Constract A New Ballistic Missile model by Line Graph and Measuring its Reliability

Amnah Waheed Sabr [1, a)] and Abeer A. Abdul-Raza [2, b)]

[1] *College of Education for Pure Sciences Thi-Qar University*
[2] *Mathematics department College of Education for Pure sciences Thi-Qar University*

a) amina waheed@utq.edu.iq
b) abeeraladub80@gmail.com

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Abstract:

This study presented a mathematical approach for analysing the reliability of a ballistic missile system using the Line Graph technique from graph theory. The missile structure was modelled by representing the relationships between its main components such as the missile body, propulsion system, fuel system, payload, control unit, guidance system, and fins. The reduction method was applied to simplify the complex network of components and identify the critical paths within the system.

Several reliability measures were employed to evaluate the system performance, including system reliability, Mean Time to Failure (MTTF), hazard rate, Shannon entropy, and extropy. These measures provide a theoretical framework for understanding system behaviour and uncertainty in failure processes. The use of line graphs also helped visualize the structural relationships between components and analyze reliability behavior over time.
Overall, the proposed model demonstrates how graph theory and reliability techniques can be combined to study complex engineering systems and support the improvement of system design and performance under operational condition

Introduction:

Ballistic missiles represent the pinnacle of modern military technology, following a parabolic trajectory driven by gravity after the initial boost phase, making reliability analysis critical for operational success.

Ballistic missiles consist of interconnected components such as the missile body (BM),propulsion system (PS),fuel (Fuel), payload, warhead(WH), control unit (CU), fins (Fins),and guidance/sensors (GS), operating in phases: boost midcourse ballistic trajectory, and re-entry

This study employs the Line Graph model from graph theory to represent these components, where each edge in the original graph becomes a vertex, and vertices are adjacent if their corresponding edges share a common endpoint; for example, BM-PS connects to PS-Fuel, clearly illustrating structural interdependenice. The model aims to compute system reliability (R_{sys}) through complex statistical equations like $R_{sys} = 1 - \left(1 - R_{Fuel-payload}\right)(1 - R_{OC-Fins})(1 - R_{GS-Sensors})$, focusing on line graph to depict time-dependent decline, MTTF, hazard rate, and entrpy , ultimately optimizing design under harsh conditions.

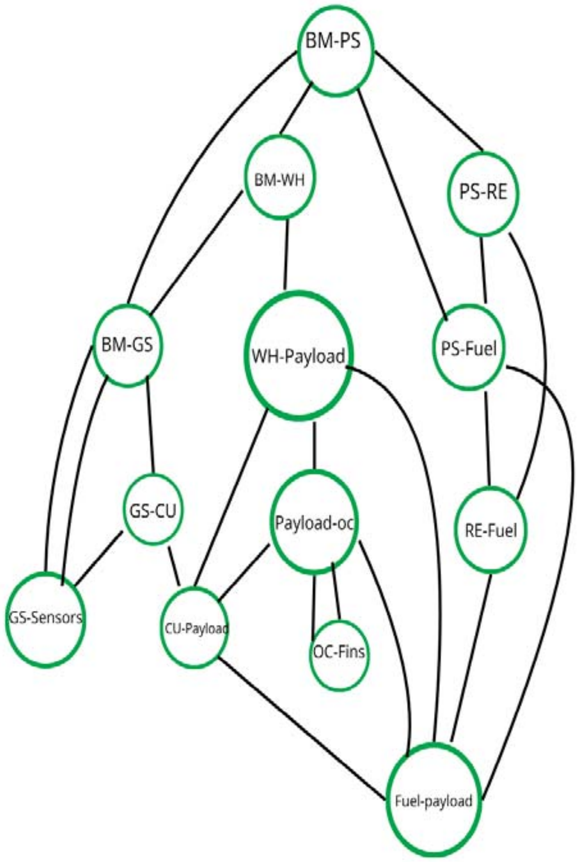

Figure 1 (Balistic Missile Structure)

BASIC CONCEPT

1. System reliability refers to the likelihood that a system will operate without failure over a specific time period.
2. A minimal path consists of components that together form a valid path, but removing any one of them breaks the path's functionality.
3. In a parallel system, the overall reliability is achieved as long as at least one component functions properly :

$$R_{sys} = 1 - \prod_{i=1}^{n}(1 - R_i) \qquad (1)$$

4. Series System
   The reliability of a series system is the probability that component succeeds and component succeeds and all of the other components in the system succeed and it compute

$$R_{sys} = \prod_{i=1}^{n} R_i \qquad (2)$$

5. Series-parallel system
   It is a system that consists of different subsystems connected in series. Each subsystem consists of components connected in parallel as shown in Figure.1.11. The reliability of the system is

   $R_S = \prod_{k=1}^{m}\left(1 - \prod_{i=1}^{n_k}(1 - R_{ik})\right)$ (3)

6. Shanoon Entropy
   Shannon's entropy quantifies the amount of information in a variable, thus providing the foundation for a theory around the notion of information.

