1	1	0	2.33	7.66				0	0
x_2	0	1	0	-1	1				0	1
s_2	0	-1	1	-1	0				0	0
← x_1	0	0	0	1	2				1	0

$$Z=(1,1,0,2.33) * \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix} = \min\{2.33, 1\} \geq 0$$. It is an optimal solution.

Max $Z_{Optimal} = 7.66$ at (2,1)

6. Results:

In this study that we are working on and as much as we have researched, Because the path to analyzing reservations is very long, we have only emphasized one multi-example, including the oldest and the latest the path in my name is (S.N.Advanced Technique). If we look closely, both approaches become optimal at the same point, but the difference is in the maximum price.

We got very good results with a very clear and beautiful difference between them.

As shown in **Table 2** below.

7. Results comparison between Chandra Sen's Techniques and S.N.Advanced Technique

Table 2.

Example	**Chandra Sen's Techniques**	**S.N.Advanced Technique**
Example	$Z_{opt.} = 3.33$ $x_1 = 2$ $x_2 = 1$	Z opt. = 7.66 $x_1 = 2$ $x_2 = 1$

8. Conclusion:

The study revealed that the results from the numerical examples were identical for both the traditional Chandra Sen Techniques and the S.N. Advanced Techniques, as shown in Table 2. However, the results from the S.N. Advanced Techniques were achieved in less time compared to the traditional methods. Therefore, we conclude that the S.N. Advanced Techniques are more efficient for application.

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