   Shannon entropy is defined by

$$H = -E(\log\left(p(x)\right) \qquad (4)$$

   Here, $p(x)$ represents the likelihood that the system occupies cell x within its phase space.

7. Extropy
   The dual measure of entropy has become wides it is known as extropy and defined

$$J(X) = J(P) = -\sum_{i=1}^{n}(1 - p)\log(1 - p_i) \qquad (5)$$

   Where p_i is the probability densety function [11,12]

Reduction Method for Reliability Analysis in the Line Graph Model

The reduction method is used to simplify the reliability analysis of complex systems represented by a line graph. In this model, each edge of the original missile structure becomes a vertex, and connections between vertices represent interactions between missile components.

To analyse the ballistic missile system, minimal paths connecting the main components were identified. Components arranged in series were reduced by multiplying their reliabilities, while parallel components were replaced by equivalent reliability expressions. By applying these reductions step by step, the complex line graph of the missile system is simplified until it becomes a single equivalent reliability function representing the overall reliability of the system.

After applied the reduction method

Structured Processes

BM-PS → PS-Fuel → Fuel-payload
P_1: $x_2\ x_9$

BM-PS → PS-RE → RE-FueL → Fuel-payload
P_2: $x_1x_5x_{16}$

BM-PS → BM-WH → WH-payload → Fuel-payload
P_3: $x_3\ x_7\ x_{11}$

BM-PS → BM-WH → WH-payload → CU-payload → Fuel-payload
P_4: $x_3\ x_7\ x_{13}\ x_{22}$

BM-PS → BM-WH → WH-payload → payload-OC → OC-Fins
P_5: $x_3x_7x_{12}x_{17}$

BM-PS → BM-GS → GS-CU → CU-payload → Fuel-payload
P_6: $x_4\ x_{14}\ x_{20}\ x_{22}$

BM-PS → BM-GS → GS-Sensors
P_7: $x_4\ x_{15}$

BM-PS → BM-WH → BM-GS → GS-CU → GS-Sensors
P_8: $x_3\ x_8\ x_{14}\ x_{21}$

Proposition

Assume the reliability of Balisty Missile

$$R_{sys} = 1 - \left(1 - R_{Fuel-paylod}\right)(1)(1 - R_{GS-Sensors})$$

$$= 1 - [(1 - R^2)(1 - R^3)(1 - R^3)(1 - R^4)(1 - R^4)](1 - R^4)[(1 - R^2)(1 - R^4)]$$

$$=1 - [(1 - R^3 - R^2 + R^5)(1 - R^4 - R^3 + R^7)(1 - R^4)](1 - R^4)[1 - R^4 - R^2 + R^6] =$$

$$= 1 - [1 - R^2 - 2R^3 - 2R^4 + 2R^5 + 3R^6 + 4R^7 - 4R^9 - 3R^{10} - 2R^{11} + 2R^{12} + 2R^{13} + R^{14} - R^{16}$$
$$- 2R^4 + 2R^6 + 4R^7 + 4R^8 - 4R^9 - 6R^{10} - 8R^{11} + 8R^{13} + 6R^{14} + 4R^{15} - 4R^{16}$$
$$- 4R^{17} - 2R^{18} + 2R^{20} - R^2 + R^4 + 2R^5 + 2R^6 - 2R^7 - 3R^8 - 4R^9 + 4R^{11} + 3R^{12}$$
$$+ 2R^{13} - 2R^{14} - 2R^{15} - R^{16} + R^{18} + 2R^6 - 2R^8 - 4R^9 - 4R^{10} + 4R^{11} + 6R^{12}$$
$$+ 8R^{13} - 8R^{15} - 6R^{16} - 4R^{17} + 4R^{18} + 4R^{19} + 2R^{20} - 2R^{22} + R^8 - R^{10} - 2R^{11}$$
$$- 2R^{12} + 2R^{13} + 3R^{14} + 4R^{15} - 4R^{17} - 3R^{18} - 2R^{19} + 2R^{20} + 2R^{21} + R^{22} - R^{24}$$
$$- R^{10} + R^{12} + 2R^{13} + 2R^{14} - 2R^{15} - 3R^{16} - 4R^{17} + 4R^{19} + 3R^{20} + 2R^{21} - 2R^{22}$$
$$- 2R^{23} - R^{24} + R^{26}]$$

$$= 2R^2 + 2R^3 + 3R^4 - 4R^5 - 9R^6 - 6R^7 + 16R^9 + 15R^{10} + 4R^{11} - 10R^{12} - 24R^{13} - 10R^{14}$$
$$+ 4R^{15} + 15R^{16} + 16R^{17} - 6R^{19} - 9R^{20} - 4R^{21} + 3R^{22} + 2R^{23} + 2R^{24} - R^{26}$$

When $R = 0.5 \quad then$

$R_{sys} = 0.7$

$$R_{sys} = 2e^{-2\lambda x} + 2e^{-3\lambda x} + 3e^{-4\lambda x} - 4e^{-5\lambda x} - 9e^{-6\lambda x} - 6e^{-7\lambda x} + 16e^{-9\lambda x} + 15e^{-10\lambda x} + 4e^{-11\lambda x}$$
$$- 10e^{-12\lambda x} - 24e^{-13\lambda x} - 10e^{-14\lambda x} + 4e^{-15\lambda x} + 15e^{-16\lambda x} + 16e^{-17\lambda x} - 6e^{-19\lambda x}$$
$$- 9e^{-20\lambda x} - 4e^{-21\lambda x} + 3e^{-22\lambda x} + 2e^{-24\lambda x} - e^{-26\lambda x}$$

Mean Time to Failure (MTTF)

The mean time to failure is defined by

$$MTTF = \int_0^t \big(2e^{-2\lambda x} + 2e^{-3\lambda x} + 3e^{-4\lambda x} - 4e^{-5\lambda x} - 9e^{-6\lambda x} - 6e^{-7\lambda x} + 16e^{-9\lambda x} + 15e^{-10\lambda x}$$
$$+ 4e^{-11\lambda x} - 10e^{-12\lambda x} - 24e^{-13\lambda x} - 10e^{-14\lambda x} + 4e^{-15\lambda x} + 15e^{-16\lambda x} + 16e^{-17\lambda x}$$
$$- 6e^{-19\lambda x} - 9e^{-20\lambda x} - 4e^{-21\lambda x} + 3e^{-22\lambda x} + 2e^{-24\lambda x} - e^{-26\lambda x}\big)dx$$

$$= -\frac{2}{2\lambda}e^{-2\lambda x} - \frac{2}{3\lambda}e^{-3\lambda x} - \frac{3}{4\lambda}e^{-4\lambda x} + \frac{4}{5\lambda}e^{-5\lambda x} + \frac{9}{6\lambda}e^{-6\lambda x} + \frac{6}{7\lambda}e^{-7\lambda x} - \frac{16}{9\lambda}e^{-9\lambda x} - \frac{15}{10\lambda}e^{-10\lambda x} - \frac{4}{11\lambda}e^{-11\lambda x} + \frac{10}{12\lambda}e^{-12\lambda x} + \frac{24}{13\lambda}e^{-13\lambda x} + \frac{10}{14\lambda}e^{-14\lambda x} - \frac{4}{15\lambda}e^{-15\lambda x} - \frac{15}{16\lambda}e^{-16\lambda x} - \frac{16}{17\lambda}e^{-17\lambda x} + \frac{6}{19\lambda}e^{-19\lambda x} + \frac{9}{20\lambda}e^{-20\lambda x} + \frac{4}{21\lambda}e^{-21\lambda x} - \frac{3}{22\lambda}e^{-22\lambda x} - \frac{2}{24\lambda}e^{-24\lambda x} + \frac{1}{26\lambda}e^{-26\lambda x}$$

$$= \left[-\frac{1}{\lambda}e^{-2\lambda t} - \frac{2}{3\lambda}e^{-3\lambda t} - \frac{3}{4\lambda}e^{-4\lambda t} + \frac{4}{5\lambda}e^{-5\lambda t} + \frac{3}{2\lambda}e^{-6\lambda t} + \frac{6}{7\lambda}e^{-7\lambda t} - \frac{16}{9\lambda}e^{-9\lambda t} - \frac{3}{2\lambda}e^{-10\lambda t} - \frac{4}{11\lambda}e^{-11\lambda t} + \frac{5}{6\lambda}e^{-12\lambda t} + \frac{24}{13\lambda}e^{-13\lambda t} + \frac{5}{7\lambda}e^{-14\lambda t} - \frac{4}{15\lambda}e^{-15\lambda t} - \frac{15}{16\lambda}e^{-16\lambda t} - \frac{16}{17\lambda}e^{-17\lambda t} + \frac{6}{19\lambda}e^{-19\lambda t} - \frac{9}{20\lambda}e^{-20\lambda t} + \frac{4}{21\lambda}e^{-21\lambda t} - \frac{3}{22\lambda}e^{-22\lambda t} - \frac{1}{12\lambda}e^{-22\lambda t} - \frac{1}{26\lambda}e^{-26\lambda t}\right] - \left[-\frac{1}{\lambda}e^{0} - \frac{2}{3\lambda}e^{0} - \frac{3}{4\lambda}e^{0} + \frac{4}{5\lambda}e^{0} + \frac{3}{2\lambda}e^{0} + \frac{6}{7\lambda}e^{0} - \frac{16}{9\lambda}e^{0} - \frac{3}{2\lambda}e^{0} - \frac{4}{11\lambda}e^{0} + \frac{5}{6\lambda}e^{0} + \frac{24}{13\lambda}e^{0} + \frac{5}{7\lambda}e^{0} - \frac{4}{15\lambda}e^{0} - \frac{15}{16\lambda}e^{0} - \frac{16}{17\lambda}e^{0} + \frac{6}{19\lambda}e^{0} + \frac{9}{20\lambda}e^{0} + \frac{4}{21\lambda}e^{0} - \frac{3}{22\lambda}e^{0} - \frac{1}{12\lambda}e^{0} + \frac{1}{26\lambda}e^{0}\right]$$

When $\lambda = 1,\quad t = 0.5$

$MTTF = 0.402$

$$f(x) = -dR(x)$$

$$= -\left(-4\lambda e^{-2\lambda x} - 6\lambda e^{-3\lambda x} - 12\lambda e^{-4\lambda x} + 20\lambda e^{-5\lambda x} + 54\lambda e^{-6\lambda x} + 42\lambda e^{-7\lambda x} - 144\lambda e^{-9\lambda x} - 150\lambda e^{-10\lambda x} - 44\lambda e^{-11\lambda x} + 120\lambda e^{-12\lambda x} + 312\lambda e^{-13\lambda x} + 140\lambda e^{-14\lambda x} - 60\lambda e^{-15\lambda x} - 240\lambda e^{-16\lambda x} - 272\lambda e^{-17\lambda x} + 144\lambda e^{-19\lambda x} + 180\lambda e^{-20\lambda x} + 84\lambda e^{-21\lambda x} - 66\lambda e^{-22\lambda x} - 48\lambda e^{-24\lambda x} + 26\lambda e^{-26\lambda x}\right)$$

COMPUTING THE ENTROPY OF Balisty Missile

In this section will compute the Shannon entropy of Balisty Missile

In equation ( ) the Shannon entropy is

$$H = -E(\log(p(x))$$

By substitute equation ( ) in equation ( ) gets

$$= -\int_0^\infty f(t)\log\big(f(t)\big)\,dt$$

$$= -\int_0^\infty \big[4\lambda e^{-2\lambda t} + 6\lambda e^{-3\lambda t} + 12\lambda e^{-4\lambda t} - 20\lambda e^{-5\lambda t} - 54\lambda e^{-6\lambda t} - 42\lambda e^{-7\lambda t} + 144\lambda e^{-9\lambda t} + 150\lambda e^{-10\lambda t} + 44\lambda e^{-11\lambda t} - 120\lambda e^{-12\lambda t} - 312\lambda e^{-13\lambda t} - 140\lambda e^{-14\lambda t} + 60\lambda e^{-15\lambda t} + 240\lambda e^{-16\lambda t} + 272\lambda e^{-17\lambda t} - 144\lambda e^{-19\lambda t} - 180\lambda e^{-20\lambda t} - 84\lambda e^{-21\lambda t} + 66\lambda e^{-22\lambda t} + 48\lambda e^{-24\lambda t} - 26\lambda e^{-26\lambda t}\big] * \log\big[4\lambda e^{-2\lambda t} + 6\lambda e^{-3\lambda t} + 12\lambda e^{-4\lambda t} - 20\lambda e^{-5\lambda t} - 54\lambda e^{-6\lambda t} - 42\lambda e^{-7\lambda t} + 144\lambda e^{-9\lambda t} + 150\lambda e^{-10\lambda t} + 44\lambda e^{-11\lambda t} - 120\lambda e^{-12\lambda t} - 312\lambda e^{-13\lambda t} - 140\lambda e^{-14\lambda t} + 60\lambda e^{-15\lambda t} + 240\lambda e^{-16\lambda t} + 272\lambda e^{-17\lambda t} - 144\lambda e^{-19\lambda t} - 180\lambda e^{-20\lambda t} - 84\lambda e^{-21\lambda t} + 66\lambda e^{-22\lambda t} + 48\lambda e^{-24\lambda t} - 26\lambda e^{-26\lambda t}\big]dt$$

$when \quad \lambda = 1, \quad t = 0.5$

$H = 0.11165$

COMPUTING THE EXTROPY OF OF Balisty Missile

The dual measure of entropy has become wides it is known as extropy, so to compute the extropy of Balisty Missile by substitute equation ( ) in equation ( ) gets

$$J = -\int_0^\infty \big(1 - f(t)\big)\ \log\big(1 - f(t)\big)\,dt$$

$$= -\int_0^\infty \big[1 - \big(4\lambda e^{-2\lambda t} + 6\lambda e^{-3\lambda t} + 12\lambda e^{-4\lambda t} - 20\lambda e^{-5\lambda t} - 54\lambda e^{-6\lambda t} - 42\lambda e^{-7\lambda t} + 144\lambda e^{-9\lambda t} + 150\lambda e^{-10\lambda t} + 44\lambda e^{-11\lambda t} - 120\lambda e^{-12\lambda t} - 312\lambda e^{-13\lambda t} - 140\lambda e^{-14\lambda t} + 60\lambda e^{-15\lambda t} + 240\lambda e^{-16\lambda t} + 272\lambda e^{-17\lambda t} - 144\lambda e^{-19\lambda t} - 180\lambda e^{-20\lambda t} - 84\lambda e^{-21\lambda t} + 66\lambda e^{-22\lambda t} + 48\lambda e^{-24\lambda t} - 26\lambda e^{-26\lambda t}\big)\big] * \log\big[1 - \big(4\lambda e^{-2\lambda t} + 6\lambda e^{-3\lambda t} + 12\lambda e^{-4\lambda t} - 20\lambda e^{-5\lambda t} - 54\lambda e^{-6\lambda t} - 42\lambda e^{-7\lambda t} + 144\lambda e^{-9\lambda t} + 150\lambda e^{-10\lambda t} + 144\lambda e^{-11\lambda t} - 120\lambda e^{-12\lambda t} - 312\lambda e^{-13\lambda t} - 140\lambda e^{-14\lambda t} + 60\lambda e^{-15\lambda t} + 240\lambda e^{-16\lambda t} + 272\lambda e^{-17\lambda t} - 144\lambda e^{-19\lambda t} - 180\lambda e^{-20\lambda t} - 84\lambda e^{-21\lambda t} + 66\lambda e^{-22\lambda t} + 48\lambda e^{-24\lambda t} - 26\lambda e^{-26\lambda t}\big)\big]\,dt$$

$When \quad \lambda = 1 \quad , t = 0.5$

$J = \ 0.25335$

CONCLUTION

The Line Graph model serves as an advanced analytical tool for evaluating ballistic missile reliability, representing key components like the missile body, propulsion system, fuel, and payload as vertices derived from edges in the original graph.

Results demonstrate a time-dependent reliability decline, with system reliability R_{sys} at 0.6 when individual component reliability R=0.5, Mean Time to Failure(MTTF) of 0.402, Hazard Rate of 0.9284, Shannon Entropy of 0.11165, and Extropy of 0.25335, quantifying failure uncertainty.

This approach provides practical insights for optimizing ballistic missile design under harsh conditions, offering significant applications in military engineering for enhanced performance and safety

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